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A CONSTRAINT-BASED TRANSFORMATION FOR VERIFYING INFINITE STATE SYSTEMS

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Abstract

We address the problem of verifying safety properties of infinite state reactive systems that use unbounded integer variables. We consider systems specified by using linear constraints over the integers and we assume that, for verifying safety properties of these systems, one uses reachability analysis techniques. Our method improves the effectiveness of forward and backward reachability analyses by preprocessing the system specification. For forward reachability our method consists in: (i) transforming the system specification into an equivalent one (with respect to the safety property of interest) by a constraint propagation technique that works backward from the constraints representing the unsafe states, and then (ii) applying to the transformed system specification a reachability analysis that works forward from the constraints representing the initial states. For backward reachability our method works as for forward reachability, by interchanging the roles of the initial states and the unsafe states. We have implemented our method by using the MAP program transformation tool. Our implementation works as a preprocessor for infinite state systems specified in FASTer, a powerful tool for verifying safety properties of infinite state systems with integer variables. Through various experiments performed on several infinite state systems, we have shown that our constraint-based transformation of the system specifications considerably increases the number of successful verifications without a significant degradation of the time performance.

Key words: Program Transformation, Software Verification, Infinite State Systems.

1. Introduction

After the development of very effective techniques for model checking of reactive systems specified by finite state automata [8], the verification community is shifting its interest to classes of more complex system specifications, where one is allowed to model reactive systems with unbounded data structures, and thus, with a possibly infinite set of reachable states.

Counter systems are among the classes of system specifications that have been recently studied: they augment a finite state control structure with unbounded integer variables (see, for instance, [2, 5, 6, 16, 22]). These systems use Presburger formulas to symbolically represent (possibly infinite) sets of states and transitions. For reasons of simplicity, in this paper we consider counter systems defined by linear constraints (that is, equalities and inequalities) over the integers, instead of full Presburger formulas.

It is straightforward to encode as counter systems many classical devices, such as Petri nets (with reset, inhibitor, and transfer arcs) and counter machines (with increment, decrement, and test-if-zero). As a consequence of well-known results in the theory of computation, many properties, like reachability, are undecidable for counter systems. Thus, in order to verify *safety* properties of counter systems, that is, properties that specify that some given states are not reachable from the initial states, a naive computation of the set of reachable states will not terminate in most cases and, for this reason, several alternative approaches have been proposed.

Several works have identified restricted classes of systems for which the reachability problem is solvable. These classes include various types of Petri nets (see [11] for some results) or, equivalently, *vector addition systems* [22]. These decidability results are useful when the system to be analysed falls within a specific class (at least in theory, as a tight complexity analysis is still required). However, they do not provide a general method and no action can be performed when the system is outside those decidable classes.

Another approach consists in providing terminating methods for computing a symbolic *over-approximation* of the set of states that are reachable from the initial states (see, for instance, [1, 7, 17]). Then a safety property is verified if this over-approximated set has an empty intersection with the set of unsafe states. Clearly, methods following this approach are inconclusive in the case where the over-approximated reachability set has a non-empty intersection with the set of unsafe states.

More sophisticated methods for the exact (symbolic) computation of the reachability set of counter systems have been proposed [2, 6, 5, 16]. In particular, these methods use *acceleration* techniques to improve the convergence of the computation of the reachability set by pre-computing new transitions that condense in one step the effect of the transitive closure of a sequence of transitions.

In this paper we focus on methods which allow the *exact* computation of the reachability set and we propose a transformation technique for improving the convergence of the symbolic computation of that set. The effect of our technique adds on the improvements due to acceleration techniques.

Our starting point is the observation that in some cases during a forward reachability analysis, the computation of the set *Reach* of states that are reachable from the initial states diverges because this set is not representable as a finite set of constraints. However, in order to verify a safety property stating that no element in a set *Unsafe* of unsafe states is reachable, we are actually interested in computing $Reach \cap Unsafe$ and checking whether or not this set is empty. In some cases the computation of this set may terminate even if the computation of *Reach* diverges (consider, for instance, the extreme case where the set *Unsafe* is empty).

The main objective of the technique presented in this paper is to exploit the information about the set *Unsafe* of the unsafe states during the computation of *Reach*. Our technique does not modify the algorithm for computing the reachability set. On the contrary, it transforms the system specification so that the reachability computation for the new specification incorporates the information about *Unsafe*. By interchanging the roles of the initial states and the unsafe states, our technique can also be applied when performing a backward reachability analysis and it allows us to exploit the information about the set *Init* of the initial states.

The contributions of this paper are the following ones.
(1) We have defined a transformation algorithm that, given a system specification *Spec* and a set *Unsafe* of unsafe states, computes a new specification *Spec'* by propagating the constraints representing *Unsafe*. The propagation process proceeds backwards from *Unsafe*, that is, in opposite direction with respect to the one for computing the set of reachable states from the initial states. Propagation is realized by means of a *generalized* PRE operator, called GENPRE, which computes constraints entailed by the states from which a given set of states is reachable in one transition step.
(2) We have proved that, under some assumptions on the GENPRE operator, the transformation algorithm terminates and produces a new specification *Spec'* which is equivalent to the given *Spec* with respect to *Unsafe*. That is, *Spec* is safe if and only if *Spec'* is safe. This preservation of equivalence is a notable difference with respect to the techniques that use approximations, such as the ones cited above [1, 7, 17].
(3) We have provided several definitions of the GENPRE operator that combine (variants of) operators and relations introduced in the field of program analysis and transformation, such as the *widening* and *convex hull* operators and the *well-quasi order* relations on constraints. We have proved that these definitions guarantee termination and correctness of the transformation algorithm.
(4) Finally, we have implemented our transformation algorithm on the MAP transformation system [23], an experimental transformation tool written in *constraint logic programming* (CLP), and we have performed experiments on several infinite state systems by using the FASTer verification tool for counter systems [4]. These experiments show that our transformation determines an increase of the number of successful verifications without a significant degradation of the time performance (actually, in some cases the verification time is highly improved).

Our paper is structured as follows. In Section 2 we present a language based on linear constraints over the integers for specifying counter systems and their safety properties. In Sections 3 and 4 we present our transformation algorithm for counter systems and we prove its termination and correctness. In Section 5 we present some experimental results using FASTer. Finally, in Section 6 we discuss related work in the field of program transformation and analysis.

2. Specifying Counter Systems

In order to specify counter systems and their safety properties we use a simplified version of the languages considered in [3, 4, 21, 25]. Our language allows us to specify systems and properties by using linear constraints over the set $\mathbb{Z}$ of the integers.

A *system specification* is a 4-tuple $\langle \mathit{Var}, \mathit{Init}, \mathit{Trans}, \mathit{Unsafe} \rangle$ defined as follows.
(i) *Var* is a *variable declaration* which is a sequence of declarations of (distinct) variables, each of which may be either: (i.1) an *enumerated* variable, or (i.2) an *integer* variable. (i.1) An enumerated variable x is declared by the statement: **enumerated** x D, meaning that x ranges over a finite set D of constants. The set D is said to be the *type* of x and it is also said to be the

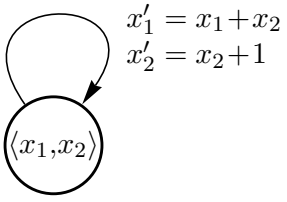
type of every constant in D. (i.2) An integer variable x is declared by the statement: **integer** x, meaning that x is a variable ranging over the set $\mathbb{Z}$ of the integers.
(ii) *Init* denotes the set of *initial states*. It is a set of constraints of the form $\{init_1(X), \ldots, init_K(X)\}$.
(iii) *Trans* denotes the *transition relation* between states. It is a set of constraints of the form $\{t_1(X, X'), \ldots, t_M(X, X')\}$.
(iv) *Unsafe* denotes the set of the *unsafe states*. It is a set of constraints of the form $\{u_1(X), \ldots, u_N(X)\}$.

Each set of constraints of Points (ii)–(iv) has to be interpreted as a *disjunction* of constraints. By X we denote the tuple $\langle x_1, \ldots, x_k, x_{k+1}, \ldots, x_n\rangle$ of variables declared in *Var*, where: (i) for $i = 1, \ldots, k$, x_i is an enumerated variable of type D_i, for some finite set D_i of constants, and (ii) for $i = k+1, \ldots, n$, x_i is an integer variable. By X' we denote the tuple $\langle x'_1, \ldots, x'_k, x'_{k+1}, \ldots, x'_n\rangle$ of the primed variables.

Constraints are defined as follows. If e_1 and e_2 are enumerated variables or constants of the same type, then $e_1 = e_2$ and $e_1 \neq e_2$ are *atomic constraints*. If p_1 and p_2 are linear polynomials with integer coefficients, then $p_1 = p_2$, $p_1 \geq p_2$, and $p_1 > p_2$ are *atomic constraints*. A *constraint* is either *true*, or *false*, or an atomic constraint, or a *conjunction* of constraints. By $c(X)$ we denote a constraint on the tuple X of variables. By $\mathcal{C}[X]$ we denote the set of all constraints on X.

Example 1. In Figure 1 we show a reactive system and its specification.

(1.1)



(1.2)

Var: **integer** x_1; **integer** x_2;
Init: $\{x_1 \geq 0 \ \wedge \ x_2 \geq 0 \ \wedge \ x_2 \leq 1\}$;
Trans: $\{x'_1 = x_1 + x_2 \ \wedge \ x'_2 = x_2 + 1\}$;
Unsafe: $\{(x_1 < 0 \wedge x_2 \leq 1), (x_2 \leq -1 \wedge x_2 \geq -2)\}$

Figure 1: A reactive system (1.1) and its specification *Spec* (1.2). A set of constraints is interpreted as a disjunction of constraints.

Now we define the notion of reachability associated with a system specification.

A *state* is an n-tuple $\langle r_1, \ldots, r_k, z_{k+1}, \ldots, z_n\rangle$ of constants in $D_1 \times \ldots \times D_k \times \mathbb{Z}^{n-k}$. By Σ we denote the set of all states. A state s will be called an *initial* (resp., *unsafe*) state if there exists $init_i(X) \in Init$ (resp., $u_i(X) \in Unsafe$) such that $init_i(s)$ (resp. $u_i(s)$) holds. A constraint $c(X)$ is *less general* than a constraint $d(X)$, or $d(X)$ is *more general* than $c(X)$, denoted $c(X) \sqsubseteq d(X)$, if for all $s \in \Sigma$, if $c(s)$ holds then also $d(s)$ holds. $c(X)$ is said to be *equivalent* to $d(X)$, denoted $c(X) \equiv d(X)$, if $c(X) \sqsubseteq d(X)$ and $d(X) \sqsubseteq c(X)$.

A *computation sequence* is a sequence of states $s_0, \ldots, s_m$, with $m \geq 0$, such that, for $i = 0, \ldots, m-1$, $t(s_i, s_{i+1})$ holds, for some $t(X, X') \in Trans$. State s_m is *reachable* from state s_0 if there exists a computation sequence $s_0, \ldots, s_m$. A system specification is *safe* if there is no unsafe state which is reachable from an initial state. Two system specifications $Spec_1$ and $Spec_2$ are said to be *equivalent* if $Spec_1$ is safe iff $Spec_2$ is safe.

In order to compute the possibly infinite set of states which are (*forward*) reachable from an initial state, available verification tools make use of *symbolic* reachability algorithms which represent set of states as Presburger formulas, or equivalent automata-theoretic notions [3, 4,

Input: A specification *Spec*.
Output: A new specification *Spec′* equivalent to *Spec*.
Phase 1: Construction of the tree *CTree* of constraints.
Let GENPRE(Λ) be the set $\{g_1(X), \ldots, g_R(X)\}$.
$CTree := \{\Lambda \xleftarrow{\{\}} g_1(X), \ldots, \Lambda \xleftarrow{\{\}} g_R(X)\}$.
while there exists a non-recurrent constraint $c(X)$ labelling a leaf of *CTree do*
let GENPRE($c(X)$) be the set $\{g_1(X), \ldots, g_S(X)\}$;
for $j = 1, \ldots, S$, let T_j be $\{t_i(X, X') \in Trans \mid \pi(i) = j\}$;
$CTree := CTree \cup \{c(X) \xleftarrow{T_1} g_1(X), \ldots, c(X) \xleftarrow{T_S} g_S(X)\}$;
od;
Phase 2: Extraction of the new specification *Spec′* from the tree *CTree* of constraints.
SPECEXTRACT(*Spec*, *CTree*, *Spec′*)

Figure 2: The Specification Transformer algorithm.

21, 25]. Similar symbolic techniques are used for computing the set of states from which an unsafe state is (*backward*) reachable. In this paper we will not need to take into consideration any specific algorithm for computing reachability sets, as our technique transforms a system specification into an equivalent one, independently of any such algorithm.

3. An Algorithm for Transforming Specifications

In this section we present our method for improving the reachability analysis of counter systems by applying constraint-based transformations of system specifications. Here we consider the forward reachability, but our method can also be applied to the case of backward reachability by interchanging the roles of initial states and unsafe states.

Our method is based on an algorithm that transforms a specification $Spec = \langle Var, Init, Trans, Unsafe\rangle$ into an equivalent specification $Spec' = \langle Var', Init', Trans', Unsafe'\rangle$ for which the reachability analysis is hopefully more effective. The objective of our transformation is to derive, from the transition relation *Trans* and the set *Unsafe* of the unsafe states, a new transition relation *Trans′* such that by using the relation *Trans′*, instead of *Trans*, the unsafe states are implicitly taken into consideration when constructing the set of the reachable states.

Our Specification Transformer algorithm (see Figure 2) consists of two phases:
(Phase 1) the construction of a tree of constraints, called *CTree*, by using the procedure GENPRE from the given specification *Spec*, and
(Phase 2) the extraction of a new specification *Spec′* from the tree *CTree* by using the procedure SPECEXTRACT.

In the tree *CTree*, (i) the root node is labelled by a distinguished symbol Λ (which stands for the constraint *false*), (ii) each non-root node is labelled by a constraint in $\mathcal{C}[X]$, and (iii) each arc is labelled by a set of transitions. When no confusion arises, we will identify a node with the constraint which labels it. An arc from a node (or constraint) $c_1(X)$ to a node (or constraint) $c_2(X)$ labelled by a set $T \subseteq Trans$ of transitions is denoted by $c_1(X) \xleftarrow{T} c_2(X)$. For simplicity, *CTree* will be represented as the set of its labelled arcs.

In the tree *CTree* the following hold: (i) the disjunction of all constraints in the non-root nodes is an over-approximation of the set of states from which an unsafe state is reachable, and

(ii) for every arc $c_1(X) \xleftarrow{T} c_2(X)$, we have that $c_2(X)$ is an over-approximation of the set of states from which a state X satisfying $c_1(X)$ can be reached in one step by using one of the transitions in T.

In order to construct *CTree* we will use the following operator.

Definition 3.1. *A* generalized PRE operator*, called* GENPRE*, is a function which, for any* $a \in \mathcal{C}[X] \cup \{\Lambda\}$*,*

- *if* $a = \Lambda$*, then it returns a set* $\{g_1(X), \ldots, g_R(X)\}$ *of constraints in* $\mathcal{C}[X]$ *and a surjective mapping* $\lambda\colon \{1, \ldots, N\} \longrightarrow \{1, \ldots, R\}$*, with* $R \leq N$ *(where* N *is the cardinality of Unsafe), such that, for* $i = 1, \ldots, N$*,* $u_i(X) \sqsubseteq g_{\lambda(i)}(X)$*;*
- *else if* $a = c(X)$ *for some* $c(X) \in \mathcal{C}[X]$*, then it returns a set* $\{g_1(X), \ldots, g_S(X)\}$ *of constraints in* $\mathcal{C}[X]$ *and a surjective mapping* $\pi\colon \{1, \ldots, M\} \longrightarrow \{1, \ldots, S\}$*, with* $S \leq M$ *(where* M *is the cardinality of Trans and both* π *and* S *may depend on* $c(X)$*), such that, for* $i = 1, \ldots, M$*,* $c(X) \wedge t_i(Y, X) \sqsubseteq g_{\pi(i)}(Y)$*.*

Note that the GENPRE operator depends on the input a, but also on: (i) the set *Trans* of transitions, and (ii) the upper portion of the tree *CTree* constructed so far. However, for reasons of simplicity, we do not explicitly indicate *Trans* and *CTree* among the arguments of the operator GENPRE and we indicate the argument a only.

The tree *CTree* is incrementally constructed as follows. Initially, *CTree* is given by the R arcs: $\Lambda \xleftarrow{\{\}} g_1(X)$, ..., $\Lambda \xleftarrow{\{\}} g_R(X)$, where $\{g_1(X), \ldots, g_R(X)\} = \text{GENPRE}(\Lambda)$.

Suppose that we have constructed an upper portion U of *CTree*. Let us introduce the following terminology.

A leaf constraint $c(X)$ of the tree U is said to be *recurrent* if there exists a non-leaf constraint $d(X)$ in U such that $c(X) \sqsubseteq d(X)$.

Now, if all leaf constraints in U are recurrent, then the construction of *CTree* terminates. Otherwise, if there exists a non-recurrent leaf constraint $c(X)$ in U, then the current upper portion U of *CTree* is expanded by adding the S arcs: $c(X) \xleftarrow{T_1} g_1(X), \ldots$, and $c(X) \xleftarrow{T_S} g_S(X)$, where:

- $\{g_1(X), \ldots, g_S(X)\} = \text{GENPRE}(c(X))$, and
- for $j = 1, \ldots, S$, $T_j = \{t_i(X, X') \in \mathit{Trans} \mid \pi(i) = j\}$. Recall that the mapping π is provided by $\text{GENPRE}(c(X))$.

In order to guarantee the termination of the construction of *CTree* we assume that our GENPRE operator is *terminating*, in the sense specified by the following definition.

Definition 3.2. *A* GENPRE *operator is said to be* terminating *if, for every infinite sequence* $c_1(X), c_2(X), \ldots$ *of constraints such that: (i)* $c_1(X) \in \text{GENPRE}(\Lambda)$*, and (ii) for every* $i > 0$*,* $c_{i+1}(X) \in \text{GENPRE}(c_i(X))$*, there exist* k *and* n*, with* $0 < k < n$*, such that* $c_n(X) \sqsubseteq c_k(X)$*.*

In the next section we define the three terminating GENPRE operators we have used in our experiments.

Theorem 3.3. *For any specification Spec and any terminating* GENPRE *operator, the construction of CTree terminates.*

From *CTree* a new system specification $\mathit{Spec}' = \langle \mathit{Var}', \mathit{Init}', \mathit{Trans}', \mathit{Unsafe}' \rangle$ can be extracted by the SPECEXTRACT procedure described below. In particular, this procedure adds new control states, by introducing an enumerated variable s.

Procedure SpecExtract(*Spec*, *CTree*, *Spec′*).

Input: A specification $Spec = \langle Var, Init, Trans, Unsafe \rangle$ and the tree *CTree* constructed in Phase 1 of the Specification Transformer.

Output: A new specification *Spec′* that is equivalent to *Spec*.

Step 1. We introduce a set *CS* of new constants (the name *CS* stands for 'control states') starting from the nodes in the tree *CTree* and *Unsafe*. We associate a constant $\mathtt{k}_\Lambda \in CS$ with the root symbol Λ. With each constraint $c(X)$ occurring in a node of *CTree* or in *Unsafe*, we associate a constant $\mathtt{k}_c \in CS$, such that for all constraints $c(X), d(X)$ occurring in a node of *CTree* or in *Unsafe*, $\mathtt{k}_c = \mathtt{k}_d$ iff $c(X) \equiv d(X)$.
We also introduce a *folding function* $\varphi : (CS - \{\mathtt{k}_\Lambda\}) \longrightarrow CS$ such that, for all constraints $c(X)$ and $g(X)$ associated with the constants $\mathtt{k}_c$ and $\mathtt{k}_g$, respectively, if $\varphi(\mathtt{k}_c) = \mathtt{k}_g$, then $c(X) \sqsubseteq g(X)$, that is, $g(X)$ is more general than $c(X)$.

Step 2. We introduce a new enumerated variable s (ranging over a subset of *CS* and declared as indicated at Step 4 below) and the corresponding primed variable s'. We define the set $\overline{Trans}$ of constraints to be

$\{s = \varphi(\mathtt{k}_d) \wedge t_i(X, X') \wedge\ c(X') \wedge s' = \mathtt{k}_c\ \mid$
$c(X) \stackrel{T}{\longleftarrow} d(X) \in CTree$ and $t_i(X, X') \in T$ and $(t_i(X, X') \wedge c(X'))$ is satisfiable$\}$.

Note that every arc arriving at $\mathtt{k}_\Lambda$ having the empty set as label, does not contribute to $\overline{Trans}$.

Step 3. We define on the set *CS* the '*depends on*' relation which is the transitive closure of the '*immediately depends on*' binary relation defined as follows.

(i) $\mathtt{k}_\Lambda$ *immediately depends on* $\varphi(\mathtt{k}_c)$ for every constraint $c(X)$ labelling a child node of Λ in the tree *CTree*.

(ii) For any two constants $\mathtt{k}, \mathtt{k}' \in CS$, we say that $\mathtt{k}'$ *immediately depends on* $\mathtt{k}$ if there exists a constraint $u(s, X, s', X') \in \overline{Trans}$, where the two equalities $s = \mathtt{k}$ and $s' = \mathtt{k}'$ occur.

Step 4. We have that *Spec′* is $\langle Var', Init', Trans', Unsafe' \rangle$, where:

– Var' is obtained by adding to *Var* the variable declaration
enumerated s CS', where $CS' = \{\mathtt{k} \in CS \mid \mathtt{k}_\Lambda$ depends on $\mathtt{k}\}$;

– $Init'$ is $\{s = \mathtt{k}_c \wedge c(X) \wedge init(X) \mid$
$\mathtt{k}_\Lambda$ depends on $\mathtt{k}_c$ and $init(X) \in Init$ and $(c(X) \wedge init(X))$ is satisfiable$\}$;

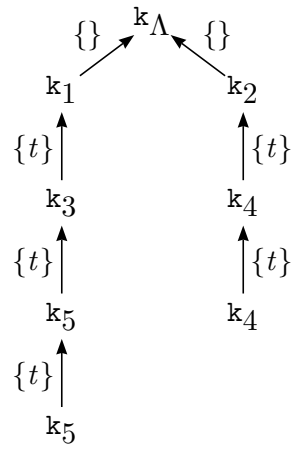
– $Trans'$ is $\{u(s, X, s', X') \in \overline{Trans} \mid$
equality $s' = \mathtt{k}$ occurs in $u(s, X, s', X')$ and $\mathtt{k}_\Lambda$ depends on $\mathtt{k}\}$;

– $Unsafe'$ is $\{s = \varphi(\mathtt{k}_u) \wedge u(X) \mid u(X) \in Unsafe\}$

Note that at Step 1 of the SpecExtract procedure we may choose φ to be a function such that $\varphi(\mathtt{k}_c) = \mathtt{k}_g$ for some constraint $g(X)$ which is *maximally general*, that is, for all $d(X)$ labelling a node of *CTree*, if $g(X) \sqsubseteq d(X)$ then $g(X) \equiv d(X)$. The function φ will be called a *maximally general folding*.

Theorem 3.4 (Correctness of the Specification Transformer) *For any specification Spec and terminating* Genpre *operator, the Specification Transformer algorithm terminates and returns a specification Spec′ which is equivalent to Spec.*

Example 2. In Figures 3 and 4 we show the two-phase construction of the new specification *Spec′* starting from the system specification of Figure 1. In the left part of Figure 3 we show



$\mathtt{k}_1:\ x_1<0\ \wedge\ x_2\leq 1$ $\qquad \varphi(\mathtt{k}_1)=\mathtt{k}_1$

$\mathtt{k}_2:\ x_2\leq -1\ \wedge\ x_2\geq -2$ $\qquad \varphi(\mathtt{k}_2)=\mathtt{k}_5$

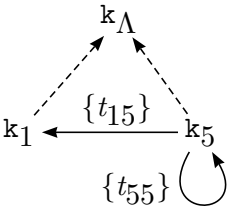
$\mathtt{k}_3:\ x_1+x_2<0\ \wedge\ x_2\leq 0$ $\qquad \varphi(\mathtt{k}_3)=\mathtt{k}_5$

$\mathtt{k}_4:\ x_2\leq -2$ $\qquad \varphi(\mathtt{k}_4)=\mathtt{k}_5$

$\mathtt{k}_5:\ x_2\leq 0$ $\qquad \varphi(\mathtt{k}_5)=\mathtt{k}_5$

t stands for $x_1'=x_2+x_1\wedge x_2'=x_2+1$

Figure 3: On the left we show the constraint tree *CTree* rooted in $\mathtt{k}_\Lambda$ for the system specification of Figure 1. Every arc $\mathtt{k}_m \xleftarrow{T} \mathtt{k}_n$ is obtained by applying the GENPRE operator with the options *Singleton* and *WidenSum*. In the middle we show the correspondence between the constraints and the associated constants $\mathtt{k}_1,\ldots,\mathtt{k}_5$. On the right we show the maximally general folding function φ.



Var′: **integer** x_1; **integer** x_2; **enumerated** s $\{\mathtt{k}_1,\mathtt{k}_5\}$;

Init′: $\{s=\mathtt{k}_5\wedge x_1\geq 0\wedge x_2=0\}$;

Trans′: $\{(s=\mathtt{k}_5\wedge x_1'=x_2+x_1\wedge x_2'=x_2+1\wedge x_1'<0\wedge x_2'\leq 1\wedge s'=\mathtt{k}_1),\ (t_{15})$

$(s=\mathtt{k}_5\wedge x_1'=x_2+x_1\wedge x_2'=x_2+1\wedge x_2'\leq 0\wedge s'=\mathtt{k}_5)\};\ (t_{55})$

Unsafe′: $\{(s=\mathtt{k}_1\wedge x_1<0\wedge x_2\leq 1),\ (s=\mathtt{k}_5\wedge x_2\leq -1\wedge x_2\geq -2)\}$

Figure 4: On the left we show the relation *Trans′*: the arc $\mathtt{k}_m \xleftarrow{\{t\}} \mathtt{k}_n$ indicates that $\mathtt{k}_m$ depends on $\mathtt{k}_n$ and $t\in$ *Trans′* (the dashed arcs show that $\mathtt{k}_\Lambda$ depends on $\mathtt{k}_1$ and $\mathtt{k}_5$). On the right we show the specification *Spec′* derived from the system specification of Figure 1. In *Trans′* and *Unsafe′* comma is interpreted as disjunction.

the *CTree* constructed at the end of Phase 1 of the Specification Transformer algorithm. For reasons of simplicity, in the nodes of the tree *CTree* in Figure 3, instead of the constraints, we have indicated their associated constants $\mathtt{k}_\Lambda,\mathtt{k}_1,\ldots,\mathtt{k}_5$ (to be introduced at Step 1 of the SPECEXTRACT procedure). The correspondence between the constraints and the constants is given in the middle part of Figure 3. Let t be an abbreviation for $x_1'=x_2+x_1\wedge x_2'=x_2+1$, which is the only constraint in *Trans*. In what follows, when no confusion arises, we will identify the constraints with their associated constants.

The specific GENPRE operator we consider, uses the option *Singleton* for partitioning and *WidenSum* for generalizing (see Section 4).

At the beginning of Phase 1 of the derivation of the new system specification, GENPRE(Λ) returns the two constraints $\mathtt{k}_1$ (that is, $x_1<0\wedge x_2\leq 1$) and $\mathtt{k}_2$ (that is, $x_2\leq -1\wedge x_2\geq -2$) and introduces the two arcs $\mathtt{k}_\Lambda \xleftarrow{\{\}} \mathtt{k}_1$ and $\mathtt{k}_\Lambda \xleftarrow{\{\}} \mathtt{k}_2$. Note that $\mathtt{k}_1$ and $\mathtt{k}_2$ are exactly the two constraints in *Unsafe* and, with reference to the Specification Transformer algorithm of Figure 2, this means that: (i) $R=N=2$, (ii) the function λ returned by GENPRE is the identity function,

and (iii) GENPRE does not make any generalization at this time, that is, $g_{\lambda(1)}(X) = u_1(X)$ and $g_{\lambda(2)}(X) = u_2(X)$. Since $\mathtt{k}_1$ and $\mathtt{k}_2$ are non-recurrent, the construction continues by further applications of GENPRE to $\mathtt{k}_1$ and $\mathtt{k}_2$, which return, respectively, the two constraints $\mathtt{k}_3$ (that is, $x_1 + x_2 < 0 \wedge x_2 \leq 0$) and $\mathtt{k}_4$ (that is, $x_2 \leq -2$) and introduces the two arcs $\mathtt{k}_1 \xleftarrow{\{t\}} \mathtt{k}_3$ and $\mathtt{k}_2 \xleftarrow{\{t\}} \mathtt{k}_4$. Now, a further application of GENPRE to $\mathtt{k}_4$ returns the same constraint and introduces the arc $\mathtt{k}_4 \xleftarrow{\{t\}} \mathtt{k}_4$, which indicates that $\mathtt{k}_4$ is recurrent. The construction continues by applying GENPRE to $\mathtt{k}_3$, which returns $\mathtt{k}_5$ (that is, $x_2 \leq 0$) and introduces the arc $\mathtt{k}_3 \xleftarrow{\{t\}} \mathtt{k}_5$. One more application of GENPRE to $\mathtt{k}_5$ returns the same constraint and introduces the arc $\mathtt{k}_5 \xleftarrow{\{t\}} \mathtt{k}_5$. Now, since also $\mathtt{k}_5$ is recurrent, the construction of the tree *CTree* is completed.

We proceed with Phase 2 by considering the maximally general folding function φ whose definition is as follows: $\varphi(\mathtt{k}_1) = \mathtt{k}_1$ and $\varphi(\mathtt{k}_2) = \varphi(\mathtt{k}_3) = \varphi(\mathtt{k}_4) = \varphi(\mathtt{k}_5) = \mathtt{k}_5$ (see the right part of Figure 3). The function φ satisfies the condition of Step 1 because $\mathtt{k}_5$ is more general than $\mathtt{k}_2$, $\mathtt{k}_3$, $\mathtt{k}_4$, and $\mathtt{k}_5$ itself, while it is incomparable with $\mathtt{k}_1$.

At Step 2 we compute the following set $\overline{\mathit{Trans}}$ of constraints (every arc in *CTree* which does not arrive at $\mathtt{k}_\Lambda$, generates an element of $\overline{\mathit{Trans}}$):

$$\begin{array}{l}
\{s = \varphi(\mathtt{k}_3) \ \wedge\ x_1' = x_2 + x_1 \ \wedge\ x_2' = x_2 + 1 \ \wedge\ x_1' < 0 \ \wedge\ x_2' \leq 1 \ \wedge\ s' = \mathtt{k}_1, \\
\ s = \varphi(\mathtt{k}_4) \ \wedge\ x_1' = x_2 + x_1 \ \wedge\ x_2' = x_2 + 1 \ \wedge\ x_2' \leq -1 \ \wedge\ x_2' \geq -2 \ \wedge\ s' = \mathtt{k}_2, \\
\ s = \varphi(\mathtt{k}_5) \ \wedge\ x_1' = x_2 + x_1 \ \wedge\ x_2' = x_2 + 1 \ \wedge\ x_1' + x_2' < 0 \ \wedge\ x_2' \leq 0 \ \wedge\ s' = \mathtt{k}_3, \\
\ s = \varphi(\mathtt{k}_4) \ \wedge\ x_1' = x_2 + x_1 \ \wedge\ x_2' = x_2 + 1 \ \wedge\ x_2' \leq -2 \ \wedge\ s' = \mathtt{k}_4, \\
\ s = \varphi(\mathtt{k}_5) \ \wedge\ x_1' = x_2 + x_1 \ \wedge\ x_2' = x_2 + 1 \ \wedge\ x_2' \leq 0 \ \wedge\ s' = \mathtt{k}_5\}
\end{array}$$

Then, at Step 3, from the set $\overline{\mathit{Trans}}$ we compute the *immediately depends on* relation which is the set $\{(\mathtt{k}_\Lambda, \mathtt{k}_1), (\mathtt{k}_\Lambda, \mathtt{k}_5), (\mathtt{k}_1, \mathtt{k}_5), (\mathtt{k}_2, \mathtt{k}_5), (\mathtt{k}_3, \mathtt{k}_5), (\mathtt{k}_4, \mathtt{k}_5), (\mathtt{k}_5, \mathtt{k}_5)\}$. We have that $\mathtt{k}_\Lambda$ *depends on* $\mathtt{k}_1$ and $\mathtt{k}_5$ and only on those constraints. The constraints on which $\mathtt{k}_\Lambda$ depends are required in the following Step 4 for constructing Trans' starting from $\overline{\mathit{Trans}}$.

Finally, at Step 4 we extract the new specification Spec' (see Figure 4) as follows:

(i) the enumerated variable s ranges over the set $\{\mathtt{k}_1, \mathtt{k}_5\}$ of constraints on which $\mathtt{k}_\Lambda$ depends,

(ii) Init' is $\{s = \mathtt{k}_5 \wedge x_2 \leq 0 \wedge \mathit{Init}\}$, where Init is the constraint $x_1 \geq 0 \wedge x_2 \geq 0 \wedge x_2 \leq 1$ (note that $s = \mathtt{k}_1 \wedge x_1 < 0 \wedge x_2 \leq 1 \wedge \mathit{Init}$ is unsatisfiable), and thus, Init' can also be rewritten as $\{s = \mathtt{k}_5 \wedge x_1 \geq 0 \wedge x_2 = 0\}$,

(iii) Trans' is built by selecting from $\overline{\mathit{Trans}}$ every constraint that assigns to the variable s' either the value $\mathtt{k}_1$ or the value $\mathtt{k}_5$: there are two of them

$$(s = \mathtt{k}_5 \ \wedge\ x_1' = x_2 + x_1 \ \wedge\ x_2' = x_2 + 1 \ \wedge\ x_1' < 0 \ \wedge\ x_2' \leq 1 \ \wedge\ s' = \mathtt{k}_1), \qquad (t_{15})$$

$$(s = \mathtt{k}_5 \ \wedge\ x_1' = x_2 + x_1 \ \wedge\ x_2' = x_2 + 1 \ \wedge\ x_2' \leq 0 \ \wedge\ s' = \mathtt{k}_5), \qquad (t_{55})$$

(iv) Unsafe' is obtained from $\mathit{Unsafe} = \{u_1(X), u_2(X)\}$, where $u_1(X)$ is $(x_1 < 0 \wedge x_2 \leq 1)$ and $u_2(X)$ is $(x_2 \leq -1 \wedge x_2 \geq -2)$. Now, $\mathtt{k}_1$ is the constant associated with u_1 and $\mathtt{k}_2$ is the constant associated with u_2 (see Figure 3). Thus, $\mathit{Unsafe}' = \{s = \varphi(\mathtt{k}_1) \wedge u_1(X),\ s = \varphi(\mathtt{k}_2) \wedge u_2(X)\}$, that is, $\mathit{Unsafe}' = \{(s = \mathtt{k}_1 \wedge x_1 < 0 \wedge x_2 \leq 1),\ (s = \mathtt{k}_5 \wedge x_2 \leq -1 \wedge x_2 \geq -2)\}$.

4. Generalized PRE Operators

In this section we present the definitions of various GENPRE operators that are used in the construction of the constraint tree *CTree*. We will use variants of generalization operators, such

as the convex-hull operator and the widening operator, which are introduced in the field of static program analysis. In our construction of the tree *CTree* these operators should act on the integers $\mathbb{Z}$ and, unfortunately, in this case they are not as efficient as in the case in which they act on the reals $\mathbb{R}$. Thus, in order to overcome this difficulty and have an efficient implementation of our generalization operators, we use a relaxation from the integers $\mathbb{Z}$ to the reals $\mathbb{R}$, that is, we use solvers on the reals $\mathbb{R}$ for performing the unsatisfiability tests, the entailment tests, and the projection operations which are required by our generalization operators.

In [15] it is shown that, for any given finite domains $D_1, \ldots, D_k$, the tree *CTree* derived by using a relaxation from $D_1 \times \ldots \times D_k \times \mathbb{Z}^{n-k}$ to $D_1 \times \ldots \times D_k \times \mathbb{R}^{n-k}$ allows us to get a specification which is *equivalent* to the given specification. Details on this point can be found in [15].

In the definition of the GENPRE operators we distinguish between: (i) atomic constraints on enumerated variables and (ii) atomic constraints on variables ranging over the integers. Thus, any constraint c can be partitioned into: (i) a (possibly empty) conjunction of equalities on enumerated variables, denoted $\mathit{fd}(c)$ (*fd* stands for 'finite domain'), and (ii) a (possibly empty) conjunction of linear inequalities on the integers, denoted $\mathit{it}(c)$ (*it* stands for 'integer type').

Note that the set of all conjunctions of equalities on enumerated variables can be viewed as a finite lattice whose underlining partial order is defined by the entailment relation $\sqsubseteq$. Given the set of constraints $\{c_1, \ldots, c_n\}$, we define their *most specific generalization*, denoted $\gamma(\{c_1, \ldots, c_n\})$, to be the conjunction of: (i) the least upper bound of the conjunctions $\mathit{fd}(c_1), \ldots, \mathit{fd}(c_n)$ of equalities on enumerated variables, and (ii) the *convex hull* [9] of the constraints $\mathit{it}(c_1), \ldots, \mathit{it}(c_n)$ on the reals $\mathbb{R}$. For $i = 1, \ldots, n$, we have that $c_i \sqsubseteq \gamma(\{c_1, \ldots, c_n\})$.

Given a set of constraints $\mathit{Cs} = \{c_1, \ldots, c_n\}$, we introduce the equivalence relation $\simeq_{\mathit{fd}}$ on *Cs* such that, for every $c_1, c_2 \in \mathit{Cs}$, $c_1 \simeq_{\mathit{fd}} c_2$ iff $\mathit{fd}(c_1)$ is equivalent to $\mathit{fd}(c_2)$. We also define the equivalence relation $\simeq_{\mathit{it}}$ on *Cs* as the reflexive, transitive closure of the relation $\downarrow_{\mathbb{R}}$ on *Cs* such that, for every $c_1, c_2 \in \mathit{Cs}$, $c_1 \downarrow_{\mathbb{R}} c_2$ iff $\mathit{it}(c_1) \wedge \mathit{it}(c_2)$ is satisfiable in $\mathbb{R}$.

For example, let the constraint c_1 be $x_1 = \mathtt{a} \wedge x_2 > 0$, for some $\mathtt{a}$ in a finite domain D, and the constraint c_2 be $x_1 = \mathtt{a} \wedge x_2 < 0$. We have that $c_1 \simeq_{\mathit{fd}} c_2$ on $\{c_1, c_2\}$. Let c_3 be the constraint $x_1 > 0 \wedge x_1 < 2$, c_4 be the constraint $x_1 > 1 \wedge x_1 < 3$, and c_5 be the constraint $x_1 > 2 \wedge x_1 < 4$. Since $c_3 \downarrow_{\mathbb{R}} c_4$ and $c_4 \downarrow_{\mathbb{R}} c_5$, we have $c_3 \simeq_{\mathit{it}} c_5$ on $\{c_3, c_4, c_5\}$. Note that $c_3 \not\simeq_{\mathit{it}} c_5$ on $\{c_3, c_5\}$ because $c_3 \wedge c_5$ is *not* satisfiable in $\mathbb{R}$.

Given a specification $\mathit{Spec} = \langle \mathit{Var}, \mathit{Init}, \mathit{Trans}, \mathit{Unsafe} \rangle$ and a leaf node L of the constraint tree *CTree*, the GENPRE operator constructs the child nodes $g_1(X), \ldots, g_S(X)$ of L in the following three steps: (1) *Unfold*, (2) *Partition*, and (3) *Generalize*.

(*Step* 1). (*Unfold*) If L is Λ, we define U to be the set $\{u_1(Y), \ldots, u_N(Y)\}$ of the constraints in *Unsafe*. If L is a constraint $c(X)$, we define U to be the set $\{d_1(Y), \ldots, d_M(Y)\}$ of $\mathbb{R}$-satisfiable constraints obtained as follows.

First, we use the transitions in the set *Trans* arriving at the node $c(X)$ and we obtain the set $\mathit{TempU} = \{c(X) \wedge t_1(Y, X), \ldots, c(X) \wedge t_M(Y, X)\}$ of constraints. Then we compute the $\mathbb{R}$-projection of each element in *TempU* on the tuple of variables Y, thereby deriving the set $\{d_1(Y), \ldots, d_M(Y)\}$ of $\mathbb{R}$-satisfiable constraints.

(*Step* 2). (*Partition*) The partition of the set U consists of the equivalence classes $E_1, \ldots, E_S$, for $S \geq 1$, induced on U by one of the following equivalence relations:

(*Singleton*) no two constraints are equivalent (every block is a singleton);

(*Chain*) for $i, j = 1, \ldots, M$, two constraints $d_i(Y)$ and $d_j(Y)$ are equivalent iff $d_i(Y) \simeq_{\mathit{fd}} d_j(Y)$ and $d_i(Y) \simeq_{\mathit{it}} d_j(Y)$ on $\{d_1(Y), \ldots, d_M(Y)\}$;

(*All*) all constraints are equivalent (a single block).

(*Step* 3). (*Generalize*) For $i = 1, \ldots, S$, the children contraints $g_1(X), \ldots, g_S(X)$ of $c(X)$ are constructed as follows. Let E_i be the set $\{d_1(X), \ldots, d_K(X)\}$ (where Y variables have been replaced by X variables):

Step 3.1 Let $b(X)$ denote the most specific generalization $\gamma(\{d_1(X), \ldots, d_K(X)\})$.
if there exists a nearest ancestor $a_1(X)$ of $c(X)$ (possibly $c(X)$ itself) in *CTree* such that $a_1(X) \simeq_{fd} c(X)$
then $b_{anc}(X) = \gamma(\{a_1(X), b(X)\})$ *else* $b_{anc}(X) = b(X)$;

Step 3.2 Let us consider a *generalization operator* $\ominus$ (for instance, the operator *WidenSum* defined below).
if in *CTree* there exists a constraint $d(X)$ such that $b_{anc}(X) \sqsubseteq d(X)$ in $\mathbb{R}$
then $g_i(X) = d(X)$
else if there exists a nearest ancestor $a_2(X)$ of $c(X)$ (possibly $c(X)$ itself) in *CTree* such that $a_2(X) \simeq_{fd} b_{anc}(X)$
then $g_i(X) = a_2(X) \ominus b_{anc}(X)$ *else* $g_i(X) = b_{anc}(X)$.

The generalization operator we have considered in our experiments is the *WidenSum* operator, denoted $\ominus_{WS}$, and it has been defined and studied in [13]. As shown in [13], it performs quite well in practice with respect to other generalization operators introduced in the literature. For convenience of the reader we now recall its definition.

We first introduce the thin well-quasi ordering $\precsim_S$. For any atomic constraint a on $\mathbb{R}$ of the form $q_0 + q_1 x_1 + \ldots + q_k x_k \lessdot 0$, where $\lessdot$ is either $<$ or $\leq$, we define *sumcoeff*(a) to be $\sum_{j=0}^{k} |q_j|$. Given two atomic constraints a_1 of the form $p_1 < 0$ and a_2 of the form $p_2 < 0$, we have that $a_1 \precsim_S a_2$ iff *sumcoeff*$(a_1) \leq$ *sumcoeff*(a_2). Similarly, if we are given the atomic constraints a_1 of the form $p_1 \leq 0$ and a_2 of the form $p_2 \leq 0$. Given any two constraints $c = a_1 \wedge \ldots \wedge a_m$ and $d = b_1 \wedge \ldots \wedge b_n$, where the a_i's and the b_i's are atomic constraints, the operator *WidenSum* returns the constraint $c \ominus_{WS} d$ which is the conjunction of the constraints in the set $\{a_h \mid 1 \leq h \leq m$ and $d \sqsubseteq a_h\} \cup \{b_k \mid b_k$ occurs in $it(d)$ and $\exists\, a_i$ occuring in $it(c), b_k \precsim_S a_i\}$. Note that it is the case that $fd(d)$ is a subconjunction of $c \ominus_{WS} d$.

5. Experimental Evaluation

In this section we present the results of the experiments we have performed for verifying safety properties of some infinite state systems taken from the literature [4, 10, 25].

We have performed our experiments by using the FASTer tool, which is designed to prove safety properties of linear counter systems [4]. FASTer performs forward reachability analysis by an exact computation of the least fixpoint of the transition relation of an *accelerated* version of the system, which is obtained by adding transitions which realize the effect of sequences of transitions.

We have run FASTer using its best options, that is, the options which make it terminate more often, among those reported on the FASTer home page. In particular, we have used the MONA system for manipulating constraints, and we have used a fixed value as the seed of the random number generator.

The constraint manipulation techniques presented in this paper were implemented on MAP [23], a tool for transforming CLP programs which uses the SICStus Prolog `clpr` library to operate on constraints on the reals. (Recall that the GENPRE operators defined in Section 4 manipulate constraints on the reals.)

All experiments were performed on an Intel Core 2 Duo E7300 2.66 GHz with 4GB of memory, under Linux. The results of our experiments are reported in Table 1, where we have indicated, for each counter system, the following times expressed in seconds: (i) the time taken by FASTer for verifying the given system (column *System*), and (ii) for each GENPRE operator, the sum of the time taken by MAP for transforming the system and the time taken by FASTer for verifying the transformed system (columns *Singleton*, *Chain*, and *All*).

	System (= *Spec*)	*Transformed System* (= *Spec′*)		
EXAMPLES		*Singleton*	*Chain*	*All*
Bakery2	24.08	0.09	0.10	0.08
Bakery3	∞	2.25	2.23	∞
Mutast	0.48	1.32	1.35	0.63
Ticket	0.27	0.94	0.94	0.34
Bounded Buffer	∞	0.21	0.23	∞
Unbounded Buffer	0.06	0.19	0.28	0.11
Selection Sort	∞	0.78	∞	∞
Scheduler2	0.20	∞	0.26	0.28
Train	8.86	1.06	11.24	8.93
TTP	112.88	∞	41.72	6.15
Number of verified properties	7	8	9	7

Table 1: Verification times (in seconds) using FASTer [4]. '∞' means 'No answer' within 5 minutes or memory limit exceeded.

The experiments show that the overall *precision*, that is, the number of verified properties, achieved by FASTer on the systems obtained by applying our transformation technique, is never worse than the precision of FASTer on the original systems.

By comparing the precision of the GENPRE operators, we observe that: (i) the *Chain* operator is the most precise, being able to prove 9 out of 10 safety properties (all, except Selection Sort); (ii) the *Singleton* operator is able to improve the precision for the three systems whose safety property could not be proved by FASTer (Bakery3, Bounded Buffer and Selection Sort), but it fails on two other systems (Scheduler and TTP); (iii) the *All* operator, with 7 properties out of 10, is the least precise (actually, it has the same precision of FASTer on the original systems).

If we consider the verification times, we have that the time in column *System* and the time in the other columns are of the same order of magnitude in almost all cases. Sometimes (and, in particular, in the case of the Bakery2 system) our method substantially reduces the total verification time.

Moreover, we have compared the total verification times of the given systems and the derived systems for which FASTer is able to prove the safety property, that is, we computed the total time of each column by summing up only the times occurring in the rows which do not have any ∞ symbol in any of the columns.

The smallest total verification time thus obtained, is the one of column *Singleton*, that is, the one relative to the systems derived by the *Singleton* operator (3.6 seconds), followed by the times relative to the systems derived by using the *All* and *Chain* operators (10.09 and 13.91 seconds, respectively). The largest overall verification time of FASTer is the one relative to the original systems (33.75 seconds).

Similar considerations also apply if we compare the time taken by FASTer on the original systems with that it takes on the systems derived by using *Chain*, which is the most precise operator (146.83 and 55.89 seconds, respectively).

Thus, the increase of precision due to the constraint-based transformation technique we have proposed, does not determine any significant degradation of the time performance.

6. Related Work and Conclusions

The transformation method presented in this paper is related to *program specialization*, which is a program transformation technique that, given a program and a specific context of use, derives a specialized program that is more effective in the given context [20]. Indeed, the improvement of precision (and performance) of forward reachability analysis is due to the fact that the transformed systems incorporate information about the specific properties holding in the unsafe states.

The use of specialization techniques for the verification of infinite state systems has been first proposed in the context of constraint logic programming (see, for instance, [12, 14, 24]). In [12, 24] infinite state systems and their properties are encoded as constraint logic programs (following an approach similar to [10, 16]) and program specialization is used as a pre-processing technique for a more effective computation of the least model (or an over-approximation thereof) of those programs. In [14] constraint logic programs provide an intermediate representation of the systems to be verified. However, the final result of the specialization is not a constraint logic program, but a new infinite state system which is then analyzed by using the ALV tool [25].

Unlike [12, 14, 24], in this paper we define a transformation method that works directly on the system specification, without the need of any constraint logic programming representation. Our new approach has the advantage of allowing the use of efficient representation and manipulation techniques for constraints, without the need for providing the representation and the manipulation of clauses, which we, thus, prove to be redundant. We have used our technique as a pre-processing of FASTer [4], instead of ALV, thereby demonstrating that our transformation method works well also in combination with *flat acceleration*.

Our transformation method is also related to some techniques for abstract interpretation [9] and, in particular, to those proposed in the field of verification of infinite state systems [1, 8]. Indeed, in Section 4 we have introduced several generalized PRE operators by combining various operators previously considered in program analysis and program specialization, such as, widening, convex hull, and well-quasi orders on the integers [9, 12, 13, 24].

The main difference between our transformation technique and abstract interpretation is that, when applied to a given system specification, the former produces an *equivalent* specification, while the latter produces a more abstract (possibly, finite state) model whose semantics is an approximation of the semantics of the given specification. Moreover, since our transformation method returns a new system specification which is written in the same language of the given specification, after transformation we may also apply abstract interpretation techniques for proving system properties.

A notable feature of our approach is that we use operators that manipulate linear constraints on real numbers, and yet, as already mentioned, we are able to preserve the equivalence of counter systems defined on the integers (obviously, this equivalence is with respect to the class of properties we consider). This feature allows us to apply at transformation time more efficient constraint solvers and more efficient manipulation techniques.

Finally, we would like to note that the transformation algorithm proposed in Section 3 is

independent of the specific interpretation of the constraints (in particular, on the integers or on the reals). Thus, our technique can easily be extended to other classes of reactive systems such as *linear hybrid systems* [18].

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