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## Automi, Linguaggi e Traduttori. 10 July 2009.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document your program of the PARSER exercise. Motivate your constructions and answers.

• PARSER [5]. Write a C++ or Java program whose time complexity is at most  $O(n)$ , for testing whether or not a given word in  $\{a, b\}^*$  of length  $n$  ( $> 0$ ) is generated by the grammar with axiom  $S$  and the following productions:

$$S \rightarrow AS \mid a \quad A \rightarrow SA \mid b$$

• REASONING [2]. Show that for every word  $w$  in  $(a + b)^*$  with an even number of  $a$ 's we have that  $A \rightarrow^+ w$  where  $A$  is the axiom of a grammar with the following productions:

$$A \rightarrow AA \mid aAa \mid Ab \mid \varepsilon$$

Hint: Show by complete induction on the number of  $b$ 's in the word  $w$  that we can generate from  $A$  every word  $w$  in the set  $\{w \mid w \in (a + b)^* \text{ and } w \text{ has an even number of } a\text{'s}\}$ .

• CONTEXT-FREE LANGUAGES [3]. Construct a pushdown automaton which recognizes *by final state* the language generated by the grammar with axiom  $S$  and the following productions:

$$S \rightarrow S S \mid a S b \mid \varepsilon$$

Show that no deterministic pushdown automaton exists that can recognize that language *by empty stack*.

• LALR [3]. Construct an  $LALR(1)$  parser, if any, for the context-free grammar  $G$  with axiom  $S$  and the following productions (do not modify the given grammar before constructing the  $LALR(1)$  parser):  $S \rightarrow a A \mid b B$

$$A \rightarrow \varepsilon \mid c A d \quad B \rightarrow \varepsilon$$

• LANGUAGES [3]. Prove that the language  $\{a, b\}^* - \{a^i b^i \mid i \geq 0\}$  is a context-free language and it is *not* regular.

• DECIDABILITY [3]. Show that the intersection of two semidecidable languages subsets of  $\{a, b\}^*$ , is a semidecidable language.

• CORRECTNESS [3]. Let  $x^n$  be defined as follows:

$$\text{if } n = 0 \text{ then } 1 \text{ else if } \text{odd}(n) \text{ then } x \times (x^{n-1}) \text{ else } (x^{n/2}) \times (x^{n/2}).$$

Show the total correctness of the following program ( $I$  is the invariant assertion of the loop):

```
{n ≥ 0}
k := n; y := 1; z := x;
I ≡ {y × zk = xn ∧ k ≥ 0}
while k ≠ 0 do if odd(k) then begin k := k - 1; y := y × z; end;           k := k / 2; z := z × z;
od
{y = xn}
```

**Nota.** Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 4+4 punti. Per la prova orale, si presenti: (i) il programma, fatto da solo/a, dell'esercizio PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi fatti da solo/a e (iii) si venga, se possibile, con un computer portatile ove siano installati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.

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## Automi, Linguaggi e Traduttori. 21 July 2009.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document your program of the PARSER exercise. Motivate your constructions and answers.

• PARSER [5]. Write a C++ or Java program whose time complexity is at most  $O(n)$ , for testing whether or not a given word in  $\{a, b\}^*$  of length  $n$  ( $> 0$ ) is generated by the grammar with axiom  $S$  and the following productions:

$$S \rightarrow AS \mid a \quad A \rightarrow SA \mid b$$

Hint (suggerimento): Find the regular expression which denotes the regular language generated by the given context-free grammar.

• REASONING [2]. Show that for every word  $w$  in  $(a + b)^*$  with an even number of  $a$ 's, we have that  $A \rightarrow^+ w$  where  $A$  is the axiom of a grammar with the following productions:

$$A \rightarrow AA \mid aAa \mid Ab \mid \varepsilon$$

Note: You should *not* show the obvious fact that every word generated by  $A$  has an even number of  $a$ 's. Instead, you should show that every word in  $(a + b)^*$  with an even number of  $a$ 's can be generated by  $A$ .

Hint: Show by complete induction on the number of  $b$ 's in the word  $w$  that we can generate from  $A$  every word  $w$  in the set  $\{w \mid w \in (a + b)^* \text{ and } w \text{ has an even number of } a\text{'s}\}$ .

• CONTEXT-FREE LANGUAGES [3]. Construct a pushdown automaton which recognizes *by final state* the language generated by the grammar with axiom  $S$  and the following productions:

$$S \rightarrow S S \mid a S b \mid \varepsilon$$

Show that no deterministic pushdown automaton exists that can recognize that language *by empty stack*.

• LALR [3]. Construct an  $LALR(1)$  parser, if any, for the context-free grammar  $G$  with axiom  $S$  and the following productions (do not modify the given grammar before constructing the  $LALR(1)$  parser):  $S \rightarrow AA$

$$A \rightarrow a A \quad A \rightarrow a$$

- LANGUAGES [3]. Prove that: (1) the language  $\{a, b\}^* - \{a^i b^i \mid i \geq 0\}$  is a context-free language and (2) it is *not* regular.
- DECIDABILITY [3]. Show that given two semidecidable languages  $L1 \subseteq \{a, b\}^*$  and  $L2 \subseteq \{a, b\}^*$ , the language  $(L1 \cup L2) \cap \{a^i b^i \mid i \geq 0\}$  is a semidecidable language.
- CORRECTNESS [3].

Show the total correctness of the following program ( $I$  is the invariant assertion of the loop):

```

{ n ≥ 0 }
k := n; y := 1; z := x;
I ≡ { y × zk = xn ∧ k ≥ 0 }
while k ≠ 0 do if odd(k) then begin k := k-1; y := y × z; end;          k := k/2; z := z × z;
od
{ y = xn }

```

**Nota.** Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 4+4 punti. Per la prova orale, si presenti: (i) il programma, fatto da solo/a, dell'esercizio PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi fatti da solo/a e (iii) si venga, se possibile, con un computer portatile ove siano installati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.

### Automati, Linguaggi e Traduttori. 15 February 2010.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document the program of the PARSER exercise. Motivate your constructions and answers.

- PARSER [5]. Write a C++ or Java program whose time complexity is at most  $O(n)$ , for testing whether or not a given word in  $\{a, b\}^*$  of length  $n$  ( $> 0$ ) is generated by the grammar with axiom  $S$  and the following productions:

$$S \rightarrow b \mid AA \quad A \rightarrow aA \mid Abb \mid \varepsilon$$

and find the regular expression which denotes the regular language generated by the given context-free grammar.

- REASONING [4]. Show that for every word  $w$  in  $(a + b)^*$  with an even number of  $a$ 's, we have that  $A \rightarrow^+ w$  where  $A$  is the axiom of a grammar with the following productions:

$$A \rightarrow AA \mid aAa \mid Ab \mid \varepsilon$$

- REASONING [4]. Show that the following two context-free grammars  $G1$  and  $G2$  generate the same language:

$G1$ :

$$S \rightarrow \varepsilon \mid aB \mid bA \quad B \rightarrow bS \mid aBB \quad A \rightarrow aS \mid bAA$$

$G2$ :

$$S \rightarrow \varepsilon \mid aSb \mid bSa \mid SS$$

Show also that all the words generated by the grammar  $G1$  have even length.

- CONTEXT-FREE LANGUAGES [3]. Construct a pushdown automaton that recognizes *by final state* the language  $L$  generated by the grammar with axiom  $S$  and the following productions:

$$S \rightarrow SS \mid aSb \mid \varepsilon$$

- LALR [3]. Construct an  $LALR(1)$  parser, if any, for the context-free grammar  $G$  with axiom  $S$  and the following productions (do not modify the given grammar before constructing the parsing table of the  $LR(1)$  parser):

$$S \rightarrow AS \quad S \rightarrow \varepsilon \quad A \rightarrow aA \quad A \rightarrow b$$

- DECIDABILITY [3]. Show that given two semidecidable languages  $L1, L2 \subseteq \{a, b\}^*$ , the language  $(L1 \cap L2) \cup \{a^h b^k c^k \mid h, k \geq 0\}$  is a semidecidable language.

- CORRECTNESS [4]. Let  $N$  be the set of natural numbers.

Let  $half \in N \rightarrow N$  be the function defined by the following equations:

$$half(0) = 0 \quad half(1) = 0 \quad half(n+2) = half(n) + 1$$

Let  $f \in N \rightarrow N$  be the function defined by the following equations:

$$f(0) = 0 \quad f(1) = 1 \quad f(n+2) = 2 \times f(n)$$

Show that the following program:

```

{ K ≥ 0 }
k := K;
if even(k) then res := 0 else res := 1;
while k > 1 do res := 2 × res; k := k-2 od
{ res = f(K) }

```

is partially correct w.r.t. the initial assertion  $\{K \geq 0\}$  and the final assertion  $\{res = f(K)\}$  and it terminates. The invariant of the loop is:  $\{k \geq 0 \wedge 2^{half(k)} \times res = f(K)\}$

**Nota.** Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 4+4 punti. Per la prova orale, si presenti il programma, fatto da solo/a, dell'esercizio PARSER con alcune prove di esecuzione.

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### Automi, Linguaggi e Traduttori. 16 February 2010.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document the program of the PARSER exercise. Motivate your constructions and answers.

- PARSER [5]. Write a C++ or Java program whose time complexity is at most  $O(n)$ , for testing whether or not a given word in  $\{a, b\}^*$  of length  $n$  ( $> 0$ ) is generated by the grammar with axiom  $S$  and the following productions:

$$S \rightarrow b \mid AS \mid A \quad A \rightarrow abA \mid Ab \mid \varepsilon$$

- REASONING [3]. Prove that there is no Turing Machine  $A$  such that: (i) for every natural number  $n$ , the Turing Machine  $A$  given in input  $n$ , returns in output the code of a Turing Machine which terminates for all inputs, and (ii) for every Turing Machine  $M$  which terminates for all inputs, there exists a natural number  $m$  such that the Turing Machine  $A$  given in input  $m$ , returns in output the code of the Turing Machine  $M$ .

- CONTEXT-FREE LANGUAGES [2]. Show that the following grammar is not an  $LL(1)$  grammar:

$$S \rightarrow aSb \mid aSc \mid d$$

- LR [2]. Construct an  $LR(1)$  parser, if there exists one, for the context-free grammar  $G$  with axiom  $S$  and the following productions:  $S \rightarrow aSb \mid \varepsilon$

- LANGUAGES [3]. Prove that the intersection of the context-free language  $\{a^i b^i \mid i \geq 0\}$  and any regular language subset of  $(a+b)^*$  is a context-free language.

- DECIDABILITY [4]. Let  $L$  be a language recognized by a Turing machine which terminates for all inputs. Show that the set  $(\Sigma^* - \{a^i b^i c^i \mid i \geq 1\}) \cup L$  is a recursive set.

- CORRECTNESS [5]. Let  $N!$  denote the factorial of the natural number  $N$ . Show that the following program:

```
{N ≥ 0}
n := N; res := 1;
while n > 0 do res := res × n; n := n - 1 od
{res = N!}
```

is totally correct (i.e., it is partially correct and it terminates).

**Nota.** Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 3+3 punti. Per la prova orale, si presenti: (i) il programma, fatto da solo/a, dell'esercizio PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi fatti da solo/a e (iii) si venga, se possibile, con un computer portatile ove siano installati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.

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### Automi, Linguaggi e Traduttori. 24 February 2010.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document the program of the PARSER exercise. Motivate your constructions and answers.

- PARSER [5]. Write a C++ or Java program whose time complexity is at most  $O(n)$ , for testing whether or not a given word in  $\{a, b\}^*$  of length  $n$  ( $> 0$ ) is generated by the grammar with axiom  $S$  and the following productions:

$$S \rightarrow ASA \mid A \quad A \rightarrow aAa \mid Ab \mid \varepsilon$$

- REASONING [3]. Show that the language generated by the grammar with axiom  $S$  and the following productions:

$$S \rightarrow b \mid AS \mid A \quad A \rightarrow abA \mid Ab \mid \varepsilon$$

is equal to the language generated by the grammar with axiom  $A$  and the following productions:

$$A \rightarrow abA \mid Ab \mid bA \mid \varepsilon$$

- CONTEXT-FREE LANGUAGES [2]. Write an ambiguous grammar which generates the following language:  $\{a^n b^m \mid 0 \leq m < n\}$ . Write a non-ambiguous grammar for that same language.

- LR [2]. Construct an  $LR(1)$  parser, if any, for the context-free grammar  $G$  with axiom  $E$  and the following productions:

$$E \rightarrow E + T \mid T \quad T \rightarrow T \times a \mid a$$

- LANGUAGES [4]. Prove that  $a(ba)^* = (ab)^*a$  and  $(a^*b)^*a \neq a(ba^*)^*$ .

- DECIDABILITY [3]. (D1) Write in pseudocode an algorithm, if there exists one, that: (i) terminates for all inputs, and (ii) given a context-free grammar  $G$ , decides whether or not  $G$  is ambiguous.
- (D2) Write in pseudocode an algorithm, if there exists one, that: (i) terminates for all inputs, and (ii) given a context-free grammar  $G$ , decides whether or not  $G$  generates a deterministic context-free language.
- (D3) Write in pseudocode an algorithm, if there exists one, that: (i) terminates for all inputs, and (ii) given a context-free grammar  $G$ , decides whether or not  $G$  is a regular grammar.

- CORRECTNESS [5]. Let  $N!$  denote the factorial of the natural number  $N$ . Show that the following program:

```

{N ≥ 0}
if N = 0 then res := 1 else
begin
n := 1; res := 1;
while n ≤ N do res := res × n; n := n + 1; od
end
{res = N!}

```

is totally correct (i.e., it is partially correct and it terminates).

**Nota.** Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 3+3 punti. Per la prova orale, si presenti: (i) il programma, fatto da solo/a, dell'esercizio PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi fatti da solo/a e (iii) si venga, se possibile, con un computer portatile ove siano installati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.

### Automi, Linguaggi e Traduttori. 13 July 2010.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document the program of the PARSER exercise. Motivate your constructions and answers.

- PARSER [5]. Write a C++ or Java program whose time complexity is at most  $O(n)$ , for testing whether or not a given word in  $\{a, b\}^*$  of length  $n$  ( $\geq 0$ ) is generated by the grammar with axiom  $S$  and the following productions:

$$S \rightarrow a B S B a \mid \varepsilon \quad B \rightarrow b B b \mid \varepsilon$$

- REASONING [2]. Consider the language  $L_1$  generated by the grammar with axiom  $S_1$  and the following productions:

$$S_1 \rightarrow a B S_1 B a \mid \varepsilon \quad B \rightarrow b B b \mid \varepsilon$$

Consider the language  $L_2$  generated by the grammar with axiom  $S_2$  and the following productions:

$$S_2 \rightarrow a B S_2 B a \mid a B B a \quad B \rightarrow b B b \mid \varepsilon$$

Show that  $L_1 = L_2 \cup \{\varepsilon\}$ .

- CONTEXT-FREE LANGUAGES [2]. Construct a pushdown automaton which recognizes *by final state* the language generated by the grammar with axiom  $S$  and the following productions:

$$S \rightarrow S S S \mid a S b \mid \varepsilon$$

- GREIBACH NORMAL FORM [3]. Give a grammar in Greibach normal form which generates the language

$$\{0^i 1^k a 2^i \mid i \geq 1 \text{ and } k \geq 1\} \cup \{0^i 1^k b 2^k \mid i \geq 1 \text{ and } k \geq 1\}$$

Show that this language is a deterministic context-free language.

- LR [3]. Construct an  $LR(1)$  parser, if any, for the context-free grammar  $G$  with axiom  $S$  and the following productions (do not modify the given grammar before constructing the parsing table of the  $LR(1)$  parser):

$$S \rightarrow A S \quad S \rightarrow \varepsilon \quad A \rightarrow a A \quad A \rightarrow b$$

- LANGUAGES [3]. Prove that the intersection of the context-free language  $\{a^i b^i \mid i \geq 0\}$  and any regular language subset of  $(a + b + c)^*$  is a context-free language.

- DECIDABILITY [3]. Show that if the language  $L \subseteq \{a, b\}^*$  is semidecidable and  $\{a, b\}^* - L$  is semidecidable, then  $L \cup \{a^h b^k \mid h, k \geq 0\}$  is a decidable language.

- CORRECTNESS [3]. Show that for any  $a, b \in N$ , for any function  $c \in N \rightarrow N$ , for any function  $f \in N \rightarrow N$  defined by the following equations:

$$f(0) = a \quad f(1) = b \quad f(n+2) = c(f(n))$$

we have that the following program:

```

{K ≥ 0}
k := K;
if even(k) then res := a else res := b;
while k > 1 do res := c(res); k := k - 2 od

```

$$\{res = f(K)\}$$

is partially correct w.r.t. the given input and output assertions and it terminates for all  $K \geq 0$ .

*Hint.* Find the invariant assertion of the loop.

**Nota.** Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 3+3 punti. Per la prova orale, si presenti: (i) il programma, fatto da solo/a, dell'esercizio PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi fatti da solo/a e (iii) si venga, se possibile, con un computer portatile ove siano installati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.

### Automati, Linguaggi e Traduttori. 8 September 2010.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document your program of the PARSER exercise. Motivate your constructions and answers.

• PARSER [5]. Write a C++ or Java program whose time complexity is at most  $O(n)$ , for testing whether or not a word in  $\{a, b\}^*$  of length  $n$  ( $\geq 0$ ) is generated by the grammar with axiom  $S$  and the following productions:

$$S \rightarrow a \mid S A A \mid \varepsilon \quad A \rightarrow a A \mid A a b \mid A b a \mid \varepsilon$$

• REGULAR LANGUAGE [3]. Show that for all  $k \geq 1$ , every context-free grammar  $\langle \{a, b\}, V_N, P, S \rangle$  such that every sentential form it generates has at most length  $k^3$ , generates a regular language.

• REGULAR EXPRESSIONS [3]. Show that the regular expressions  $a(ba)^*$  and  $(ab)^*a$  denote the same subsets of  $\{a, b\}^*$ .

• CONTEXT-FREE [3]. Show that the language  $\{a, b\}^* - \{a^i b^i \mid i \geq 0\}$  is context-free.

• GREIBACH [2]. Compute the Greibach normal form of the following grammar:

$$S \rightarrow S A \mid a \quad A \rightarrow S \mid A S \mid a \mid \varepsilon$$

• LALR [3]. Construct an *LALR*(1) parser, if any, for the context-free grammar  $G$  with axiom  $S$  and the following productions:

$$S \rightarrow B B a \quad B \rightarrow B b \mid c$$

• DECIDABILITY [2]. Show that the set of all pairs  $\langle \text{Java program } P, n \rangle$  such that  $P$  does not halt for the input  $n$  is not semidecidable.

• CORRECTNESS [3]. Show the total correctness of the following program which takes in input the two integer values  $X$  and  $Y$  and computes their product.

```

{X > 0 ∧ Y > 0}
x := X; y := Y; p := 0;
while x > 1 do if even(x) then begin x := x/2; y := 2y; end;
                else begin x := x - 1; p := p + y; end;
od
p := p + y;
{p = XY}

```

**Nota.** Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 3+3 punti. Per la prova orale, si presenti: (i) il programma, fatto da solo/a, dell'Esercizio del PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi fatto da solo/a e (iii) si venga, se possibile, con un computer portatile ove siano installati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.

### Automati, Linguaggi e Traduttori. 5 July 2011.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document your program of the PARSER exercise. Motivate your constructions and answers.

• PARSER [6]. Write a C++ or Java program whose time complexity is at most  $O(n)$ , for testing whether or not a word in  $\{a, b\}^*$  of length  $n$  ( $\geq 0$ ) is generated by the grammar with axiom  $S$  and the following productions:

$$S \rightarrow A S A \mid a \quad A \rightarrow a A a \mid A b \mid \varepsilon$$

• REASONING [2]. Show that for every word  $w$  in  $(a + b)^*$  with an even number of  $a$ 's, we have that  $A \rightarrow^+ w$  where  $A$  is the axiom of a grammar with the following productions:

$$A \rightarrow A A \mid a A A a \mid A b \mid \varepsilon$$

*Hint:* Show by complete induction on the number of  $b$ 's in the word  $w$  that  $A$  can generate every word  $w$  in the set  $\{w \mid w \in (a + b)^* \text{ and } w \text{ has an even number of } a\text{'s}\}$ .

• LANGUAGES [2]. Prove that the context-free language  $\{a^i b^i \mid i \geq 0\}$  is a deterministic context-free language.

• GREIBACH [2]. Compute the Greibach normal form of the following grammar:

$$S \rightarrow A S \mid a \quad A \rightarrow a A \mid A a b \mid \varepsilon$$

• LALR [3]. Construct an *LALR*(1) parser, if any, for the context-free grammar  $G$  with axiom  $S$  and the following productions:  $S \rightarrow B B \quad B \rightarrow b B \mid a$

- DECIDABILITY [3]. Show that it is undecidable whether or not the intersection of a regular language with a context-sensitive language is the empty language. (Recall that the emptiness problem of a context-sensitive language is undecidable.)

- CORRECTNESS [4]. Prove the total correctness of the following program:

```

{N ≥ 0}
n := N;
if even(n) then res := a else res := b;
I ≡ {n ≥ 0 ∧ chalf(n)(res) = h(N)}
while n > 1 do begin res := c(res); n := n - 2 end
{res = h(N)}

```

where  $h(n)$  for all  $n \geq 0$  is defined as follows:

$$h(0) = a \quad h(1) = b \quad h(n+2) = c(h(n))$$

$I$  is the invariant. The function  $half$  is defined as follows:

$$half(0) = 0 \quad half(1) = 0 \quad half(n+2) = half(n) + 1$$

**Nota.** Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 3+3 punti. Per la prova orale, si presenti: (i) il programma, *fatto da solo/a*, dell'Esercizio del PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi *fatto da solo/a* e (iii) si venga, se possibile, con un computer portatile ove siano installati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.

### Automati, Linguaggi e Traduttori. 8 September 2011.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document your program of the PARSER exercise. Motivate your constructions and answers.

- PARSER [6]. Write a C++ or Java program whose time complexity is at most  $O(n)$ , for testing whether or not a word in  $\{a, b\}^*$  of length  $n (\geq 0)$  is generated by the grammar with axiom  $S$  and the following productions:

$$S \rightarrow SA \mid a \quad A \rightarrow aAa \mid bA \mid \varepsilon$$

- REASONING [2]. Show that for every word  $w$  in  $(a+b)^*$  with an even number of  $a$ 's, we have that  $A \rightarrow^+ w$  where  $A$  is the axiom of a grammar with the following productions:

$$A \rightarrow AA \mid aAa \mid bA \mid \varepsilon$$

- LANGUAGES [3]. (i) Prove that the context-free language  $\{a, b, c\}^* - \{a^i b^i \mid i \geq 0\}$  is a deterministic context-free language.

(ii) Find a language  $L \subseteq \{a, b\}^*$  which is not regular such that  $L^*$  is regular.

- GREIBACH [2]. Compute the Greibach normal form of the following grammar:

$$S \rightarrow AS \mid b \mid \varepsilon \quad A \rightarrow SA \mid b$$

- LALR [3]. Construct an  $LALR(1)$  parser, if any, for the context-free grammar  $G$  with axiom  $S$  and the following productions:  $S \rightarrow AAb \quad A \rightarrow Ab \mid a$

- DECIDABILITY [3]. Show that from the undecidability of the problem of establishing whether or not the intersection of two context-free languages is a context-free language does not follow the undecidability of the problem of establishing whether or not the intersection of a deterministic context-free language and a context-free language is a context-free language.

- CORRECTNESS [3]. Prove the total correctness of the following program:

```

{N ≥ 0}
n := N; res := a;
while n > 0 do begin res := c(res); n := n - 1 end
{res = h(N)}

```

where  $h(n)$  for all  $n \geq 0$  is defined as follows:

$$h(0) = a \quad h(n+1) = c(h(n))$$

**Nota.** Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 3+3 punti. Per la prova orale, si presenti: (i) il programma, *fatto da solo/a*, dell'Esercizio del PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi *fatto da solo/a* e (iii) si venga, se possibile, con un computer portatile ove siano installati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.