Informatica Teorica. 24 February 2009.

1. Prove that the set $[A \to B]$ of the continuous functions from a cpo A to a cpo B is itself a cpo using the partial order which is defined as follows: for all functions $f, g \in [A \to B]$, we state that $f \sqsubseteq g$ iff for all $a \in A$, $f(a) \sqsubseteq g(a)$.

2. Give the operational and the denotational semantics of the recursive language REC with call-by-name semantics. In the language REC we have function definitions of the form:

$$f_1(x_1, \ldots, x_n) = t_1$$
$$\vdots$$
$$f_k(x_1, \ldots, x_n) = t_k$$

and we want to evaluate terms of the form:

$t ::= n \mid x \mid t_1 \textbf{ op } t_2 \mid \textbf{if } t_0 \textbf{ then } t_1 \textbf{ else } t_2 \mid f_i(t_1, \ldots, t_n)$ where $\textbf{op} \in \{+, -, \times\}$.

3. Give the rules for the construction of the least Herbrand Model of a definite logic program.

4. Compute the minimal and the maximal fixpoints of the equation $X = aX + b$, where the unknown X is a CCS term.

5. Prove that the set $\{A \in Assn \mid \models A\}$ is not r.e.

6. Let N denote the set of natural numbers. Write an iterative program using assignments, ';', *if-then-else*, and *while-do* (but not recursive calls) which, given in input the value n (≥ 0), stores in the variable z the value of $h(n)$, where the function $h : N \to N$ is recursively defined as follows:

$h(0) = a \qquad h(1) = b \qquad h(n+2) = c(h(n))$

where $a, b \in N$ and $c : N \to N$. Prove, by using Hoare's triples, the partial correctness of that iterative program w.r.t. the precondition $n \geq 0$ and the postcondition $z = h(n)$.

7. Find *all* (not *some*) formulas $P(x, y)$, which may depend on x and y, such that the Hoare's triple

$\{y > 1\}$ $x := 0$; while $y > x \wedge P(x, y)$ do $x := y - 1$ $\{x = 0 \wedge y > 1\}$ holds.

8. Show that the eager semantics of *let* $x \Leftarrow e$ *in* t is equal to the eager semantics of $(\lambda x.t)e$.
Assume that:

$$[\![(t_1 t_2)]\!]\rho = let\ \varphi \Leftarrow [\![t_1]\!]\rho,\ v \Leftarrow [\![t_2]\!]\rho.\ \varphi(v)$$
$$[\![\lambda x.t]\!]\rho = \lfloor \lambda v.[\![t]\!]\rho[v/x] \rfloor$$
$$[\![let\ x \Leftarrow e\ in\ t]\!]\rho = let\ v \Leftarrow [\![e]\!]\rho.[\![t]\!]\rho[v/x]$$

9. Recall that a set $P \subseteq D_\perp$ is said to be *inclusive* iff for all ω-chains $d_0 \sqsubseteq d_1 \sqsubseteq \ldots$ in $D_\perp$ we have that *if* for all $i \geq 0$, $d_i \in P$ *then* $(\bigsqcup_{i \geq 0} d_i) \in P$.

Show that, for given any continuous function $f : D \to E$, we have that if $Q \subseteq E$, is an inclusive set then so is $f^{-1}(Q) \subseteq D$.

The result of this test is combined with that of the take-home exam.