Automi, Linguaggi e Traduttori. 13 February 2012.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document your program of the PARSER exercise. Motivate your constructions and answers.

• PARSER [5]. Write a C++ or Java program whose time complexity is at most $O(n)$, for testing whether or not a word in $\{a,b\}^*$ of length n (>0) is generated by the grammar with axiom S and the following productions:

$S \to S\,A \mid a \quad A \to a\,A\,a \mid b\,A \mid \varepsilon$

• REASONING [4]. Show that for every word w in $(a+b)^*$ with an even number of a's, we have that $A \to^+ w$ where A is the axiom of a grammar with the following productions:

$A \to A\,A \;\mid\; a\,A\,a \;\mid\; a\,a\,A \;\mid\; b\,A \;\mid\; \varepsilon$

• LANGUAGES [3]. (i) Prove that the context-free language $\{a,b,c\}^* - \{a^i b^i \mid i \geq 0\}$ can be recognized by final state by a deterministic pushdown automaton.
(ii) Find a language $L \subseteq \{a,b\}^*$ which is not regular such that L^* is regular.

• CHOMSKY [2]. Compute the Chomsky normal form of the following grammar:

$T \to T \times F \;\mid\; F \qquad\qquad F \to (T) \;\mid\; b$

• LALR [3]. Construct an $LALR(1)$ parser, if any, for the context-free grammar G with axiom S and the following productions: $S \to A\ A\ b \mid A\ B\ b \quad A \to A\ b \mid a \quad B \to A\ b \mid a$

• DECIDABILITY [3]. (i) Give an example of a problem P which is undecidable and its negation is undecidable. (ii) Give an example of a problem P which is semidecidable and its negation is undecidable. (iii) Prove that if problem is decidable, then its negation is decidable.

Prove all your assertions. Recall that, given the problem: "Is n an even number?", its negation is: "Is n not an even number?".

• CORRECTNESS [4]. Let x^n be defined as follows: **if** $n=0$ **then** 1 **else if** $odd(n)$ **then** $x\,(x^{n-1})$ **else** $(x^{n/2})^2$. Show the total correctness of the following program:

```
        {n ≥ 0}
k := n;  y := 1;  z := x;
while k ≠ 0 do  if odd(k) then begin k := k−1;  y := y z;  end;
                k := k div 2;  z := z z;
          od
        {y = x^n}
```

Nota. Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 3+3 punti. Per la prova orale, si presenti: (i) il programma, *fatto da solo/a*, dell'Esercizio del PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi *fatto da solo/a* e (iii) si venga, se possibile, con un computer portatile ove siano istallati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.

Automi, Linguaggi e Traduttori. 10 July 2012.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document your program of the PARSER exercise. Motivate your constructions and answers.

• PARSER [6]. Write a C++ or Java program whose time complexity is at most $O(n)$, for testing whether or not a word in $\{a,b\}^*$ of length n (≥ 0) is generated by the grammar with axiom S and the following productions:

$S \to A\,S\,A \;\mid\; A \quad A \to a\,A\,a \;\mid\; a\,A\,a\,b \;\mid\; b\,A \;\mid\; \varepsilon$

• REASONING [3]. Show that the following two grammars G_1 and G_2 with axiom A are equivalent:

$G_1: \quad A \to \varepsilon \;\mid\; A\,B \quad B \to 0\,C \quad C \to 1 \;\mid\; 0\,C\,C$

$G_2: \quad A \to \varepsilon \;\mid\; B\,A \quad B \to 0\,C \;\mid\; 0\,0\,D \quad C \to 1 \;\mid\; 0\,C\,C \quad D \to C\,C$

• LANGUAGES [2]. Show that $a^* \cup \{a^m\, b^n \;\mid\; m > n \geq 0\}^*$ is not a regular language and it satisfies the pumping lemma for regular languages.

• GREIBACH [2]. Compute a grammar in Greibach normal form of the following grammar:

$S \to A\,S \;\mid\; a \qquad\qquad A \to S \;\mid\; A\,S \;\mid\; a \;\mid\; \varepsilon$

• LALR [2]. Construct an $LALR(1)$ parser, if any, for the context-free grammar G with axiom S and the following productions: $S \to A\ S \;\mid\; \varepsilon \quad A \to a\ S\ b$

• DECIDABILITY [3]. Define two sets P and Q of words (that is, two problems) such that: (i) P is undecidable and not semidecidable, (ii) Q is undecidable and semidecidable, and (iii) $Q \subset P$.

• HOARE TRIPLES and CORRECTNESS [6]. (i) Determine the set of all boolean expressions B and programs P which satisfy the triple $\{\mathit{false}\}$ if B then P $\{\mathit{true}\}$.
(ii) Determine the set of all programs P and Q which satisfy the triple $\{\mathit{true}\}$ $P\,;\,Q$ $\{\mathit{false}\}$.
(iii) Let $N!$ denote the factorial of the natural number N. Show that the following program:

$\{N \geq 0\}$

$n := N;\ res := 1;$

while $n > 1$ **do** $res := (n^2 - n) \times res;\ \ n := n - 2$ **od**

$\{res = N!\}$

is totally correct (i.e., it is partially correct and it terminates).

Nota. Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 3+3 punti. Per la prova orale, si presenti: (i) il programma, *fatto da solo/a*, dell'Esercizio del PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi *fatto da solo/a* e (iii) si venga, se possibile, con un computer portatile ove siano istallati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.

Automi, Linguaggi e Traduttori. 11 September 2012.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document your program of the PARSER exercise. Motivate your constructions and answers.

• PARSER [6]. Write a C++ or Java program whose time complexity is at most $O(n)$, for testing whether or not a word in $\{a, b\}^*$ of length n (≥ 0) is generated by the grammar with axiom S and the following productions:

$S \rightarrow A\,S \ \mid\ A \quad A \rightarrow a\,A\,a\,S \ \mid\ b\,A \ \mid\ \varepsilon$

• REASONING [3]. Show that the context-sensitive languages are recursive sets.

• LANGUAGES [2]. Show that $\{a^m b^n \mid m \neq n \geq 0\}$ is not a regular language and it satisfies the pumping lemma for regular languages.

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A different version.
Show that $a^* \cup \{a^m b^n \mid m \neq n \geq 0\}$ is not a regular language and it satisfies the pumping lemma for regular languages.

• GREIBACH [3]. Give a grammar in Greibach normal form which generates the following language:

$\{0^i\, 1^k\, a\, 2^i \ \mid\ i, k > 0\} \ \cup\ \{0^i\, 1^k\, b\, 2^k \ \mid\ i, k > 0\}$

Show that every deterministic pda which recognizes this language has to make ε-moves.

• LALR [1]. Construct an $LALR(1)$ parser, if any, for the context-free grammar G with axiom S and the following productions: $S \rightarrow a\ A \ \mid\ b\ A \rightarrow \varepsilon \ \mid\ a\ A\ b \ \mid\ a\ a$

• DECIDABILITY [3]. (1) Define two sets $P1$ and $P2$ of words (that is, two problems) such that: (i) $P1$ is undecidable and not semidecidable, (ii) $P2$ is undecidable and not semidecidable, and (iii) $P1 \subset P2$.
(2) Consider two sets $P1$ and $P2$ of words (that is, two problems) such that: (i) $P1$ is undecidable and not semidecidable, and(ii) $P2$ is undecidable and not semidecidable. State a condition on $P1$ and $P2$ so that $P1 \cup P2$ is decidable.

• HOARE TRIPLES and CORRECTNESS [6]. (i) Determine the set of all programs P, Q, and R which satisfy the triple $\{true\}\ P\,;$ **if** b **then** Q **else** R $\{false\}$.
(ii) Let us consider the function h from natural numbers to natural numbers:
$h(0) = a$
$h(1) = b$
$h(n+2) = c(h(n))$, for any $n \geq 0$,
where a and b are natural numbers, and c is a function from natural numbers to natural numbers.

Show that the following program:

$\{K \geq 0\}$

if $even(K)$ **then** $res := a$ **else** $res := b$;

if $even(K)$ **then** $n := 0$ **else** $n := 1$;

while $n < K$ **do** $res := c(res)$; $n := n+2$ **od**

$\{res = h(K)\}$

is totally correct (i.e., it is partially correct and it terminates).
Nota. Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 3+3 punti. Per la prova orale, si presenti: (i) il programma, *fatto da solo/a*, dell'Esercizio del PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi *fatto da solo/a* e (iii) si venga, se possibile, con un computer portatile ove siano istallati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.

Automi, Linguaggi e Traduttori. 12 February 2013.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document your program of the PARSER exercise. Motivate your constructions and answers.

• PARSER [5]. Write a C++ or Java program whose time complexity is at most $O(n)$, for testing whether or not a given word in $\{a,b\}^*$ of length n (≥ 0) is generated by the grammar with axiom S and the following productions:

$S \to a\,B\,S\,B\,B\,a \mid \varepsilon \qquad B \to b\,B\,b \mid \varepsilon$

• REASONING [3]. Prove that there is no Turing Machine A such that: (i) for every natural number n, the Turing Machine A given in input n, returns in output the code of a Turing Machine which terminates for all inputs, and (ii) for every Turing Machine M which terminates for all inputs, there exists a natural number m such that the Turing Machine A given in input m, returns in output the code of the Turing Machine M.

• LANGUAGES [2]. Show that $a^* \cup \{a^m b^n \mid m \neq n \wedge m \geq 0 \wedge n \geq 0\}^*$ is a regular language and it satisfies the pumping lemma for regular languages.
Solution. We have that $\{a,b\}^* \subseteq \{a^m b^n \mid m \neq n \wedge m \geq 0 \wedge n \geq 0\}^*$, because $a = a^1 b^0$ and $b = a^0 b^1$.

• GREIBACH [2]. Compute a grammar in Greibach normal form of the language generated by the following grammar: $S \to A\,S \mid a \qquad A \to S \mid A\,S \mid a \mid \varepsilon$

• LR [3]. Construct an $LR(1)$ parser, if any, for the context-free grammar G with axiom S and the following productions (do not modify the given grammar before constructing the parsing table of the $LR(1)$ parser):

$S \to a\,A \mid b\,B \qquad A \to \varepsilon \mid c\,A\,c \qquad B \to \varepsilon$

• DECIDABILITY [3]. Let Σ be the set $\{a,b\}$. Let $\mathcal{P}(\Sigma^*)$ denote the set of all subsets of Σ^*. Give a function f from $\mathcal{P}(\Sigma^*)$ to $\mathcal{P}(\Sigma^*)$ such that for all subsets P of Σ^*, if P is not semidecidable and not decidable, then $f(P)$ is a decidable set and $P \subseteq f(P)$. Show that *any* such function f is not an injection.

• HOARE TRIPLES and CORRECTNESS [6]. (i) Determine the set of all boolean expressions B, all C-like programs P and Q which satisfy the triple $\{true\}\ P;\ (\textbf{if}\ B\ \textbf{then}\ P;)\ Q;\ \{false\}$ (Assume that the evaluation of B returns either *true* or *false* without modifying the store).
(ii) Same as (i) with the precondition *false*, instead of *true*.
(iii) Let $N!$ denote the factorial of the natural number N. Show that the following program:

$\{N \geq 0\}$
$n := N;$
$\textbf{if}\ N = 0 \vee odd(N)\ \textbf{then}\ res := 1\ \textbf{else}\ res := 2;$
$\textbf{while}\ n > 2\ \textbf{do}\ res := (n^2 - n) \times res;\ \ n := n - 2\ \ \textbf{od}$
$\{res = N!\}$

is totally correct (i.e., it is partially correct and it terminates).

Nota. Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 3+3 punti. Per la prova orale, si presenti: (i) il programma, *fatto da solo/a*, dell'Esercizio del PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi *fatto da solo/a* e (iii) si venga, se possibile, con un computer portatile ove siano istallati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.

Automi, Linguaggi e Traduttori. 04 July 2013.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document your program of the PARSER exercise. Motivate your constructions and answers.

• PARSER [5]. Write a C++ or Java program whose time complexity is at most $O(n)$, for testing whether or not a given word in $\{a,b\}^*$ of length n (≥ 0) is generated by the grammar with axiom S and the following productions:

$S \to B\,a\,B\,S\,B\,a \mid \varepsilon \qquad B \to b\,B\,b \mid \varepsilon$

• REASONING [4]. (i) Show that the following two grammars G_1 and G_2 with axiom A are equivalent:

$G_1: \quad A \to \varepsilon \mid a\,A\ G_2: \quad A \to \varepsilon \mid a \mid A\,A\,A$

(ii) Prove that there is no Turing Machine A such that: (i) for every natural number n, the Turing Machine A given in input n, returns in output the code of a Turing Machine which terminates for all inputs, and (ii) for every Turing Machine M which terminates for all inputs, there exists a natural number m such that the Turing Machine A given in input m, returns in output the code of the Turing Machine M.

• LANGUAGES [2]. Show that $a^* \cup \{a^m b^n \mid m \neq n \geq 0\}$ is not a regular language and it satisfies the pumping lemma for regular languages.

• CONTEXT-SENSITIVE [2]. Give a context-sensitive grammar, if any, which generates the language
$L = \{a^n b^n c^n \mid n \geq 1\}$ and show that L is not context-free.

• GREIBACH [2]. Compute a grammar in Greibach normal form, if any, of the language
$\{a^i\, b^j\, c^k \mid i \geq 1 \ \wedge\ j + k = 2\,i\}$.

• LR [2]. Construct an $LALR(1)$ parser, if any, for the context-free grammar G with axiom S and the following productions (do not modify the given grammar before constructing the parsing table of the $LALR(1)$ parser):
$S \to B\,B\,a \qquad\qquad B \to c \mid B\,b$

• DECIDABILITY [3]. Let Σ be the set $\{a, b\}$. Let $\mathcal{P}(\Sigma^*)$ denote the set of all subsets of Σ^*. Give a function f from $\mathcal{P}(\Sigma^*)$ to $\mathcal{P}(\Sigma^*)$ such that for all subsets P of Σ^*, if P is not semidecidable and not decidable, then $f(P)$ is a decidable set and $P \supseteq f(P)$. Show that that function f cannot be a bijection.

• HOARE TRIPLES and CORRECTNESS [6]. (i) Determine the set of all boolean expressions B, all C-like programs P and Q which satisfy the triple $\{true\}$ (**if** B **then** P;) Q; (**if** B **then** P;) $\{false\}$ (Assume that the evaluation of B returns either *true* or *false* without modifying the store).
(ii) Same as (i), with the precondition *false*, instead of *true*.
(iii) Same as (i), with the postcondition *true*, instead of *false*.
(iv) Let $N!$ denote the factorial of the natural number N. Show that the following program:

$\{N \geq 1\}$
$n := N;$
if $odd(N)$ **then** $res := 1$ **else** $res := 0;$
while $n > 2$ **do** $res := (n^2 - n) \times res;\ \ n := n-2$ **od**
$\{res = \mathit{if}\ odd(N)\ \mathit{then}\ N!\ \mathit{else}\ 0\}$

is totally correct (i.e., it is partially correct and it terminates) w.r.t. the precondition $N \geq 1$ and the postcondition $res = \mathit{if}\ odd(N)\ \mathit{then}\ N!\ \mathit{else}\ 0$.

Nota. Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 3+3 punti. Per la prova orale, si presenti: (i) il programma, *fatto da solo/a*, dell'Esercizio del PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi *fatto da solo/a* e (iii) si venga, se possibile, con un computer portatile ove siano istallati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.

Automi, Linguaggi e Traduttori. 10 September 2013.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document your program of the PARSER exercise. Motivate your constructions and answers.

• PARSER [5]. Write a C++ or Java program whose time complexity is at most $O(n)$, for testing whether or not a given word in $\{a, b\}^*$ of length n (≥ 0) is generated by the grammar with axiom A and the following productions:
$A \to a\,A\,B\,a \mid \varepsilon \qquad\qquad B \to b\,B\,b \mid b\,B \mid \varepsilon$

• REASONING [4]. (i) Prove by rule induction that the grammar with axiom A and productions:
$A \to 0\,A\,1 \mid \varepsilon \mid A\,A$
is equivalent to the grammar with axiom S and productions:
$S \to \varepsilon \mid 0\,B\,S \ B \to 1 \mid 0\,B\,B$
(ii) Write in pseudocode an algorithm, if there exists one, that given a context-free grammar G, decides whether or not G is a regular grammar.

• LANGUAGES [2]. Prove that for all languages A and B that are regular subsets of $(a+b+c)^*$, we have that $(\{a^i b^i \mid i \geq 0\} \cup A) \cap B$ is a context-free language.

• GREIBACH [2]. Compute a context-free grammar in Greibach normal form that generates the language
$\{a^i\, b^j\, c^k \ \mid\ i+j = 2\,k \ \wedge\ k \geq 1\}$.

• LR [2]. Construct an $LALR(1)$ parser, if any, for the context-free grammar G with axiom S and the following productions (do not modify the given grammar before constructing the parsing table of the $LALR(1)$ parser):
$E \to E{+}T \ \mid\ T \qquad\qquad T \to a \ \mid\ (E)$

• DECIDABILITY [3]. Show that it is undecidable whether or not the intersection of a context-free language with a context-sensitive language is the empty language. (Recall that the emptiness problem of a context-sensitive language is undecidable.)

• HOARE TRIPLES and CORRECTNESS [6]. (i) Determine the set of all boolean expressions B, all C-like programs P and Q which satisfy the triple $\{true\}$ **if** B **then** P; **if** B **then** Q $\{x \geq 0\}$ (Assume that the evaluation of B returns either *true* or *false* and it does not modify the store, but the evaluation of P may modify the store).
(ii) Same as (i), with the precondition *false*, instead of *true*.
(iii) Same as (i), with the postcondition *true*, instead of $x \geq 0$.
(iv) Let a be the array $a[0], \ldots, a[N-1]$ with N elements. Show that the following program:

$i := 0;$
while $i < N$ **do** $a[i] := 0;\ \ i := i+1$ **od**

is totally correct (that is, it is partially correct and it terminates) with respect to the precondition $N \geq 1$ and the postcondition $\forall j.\ \mathit{if}\ 0 \leq j < N\ \mathit{then}\ a[j] = 0$. Hint: find a suitable invariant for the while-do loop.

Nota. Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 3+3 punti. Per la prova orale, si presenti: (i) il programma, *fatto da solo/a*, dell'Esercizio del PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi *fatto da solo/a* e (iii) si venga, se possibile, con un computer portatile ove siano istallati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.

Automi, Linguaggi e Traduttori. 04 February 2014.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document your program of the PARSER exercise. Motivate your constructions and answers.

• **PARSER** [5]. Write a C++ or Java program whose time complexity is at most $O(n)$, for testing whether or not a given word in $\{a,b\}^*$ of length n (≥ 0) is generated by the grammar with axiom S and the following productions:

$S \;\to\; A\,S \;\mid\; A \qquad\qquad A \;\to\; a\,S\,A\,a\,S \;\mid\; b\,A \;\mid\; \varepsilon$

• **REASONING** [4]. Let N be the set of the natural numbers. Show by Mathematical Induction the correctness of the following deduction rule:

$$\frac{p(0,0) \qquad \forall x\!\in\! N.\; p(x,0) \to p(s(x),0) \qquad \forall x\!\in\! N. \forall y\!\in\! N.\; p(x,y) \to p(x,s(y))}{\forall x\!\in\! N. \forall y\!\in\! N. p(x,y)}$$

• **LANGUAGES** [2]. Let us consider the context-free grammar with axiom A and the following productions:

$A \;\to\; \varepsilon \;\mid\; A\,A \;\mid\; 0\,A\,1$

Let us also consider the context-free grammar with axiom B and the following productions:

$B \;\to\; 1 \;\mid\; 0\,B\,B$

Prove by rule induction that: $L(A) \cdot L(B) \subseteq L(B)$.

• **LR(1)** [2]. Construct a context-free grammar such that: (i) it is not $LR(1)$, and (ii) it is equivalent to the following grammar with axiom S which generates the language of balanced parentheses (a is the open parenthesis and b is the closed parenthesis):

$S \to A\,S \;\mid\; \varepsilon \qquad\qquad A \to a\,S\,b$

Show that the grammar you propose is not $LR(1)$ by constructing its LR parsing table.

• **GREIBACH** [1]. Compute the Greibach normal form of a grammar *equivalent* to the one with axiom S and the following productions:

$S \to S\,A \mid B \mid a \qquad\qquad A \to A\;a\;a \mid a \mid \varepsilon \qquad\qquad B \to B\,b\,B\,b \mid \varepsilon$

• **DECIDABILITY** [4]. (i) Prove that for all *finite* sets S of context-free grammars, there exists a Turing Machine M such that for all grammars G_1, G_2 in S, we have that $M(G_1, G_2)$ terminates and answers "yes" (by entering a final state) iff $L(G_1) = L(G_2)$.
(As usual $M(G_1, G_2)$ denotes the result computed by the Turing Machine M when acting of some encoding of the input grammars G_1 and G_2.)
(ii) Show that the membership problem for type 0 grammars is semidecidable and not decidable.

• **HOARE TRIPLES** [6]. (i) Determine the set of all boolean expressions B and all C-like programs P which satisfy the triple $\{x\!\geq\!0\}$ **if** B **then** P; **if** B **then** P $\{x\!\geq\!0\}$ Assume that the evaluation of B always terminates and returns either *true* or *false* and it does not modify the store. The evaluation of P may modify the store.
(ii) Same as (i), with the precondition and postcondition *true*, instead of $x\!\geq\!0$.
(iii) Let a be the array $a[0], \ldots, a[N-1]$ with N elements. Show that the following program:

$\{N\!\geq\!1\}$
$i := 0;$
while $i\!<\!N$ **do** $\quad a[i] := 0; \quad i := i\!+\!2$ **od**
$\{\forall j.\; if\;\; even(j) \wedge 0\!\leq\! j\!<\!N\;\; then\; a[j] = 0\}$

is totally correct (that is, it is partially correct with respect to the given assertions and it terminates).

Nota. Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 3+3 punti. Per la prova orale, si presenti: (i) il programma, *fatto da solo/a*, dell'Esercizio del PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi *fatto da solo/a* e (iii) si venga, se possibile, con un computer portatile ove siano istallati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.

Automi e Linguaggi. 10 April 2014.

Precision and clarity are important. The PARSER exercise is compulsory. Use comments to document your program of the PARSER exercise. Motivate your constructions and answers.

• **PARSER** [4].Write a C++ or Java program whose time complexity is at most $O(n)$, for testing whether or not a given word in $\{a,b,c\}^*$ of length n (≥ 0) is generated by the grammar with axiom S and the following productions:

$S \rightarrow A\,S \mid A \qquad\qquad A \rightarrow a\,A\,a \mid b\,S\,c \mid \varepsilon$

• **REASONING** [4]. Let N be the set of the natural numbers. Give a deduction rule based on Mathematical Induction that allows us to conclude that $\forall x \in N.\, \forall y \in N.\, \forall z \in N.\, p(x,y,z)$ for any predicate p of three arguments. Prove the correctness of the rule.

• **LANGUAGES** [2]. (i) Let us consider the following grammar with axiom A and productions:

$A \rightarrow a\,b\,A \mid a$

Let us also consider the following grammar with axiom B and productions:

$B \rightarrow B\,b\,a \mid a$

Prove by rule induction that: $L(A) = L(B)$.

• **PDA's** [2]. Prove that for nondeterministic pda's we have that acceptance by final state is equivalent to acceptance by empty stack.

• **LR(1)** [2]. (i) Construct an $LR(1)$ parser, if any, for the context-free grammar G with axiom S' and the following productions (do not modify the productions for S, A, and B before constructing the parsing table of the $LR(1)$ parser):

$S \rightarrow a\,A \mid b\,B \qquad\qquad A \rightarrow \varepsilon \mid a\,A\,c \qquad\qquad B \rightarrow \varepsilon$

(ii) Prove that the property of *not* being an $LR(1)$ grammar is *not* preserved by unfolding one of its productions. Hint. From $S \rightarrow A \mid a,\;\; A \rightarrow a$ by unfolding A we get: $S \rightarrow a$.

• **DECIDABILITY** [4]. (i) Prove that for all *finite* sets $\mathcal{S}_{fin}$ of Turing Machines it is decidable whether or not given any two Turing Machines M_1 and M_2 in $\mathcal{S}_{fin}$ they recognize the same language, that is, $L(M_1) = L(M_2)$. (ii) Let us consider a fixed character c. Prove that the problem of deciding whether or not a Turing Machine prints c is semidecidable and not decidable.

• **HOARE TRIPLES** [6]. Let a be the array $a[0], \ldots, a[N-1]$ with N (≥ 1) elements. Show that the following program:

$\{N \geq 1 \text{ and } odd(N)\}$

$i := 0;$
$M := (N-1)/2;$
while $i \leq M$ **do** $\quad a[M-i] := 0; \quad a[M+i] := 0; \quad i := i+1$ **od**;

$\{\forall j.\ \textit{if}\ 0 \leq j < N\ \textit{then}\ a[j] = 0\}$

is totally correct (that is, it is partially correct with respect to the given assertions and it terminates).

Nota. Tra parentesi quadre sono indicati i punti per ogni esercizio. Le due prove in itinere valgono, di norma, 3+3 punti. Per la prova orale, si presenti: (i) il programma, *fatto da solo/a*, dell'Esercizio del PARSER con alcune prove di esecuzione, (ii) un elaborato con la soluzione degli altri esercizi *fatto da solo/a* e (iii) si venga, se possibile, con un computer portatile ove siano istallati e pronti per l'esecuzione i programmi relativi alle due prove in itinere.