

3.2. Chop-and-Expand Context-Free Parser in Java

In this section we present the two Java programs: `List.java` and `CFParser.java`, which realize the context-free parsing using the Chop-and-Expand algorithm illustrated in Section 2.3.2 on page 31 and Section 3.1 on page 35. In those sections that algorithm is presented using a functional language.

The C++ versions of those programs can be found in [19, page 69]. A similar version of the program `List.java` can be found (with the same name) in Section 7.4 on page 190.

```

/**
 * =====
 *          Generic Class List <T>      (class used for parsing)
 * Filename: "List.java"
 *
 * Every object of the class List<T> is a list of elements of type T.
 * The following methods are available:
 *
 *   cons(T d), head(), tail(), isSingleton(), isEmpty(),
 *   append(List<T> list2), makeEmpty(), copy(), and listPrint().
 *
 * Also cloneCopy() is available if we uncomment line (***) below and
 * comment the previous line.
 *
 * The head of the list is to the right, not to the left.
 *
 * Note that tail() is destructive.
 * After a tail operation, if we need the original value of the list, we
 * should reconstruct it by 'consing' the head of the list.
 * =====
 */

import java.util.*;          // needed for using ArrayList<T> and Iterator

public class List<T> {      // it is ok if we do not use cloneCopy()

// public class List<T> implements Cloneable { // (***) In order to use
//     cloneCopy(), uncomment this line and comment the previous one.

    private ArrayList<T> list;

    public List() {          // constructor
        list = new ArrayList<T>();
    }

    public void cons(T datum) { // the head of the list is to the right,
        list.add(datum);      // not to the left.
    }                          // add(_) is a method of ArrayList<T>

    public T head() {        // the head of the list is to the right.
        if (list.isEmpty()) // It is the last element: >-----+
            {System.out.println("Error: head of empty list!");};// |
            T obj = list.get(list.size() - 1); // <-----+
        return obj;         // isEmpty(), size(), and get(int i)
    }                       // are methods of ArrayList<T>
}

```

```

public void tail() {                // destructive tail() method
    if (list.isEmpty())
        {System.out.println("Error: tail of empty list!");};
    list.remove(list.size() - 1); // isEmpty(), size(), and remove(int i)
}                                  // are methods of ArrayList<T>

public boolean isSingleton() {
    return list.size() == 1;      // size() is a method of ArrayList<T>
}

public boolean isEmpty() {
    return list.isEmpty();        // isEmpty() is a method of ArrayList<T>
}

// As usual, the head of the list 'this.append(list2)', after appending
// the 'list2' to the list 'this', is the head of the list 'this', before
// the append operation. Thus, 'this' = 'this' <> 'list2'.

public List<T> append(List<T> list2) {
    List<T> l2 = list2.copy();    // l2 is an argument to a recursive call
                                // to append. list2 should be copied!
    if (this.isEmpty()) {return l2;}
    else {T a = this.head(); this.tail();
        List<T> l = this.append(l2); l.cons(a);
        this.cons(a);            // reconstructing the list 'this'
        return l;}
}

/**
 * ----- Examples of isEmpty() -----
 * System.out.println(new List<Integer>().isEmpty());    prints true
 * System.out.println(new List().isEmpty());             prints true
 * List <Integer> v = new List();
 * System.out.println(v.isEmpty());                     prints true
 * System.out.println(v == null);                       prints false
 * -----
 */
public void makeEmpty() {
    list.clear();                                        // clear() is a method of ArrayList<T>
}

// ----- Printing a list -----
// We assume that an element of the list has a toString() method.
public void listPrint() {
    System.out.print("[ ");
    for (Iterator iter = list.iterator(); iter.hasNext(); ) {
        System.out.print((iter.next()).toString() + " ");
    };
    System.out.print("]");
}

// ----- Making a copy of a list without clone() -----
public List<T> copy() {
    List<T> copyList = new List<T>();
    for (Iterator<T> iter = list.iterator(); iter.hasNext(); ) {
        copyList.list.add(iter.next());
    };
    return copyList;
}

```

```

// ----- Making a copy of a list with clone() -----
public List<T> cloneCopy() { // see (***) above
    try { List<T> copyList = (List<T>)super.clone();
        copyList.list = (ArrayList<T>)list.clone();
        return copyList;
    } catch (CloneNotSupportedException e) {
        throw new InternalError();
    }
}
}
}
/**
 * -----
 * If we have the program fragment:
 *
 * Integer k = new Integer(6);
 * List<Integer> la = new List<Integer>();
 * la.cons(k);
 * System.out.print("\nla: "); la.listPrint();
 * la.cons(new Integer(4)); la.cons(new Integer(9));
 * // in Java 1.5: we can write: 9, instead of: new Integer(9)
 * List<Integer> lb = new List<Integer>();
 * lb = la.copy();
 * System.out.print("\nlb: "); lb.listPrint();
 * System.out.print("\nla: "); la.listPrint();
 * lb.tail(); List<Integer> lc = la.append(lb.append(lb));
 * System.out.print("\nlc: "); lc.listPrint();
 *
 * After compilation we get:
 * Note: ./List.java uses unchecked or unsafe operations.
 * Note: Recompile with -Xlint:unchecked for details.
 *
 * When running the program we get:
 * la: [ 6 ]
 * lb: [ 6 4 9 ]
 * la: [ 6 4 9 ]
 * lc: [ 6 4 6 4 6 4 9 ]
 * -----
 * Note: the head of the list is to the right.
 * -----
 */

```

Now we will present the program `CFParser.java` which, together with the above `List.java` program, implements the context-free parsing algorithm that we have presented using a functional language in Section 3.1 on page 35. In particular, the method `exists`, which is listed immediately above the main program, implements in Java the *exists* function of the Chop-and-Expand parser presented on page 38.

```

/**
 * =====
 *                CHOP-AND-EXPAND PARSER FOR CONTEXT FREE GRAMMARS
 * Filename:"CFParser.java". Thanks to R. M. Burstall and E. W. Dijkstra.
 *
 * -----
 *                --- TYPES ---
 * List<char>  = String
 * State      = <sentential_form, string>
 * List<State> = list of states
 * Rsides     = list of String : list of right hand sides
 * SyRs       = <Sy,Rs> = <char:symbol, Rsides:Rs = its right hand sides>
 * Gram       = List<SyRs> : list of SyRs pairs (type of a grammar)
 * -----
 * We use GENERIC TYPES to avoid the redefinition of the functions
 * head(), tail(), cons(), isEmpty(), makeEmpty() for:
 *
 * - list of String (type: List<String>). It is the type of the list of all
 *   right hand sides of a nonterminal. This type is also called: Rsides.
 * - list of States (type: List<State>)
 * - list of SyRs (type: List<SyRs>). It is the type of a grammar. It is
 *   the type of the list of <nonterminal, list of its right hand sides>.
 *   This type is also called: Gram.
 * Redefinition of functions is necessary for printing lists of different
 * types.
 * -----
 * The input grammar is given as a string named 'gg' of type 'Gram',
 * according to the following productions:
 *
 * character ::= 'a'..'d' | 'f'..'z' | 'A'..'Z'
 *            ('e' is reserved: see below)
 * charlist  ::= character | character charlist (that is, String)
 * Rsides    ::= 'e' | charlist | charlist '|' Rsides
 * SyRs      ::= char '>' Rsides
 * grammar   ::= syRs | syRs ';' grammar
 *
 * The empty productions, also called epsilon-rules, are allowed.
 * The character 'e' is reserved for denoting the right hand side of
 * an empty production, that is, 'e' denotes the empty charlist.
 * For instance:
 * gg="P>b|aQQ;Q>e|b|bP" stands for: P->b, P->aQQ, Q->e, Q->b, Q->bP
 * In P>b|aQQ;Q>e|b|bP we have that:
 *   \_/ aQQ is a string: "aQQ"
 *   \___/ b|aQQ is a list of elements of type String, i.e., an Rsides.
 *           It is: ["b", "aQQ"]
 *   \_____/ P>b|aQQ is a SyRs, i.e., a pair of a symbol and its
 *           right hand sides. It is: <'P', ["b", "aQQ"]>
 * The axiom of the grammar is the nonterminal of the left hand side
 * of the first production of the grammar.
 *
 * The string to parse, named 'stp', is either a non-empty sequence of
 * characters from the set {'a',..., 'd', 'f',..., 'z'} or is the empty
 * sequence denoted by 'e'.
 *
 * ----- ASSUMPTIONS ON THE INPUT -----
 * (1) When writing the input grammar, the productions relative to the same
 *     nonterminal should be grouped together. For instance, we should
 *     write "P>a|aQQ;Q>e|b|bP", instead of "P>a;Q>e;P>aQQ;Q>b;Q>bP".

```

```

* (2) The strings denoting the input grammars should be written according
*     to the rules we have stated above for the nonterminal 'grammar'.
* -----
* The grammar should not be left recursive, that is, it should not be the
* case that a nonterminal A exists such that A r ->* A s, for some strings
* r and s of terminal and/or nonterminal symbols of the grammar.
* -----
* This context-free parser works by exploring in a depth first fashion all
* possible words generated by the grammar and consistent with the
* character at hand in the string to parse 'stp'.
* -----
* There is a global variable 'tracoon'. If it is set to 'true' then we
* can see the various steps through the execution of the program, and in
* particular, the sequence of the lists of states which are generated
* during the depth first visit of search space.
* -----
* After performing a tail() operation on a list we need to reconstruct
* the list which, otherwise, is modified (see statements marked by (***)).
* =====
*/
import java.util.*;

// -----
// string with an index pointing at the next available character.
class IndexString {
    public String string;
    public int index;

    public IndexString(String string) { // constructor
        this.string = string;
        this.index = 0; // index points to the next character to be read
    }
    public void back(){
        index--;
    }
    public char getCh() {
        char x;
        if (index < string.length()) { x = string.charAt(index); }
        else x = '.';
        index++; return x; // index is incremented by one unit
    }
}
//-----
// state : <String:sentential_form, String:input_string>
class State {
    public String sentForm;
    public String inString;

    public State() {
        this.sentForm = "";
        this.inString = ""; // -----
    } // state : printing function
    // for reasons of readability the empty string is printed as "e"
    public void statePrint() {
        String sentForm1 = sentForm.equals("") ? "e" : sentForm;
        String inString1 = inString.equals("") ? "e" : inString;
        System.out.print("(" + sentForm1 + ", " + inString1 + ")");
    }
}

```

```

// -----
// List<State> : printing functions
public static void stlPrintNE(List<State> L) {
    if (!L.isEmpty())
        {State h = L.head(); h.statePrint(); L.tail();
         if (!L.isEmpty()) {System.out.print(" ");}; stlPrintNE(L);
         L.cons(h); //reconstructing the list L of states (***)
        };
}
public static void stlPrint(List<State> L) {
    if (L.isEmpty()){System.out.print(" []");}
    else {System.out.print(" ["); stlPrintNE(L); System.out.print("]");}
}
}

// -----
// SyRs: <char:symb (left hand side), List<String>:rhss (right hand sides)>
class SyRs {
    public char symb;
    public List<String> rhss;

    public SyRs() {
        this.symb = '-';
        this.rhss = null;
    }
// -----
// List<String> = R sides:printing functions
public static void rPrintNE(List<String> L) {
    if (!L.isEmpty()) {
        String h = L.head();
        if (h.equals("")) {System.out.print("e");}
        else {System.out.print(h);};
        L.tail();
        if (!L.isEmpty()) {System.out.print(" | ");};
        rPrintNE(L);
        L.cons(h); // reconstructing the list L (***)
    }
}

public static void rPrint(List<String> L) {
    if (L.isEmpty()) {System.out.print("<>");}
    else {System.out.print(" "); rPrintNE(L); System.out.print(" ");}
}

// -----
// SyRs : printing function
public static void SyRsPrint(SyRs r) {
    System.out.print(r.symb + " -> "); rPrint(r.rhss);
}
}

// -----
// Gram : List<SyRs> (it is the type of a grammar)

class Gram {
    public List<SyRs> gg;

    public Gram() {
        this.gg = new List<SyRs>();
    }
}

```

```

// -----
// List<SyRs> = Gram: printing functions
public static void printGramNE(List<SyRs> L) {
    if (!L.isEmpty())
        {SyRs h = L.head(); SyRs.SyRsPrint(h);
        System.out.print("\n"); L.tail();
        if (!L.isEmpty()) {System.out.print(" ");}; printGramNE(L);
        L.cons(h);
    }
}

public static void printGram(List<SyRs> L) {
    if (L.isEmpty()) {System.out.print("{ }");}
    else {System.out.print("{ "); printGramNE(L); System.out.print("}");}
}
}

// -----
public class CFParse {
// -----
/*
    RECURSIVE DESCENT PARSING
    *
    * for making the grammar from the string gg
    * -----
    * The function 'parseString' gets one of the alternatives (as a list of
    * characters) of the right hand sides of the productions of a nonterminal.
    */
public static String parseString(IndexString f) {
    char x = f.getCh();
    String chl;
    if (('a'<=x && x<='z')||('A'<=x && x<='Z'))
        { chl = x + parseString(f); }
    else { chl = "";
        f.back(); }
    return chl; // At the end index points to either '>' or '|' or ';' or '.'
}

// -----
public static List<String> parseRsides(IndexString f) {
    List<String> rs = new List<String>();
    String chl2 = parseString(f);
//this is the case of the epsilon production
    if (chl2.equals("e")) {chl2 = ""};
    char x = f.getCh();
    if (x=='|') { rs = parseRsides(f); rs.cons(chl2); return rs; }
    else { //Here x can be either ';' or '.'. No other characters are possible.
        f.back(); // ready to get again either ';' or '.'.
        rs.cons(chl2); return rs; }
// At the end in rs we have all the productions of a nonterminal.
}

// -----
public static SyRs parseProds(IndexString f)
throws RuntimeException {
    SyRs sRs1 = new SyRs(); sRs1.symb = f.getCh();
    char x = f.getCh();
    if (x=='>') { List<String> rs1 = new List<String>();
        rs1 = parseRsides(f); sRs1.rhss=rs1; return sRs1; }
    else { // x should have been '>'. If it is not '>' we throw an exception.
        throw new RuntimeException("Error in production!\n"); }
}
}

```

```

// -----
public static List<SyRs> parseGram(IndexString f)
                                throws RuntimeException {
    SyRs sRs1 = parseProds(f);
    char x = f.getCh();
    // Here x can be either ',' or '.'. No other characters are possible.
    if (x==',' ) { List<SyRs> gram2 = new List<SyRs>();
                  gram2 = parseGram(f); gram2.cons(sRs1);
                  return gram2; }
    else if (x=='.') { List<SyRs> gram1 = new List<SyRs>(); gram1.cons(sRs1);
                     return gram1; }
    else { throw new RuntimeException("Error parseGram!"); }
}

//-----
//          trace function

public static void trace(boolean traceon, String s, List<State> stl) {
    if (traceon)
        {System.out.print("\n" + s + ":\n"); State.stlPrint(stl);}
}
//-----
//  visiting the search space: constructing new lists of states

public static List<String> rightSides(char s, List<SyRs> gr)
                                throws RuntimeException {
    if (gr.isEmpty())
        { throw new RuntimeException ("No production for "+ s +
                                     " in the grammar.\n");}
    else {SyRs sR = new SyRs(); sR = gr.head();
          if (sR.symb == s) { return sR.rhss; }
          else { List<String> rs = new List<String>();
                gr.tail(); rs = rightSides(s,gr);
                gr.cons(sR); // reconstructing the grammar gr (***)
                return rs; }
    }
}

//-----
//          final function

public static boolean isFinal(State st) {
    return (st.sentForm.equals("")) && (st.inString.equals(""));
}
//-----
//          chop function

public static void chop(State st) { // The state st is modified. The first
                                // characters of the strings are chopped
    st.sentForm = st.sentForm.substring(1, st.sentForm.length());
    st.inString = st.inString.substring(1, st.inString.length());
}
//-----
//          expand function

public static List<State> expand(List<String> rs, String v, String w) {
    State st = new State();
    List<State> stl = new List<State>();
    if (rs.isEmpty()) { stl.makeEmpty(); return stl; }
}

```



```

else {String cs = rs.head();
    rs.tail(); stl = expand(rs,v,w);
    rs.cons(cs); // reconstructing the list rs (***)
    st.sentForm = cs.concat(v);
    st.inString = w;
    stl.cons(st); return stl; }
}
//-----
//
//          exists function
//
// This code is directly derived from the code of the exists function
// in functional style. The search space is visited in a depth first manner.
//
private static boolean exists (boolean traceon, List<State> stl1,
                               List<SyRs> gr) {
    if (stl1.isEmpty()) { return false; }
    else
    { State st1 = new State(); st1 = stl1.head();
      String st1sentForm = st1.sentForm;
      String st1inString = st1.inString;
      if (isFinal(st1)) { return true; }
      else if (!(st1sentForm.equals("")) &&
                ('A'<=st1sentForm.charAt(0) && (st1sentForm.charAt(0)<='Z')))
      { List<State> stl2 = new List<State>();
        List<String> rs = new List<String>();
        stl1.tail(); // rs : rhs's of the nonterminal symbol
        rs = rightSides(st1sentForm.charAt(0),gr);
        // delete the nonterminal symbol
        st1sentForm = st1sentForm.substring(1, st1sentForm.length());
        // EXPAND the nonterminal symbol
        stl2 = expand(rs,st1sentForm,st1inString);
        stl1 = stl2.append(stl1); // stl1 = stl2 <> stl1
        trace(traceon,"expand",stl1); // tracing ---
        return exists(traceon,stl1,gr); }
      else if (!(st1sentForm.equals("")) && !(st1inString.equals(""))
                && (((('a'<=st1sentForm.charAt(0) && (st1sentForm.charAt(0)<='d')) ||
                    (('f'<=st1sentForm.charAt(0) && (st1sentForm.charAt(0)<='z'))))
                    && (st1sentForm.charAt(0) == st1inString.charAt(0)))
                { chop(st1); stl1.tail(); stl1.cons(st1); // CHOP the terminal symbol
                  trace(traceon,"chop",stl1); // tracing ---
                  return exists(traceon,stl1,gr); }
      else { stl1.tail();
            trace(traceon,"fail",stl1); // tracing ---
            return exists(traceon,stl1,gr); }
    }
}
//-----
//
//          main program
//
public static void main(String[] args) {
/** -----
 * gg : given-grammar
 * stp : string-to-parse
 * -----
 */
String gg = "P>b|aQQ;Q>e|b|bP"; // Example 0 (with an empty production)
String stp="ab"; // true
boolean traceon = true; // variable for tracing the execution

```

```

//-----
// From the string gg to the List<SyRs> gr0.
// We first construct the IndexString f from the String gg.
// gr0 is the grammar encoded by the given string gg.
//-----
IndexString f = new IndexString(gg);
List<SyRs> gr0 = new List<SyRs>();

char axiom = gg.charAt(0);
if (('A'<=axiom) && (axiom<='Z'))
    { System.out.print("\nThe input grammar is ");
      System.out.print("\(\"e\" denotes the empty string in the rhs):\n");
      gr0 = parseGram(f);
    }
/**
 * -----
 * In order to avoid parsing, the above statement 'gr0 = parseGram(f);' can
 * be replaced by the following statements (erase the *'s, please):
 *
 * SyRs Prs = new SyRs(); Prs.symb='P'; // P
 * String chl1P = "b"; // b
 * String chl2P = "aQQ"; // |aQQ
 * List<String> rsP = new List<String>();
 * rsP.cons(chl2P); rsP.cons(chl1P); // b|aQQ
 * Prs.rhss = rsP; // P>b|aQQ
 * SyRs Qrs = new SyRs(); Qrs.symb='Q'; // Q
 * String chl1Q = ""; // e
 * String chl2Q = "b"; // |b
 * String chl3Q = "bP"; // |bP
 * List<String> rsQ = new List<String>();
 * rsQ.cons(chl3Q); rsQ.cons(chl2Q); rsQ.cons(chl1Q); // e|b|bP
 * Qrs.rhss = rsQ;
 * ----- gr0 corresponding to gg="P>b|aQQ;Q>e|b|bP"
 * gr0.cons(Qrs); gr0.cons(Prs);
 * [ <symb, rhss> <symb, rhss> ]
 * | / \ | / | \
 * gr0 is the list of pairs: [<'P', ["b", "aQQ"]>, <'Q', ["", "b", "bP"]>]
 * "" is the empty string.
 * -----
 */
Gram.printGram(gr0); }
else System.out.print("Not a grammar in input.");

System.out.print("\n\nThe axiom is " + axiom + ".\n");
State st = new State();
st.sentForm = axiom + ""; // from char to String (of length 1).
st.inString = stp;
List<State> stl = new List<State>();
stl.cons(st);
System.out.print("The initial list of states is:\n");
State.stlPrint(stl);

boolean ans = exists(traceon,stl,gr0);
System.out.print("\n\nThe following string of characters:\n ");
System.out.print(stp + "\nis ");
if (!ans) {System.out.print("NOT ");};
System.out.print("generated by the given grammar.\n");
} // end of main
}

```

```

/**
 * When printing the statelist, the head is to the left.
 * input:  output:
 * -----
 * javac CFParse.java
 * java  CFParse
 *
 *      The input grammar is ("e" denotes the empty string in the rhs):
 *      { P -> b | aQQ
 *        Q -> e | b | bP
 *      }
 *
 *      The axiom is P.
 *      The initial statelist is:
 *      [(P, ab)]
 *      expand:
 *      [(b, ab) (aQQ, ab)]
 *      fail:
 *      [(aQQ, ab)]
 *      chop:
 *      [(QQ, b)]
 *      expand:
 *      [(Q, b) (bQ, b) (bPQ, b)]
 *      expand:
 *      [(e, b) (b, b) (bP, b) (bQ, b) (bPQ, b)]
 *      fail:
 *      [(b, b) (bP, b) (bQ, b) (bPQ, b)]
 *      chop:
 *      [(e, e) (bP, b) (bQ, b) (bPQ, b)]
 *
 *      The following string of characters:
 *      ab
 *      is generated by the given grammar.
 * -----
 *
 *                               Various Examples
 *
 * String gg="P>b|aQQ;Q>e|b|bP";      // Example 0 (empty production in gg)
 * String stp="a";                      // true
 * String stp="abaa";                   // false
 * String stp="aba";                    // true
 * String gg="P>a|aQ;Q>b|bP";           // Example 1 -----
 * String stp="ab";                     // true
 * String stp="ababa";                  // true
 * String stp="aaba";                   // false
 * String gg="P>a|bQ;Q>b|aQ";           // Example 2 -----
 * String stp="baab";                   // true
 * String stp="bbaaba";                 // false
 * String gg="S>a|aAS;A>SbA|ba|SS";     // Example 2.7 of Hopcroft-Ullman ---
 * String stp="aabaaa";                 // true
 * String stp="aabba";                  // false
 * String stp="aabbaa";                 // true
 * String gg="S>aB|bA;A>bAA|a|aS;B>aBB|b|bS"; // same numbers of a's and b's
 * String stp="bbabaaaab";              // true
 * String stp="babaaaab";               // false
 * -----
 */

```

3.3. Chop-and-Expand Context-Free Parser in a Logic Language

Here is a logic program which realizes a Chop-and-Expand Parser.

Chop-and-Expand Parser

1. $parse(G, [], []) \leftarrow$
2. $parse(G, [A|X], [A|Y]) \leftarrow terminal(A), parse(G, X, Y) \quad // \text{ CHOP}$
3. $parse(G, [A|X], Y) \leftarrow nonterminal(A), member(A \rightarrow B, G), \quad // \text{ EXPAND}$
 $\quad\quad\quad append(B, X, Z), parse(G, Z, Y)$
4. $member(A, [A|X]) \leftarrow$
5. $member(A, [B|X]) \leftarrow member(A, X)$
6. $append([], L, L) \leftarrow$
7. $append([A|X], Y, [A|Z]) \leftarrow append(X, Y, Z)$

Together with these seven clauses we also need the clauses which define the terminal symbols and the nonterminal symbols. Recall that in logic programming $[A|X]$ represents the list whose head is the element A and whose tail is the list X . In particular, if $[A|X] = [2, 3, 1, 1]$ then $A = 2$ and $X = [3, 1, 1]$. All constants in Prolog should begin with a lower case letter, so that, in particular, the axiom S is represented by the character s .

The first argument of $parse$ is a context-free grammar, the second argument is a list of terminal symbols and nonterminal symbols (that is, a sentential form), and the third argument is a word represented as a list of terminal symbols. We assume that context-free grammars are represented as lists of productions of the form $x \rightarrow y$, where x is a nonterminal symbol and y is a list of terminal and nonterminal symbols.

We have that $parse(G, [s], W)$ holds iff from the symbol s we can derive the word W using the grammar G , that is, W is a word of the language generated by G .

EXAMPLE 3.3.1. Let us consider the context-free grammar $G = \langle \{a, b\}, \{S\}, \{S \rightarrow aSb, S \rightarrow ab\}, S \rangle$. It is represented by the clauses:

```
terminal(a) ←
terminal(b) ←
nonterminal(s) ←
```

together with the list $[s \rightarrow [a, s, b], s \rightarrow [a, b]]$ which represents its productions. The left hand side of the first production is assumed to be the start symbol s . For this grammar G the query $\leftarrow parse([s \rightarrow [a, s, b], s \rightarrow [a, b]], [s], [a, a, b, b])$ evaluates to true, and this means that the word $aabb$ belongs to the language generated by G .

Note that in the clause $member(A, [B|X]) \leftarrow member(A, X)$, we do not require that A is different from B . Indeed with this clause, the query of the form $\leftarrow member(A, l)$, where A is a variable and l is a ground list, generates by backtracking all members of the list l with their multiplicity. \square

More examples of Prolog programs for parsing words generated by grammars can be found in Section 10.1 on page 249.