Jieh Hsiang: *Refutational Theorem Proving Using Term-Rewriting Systems*, Artificial Intelligence, 25 (1985) 255–300.

From $\langle \vee, \wedge, ' \rangle$ to $\langle +, * \rangle$ and vice versa:

$x \vee y = x + y + (x * y)$ $x \wedge y = x * y$ $x' = 1 + x$	$x + y = (x' \wedge y) \vee (x \wedge y')$ $x * y = x \wedge y$

Axioms for a Boolean Ring:

$x + 0$	$=$	x
$x + (-x)$	$=$	0
$x + y$	$=$	$y + x$
$x + (y + z)$	$=$	$(x + y) + z$
$x * 1$	$=$	x
$x * x$	$=$	x
$(x + y) * z$	$=$	$(x * z) + (y * z)$
$x * (y * z)$	$=$	$(x * y) * z$
$x * y$	$=$	$y * x$
$x + x$	$=$	0

Canonical Term Rewriting System for a Boolean Algebra, modulo *associativity* and *commutativity* of $+$ and $*$:

$x \vee y$	$\Rightarrow$	$(x * y) + x + y$
$x \rightarrow y$	$\Rightarrow$	$(x * y) + x + 1$
$x \leftrightarrow y$	$\Rightarrow$	$x + y + 1$
x'	$\Rightarrow$	$1 + x$
$x + 0$	$\Rightarrow$	x
$x + x$	$\Rightarrow$	0
$x * 1$	$\Rightarrow$	x
$x * x$	$\Rightarrow$	x
$x * 0$	$\Rightarrow$	0
$x * (y + z)$	$\Rightarrow$	$(x * y) + (x * z)$

Theorem. A boolean term is valid iff its irreducible expression is 1; unsatisfiable iff its irreducible expression is 0; and satisfiable but not valid iff its irreducible expression is neither 0 nor 1.

Exercise. Let us show that: $p \vee (p \wedge q) \leftrightarrow p$. Indeed, we have the following rewriting:

$$\begin{array}{llll}
 & p \vee (p \wedge q) \leftrightarrow p & \Rightarrow (p * (p * q) + p + (p * q)) + p + 1 & \Rightarrow \\
\Rightarrow & ((p * q) + p + (p * q)) + p + 1 & \Rightarrow (0 + p) + p + 1 & \Rightarrow \\
\Rightarrow & p + p + 1 & \Rightarrow 0 + 1 & \Rightarrow \\
\Rightarrow & 1 & &
\end{array}$$