Verification of Array Manipulating Programs: A Transformational Approach

Emanuele De Angelis[1,3], Fabio Fioravanti[1],
Alberto Pettorossi[2], and Maurizio Proietti[3]

[1]University of Chieti-Pescara 'G. d'Annunzio', Italy
[2]University of Rome 'Tor Vergata', Italy
[3]CNR - Istituto di Analisi dei Sistemi ed Informatica, Rome, Italy

First International Workshop
on Parameterized Verification
Rome, Sept. 6, 2014

Outline

- Proving partial correctness of imperative programs by transforming constraint logic programs (or CLP programs).
- Encoding partial correctness of array manipulating programs into CLP.
- Generating programming language independent verification conditions.
- Checking the satisfiability of verification conditions via CLP transformations.
- Constraint manipulation in the theory of arrays.
- Experimental evaluation.

Outline

- Proving partial correctness of imperative programs by transforming constraint logic programs (or CLP programs).
- Encoding partial correctness of array manipulating programs into CLP.
- Generating programming language independent verification conditions.
- Checking the satisfiability of verification conditions via CLP transformations.
- Constraint manipulation in the theory of arrays.
- Experimental evaluation.

Outline

- Proving partial correctness of imperative programs by transforming constraint logic programs (or CLP programs).
- Encoding partial correctness of array manipulating programs into CLP.
- Generating programming language independent verification conditions.
- Checking the satisfiability of verification conditions via CLP transformations.
- Constraint manipulation in the theory of arrays.
- Experimental evaluation.

Outline

- Proving partial correctness of imperative programs by transforming constraint logic programs (or CLP programs).
- Encoding partial correctness of array manipulating programs into CLP.
- Generating programming language independent verification conditions.
- Checking the satisfiability of verification conditions via CLP transformations.
- Constraint manipulation in the theory of arrays.
- Experimental evaluation.

Outline

- Proving partial correctness of imperative programs by transforming constraint logic programs (or CLP programs).
- Encoding partial correctness of array manipulating programs into CLP.
- Generating programming language independent verification conditions.
- Checking the satisfiability of verification conditions via CLP transformations.
- Constraint manipulation in the theory of arrays.
- Experimental evaluation.

Outline

- Proving partial correctness of imperative programs by transforming constraint logic programs (or CLP programs).
- Encoding partial correctness of array manipulating programs into CLP.
- Generating programming language independent verification conditions.
- Checking the satisfiability of verification conditions via CLP transformations.
- Constraint manipulation in the theory of arrays.
- Experimental evaluation.

Proving partial correctness of imperative programs through transformation of CLP

Proving Partial Correctness of Imperative Programs

Consider a **program** and a **partial correctness triple**:

```
prog : while(x < n) {
           x = x+1 ;
           y = y+2 ;
       }
```

$$\{\, x=0 \land y=0 \land n \geq 1 \} \;\; prog \;\; \{ y > x \}$$

(A) Generate the **Verification Conditions** (VC's)

1.	$x=0 \land y=0 \land n \geq 1 \rightarrow \mathrm{P}(x,y,n)$	Initialization
2.	$\mathrm{P}(x,y,n) \land x < n \rightarrow \mathrm{P}(x+1,y+2,n)$	Loop
3.	$\mathrm{P}(x,y,n) \land x \geq n \land y \leq x \rightarrow \mathit{False}$	Exit

(B) If the VC's are **satisfiable** (i.e., there is an interpretation for P that makes 1,2,3 true), then the **partial correctness triple** holds.

Automatic Proofs of Satisfiability of VC's

Various methods (incomplete list):

- Verification of safety of infinite state systems in Constraint Logic Programming (CLP) [Delzanno-Podelski]
- CounterExample Guided Abstraction Refinement (CEGAR), Interpolation, Satisfiability Modulo Theories [Podelski-Rybalchenko, Bjørner, McMillan, Alberti et al.]
- Symbolic execution of Constraint Logic Programs [Jaffar et at.]
- Static Analysis and Transformation of Constraint Logic Programs [Gallagher et al., Albert et al., De Angelis et al.]

The CLP Transformation Method

(A) Generate the VC's as a CLP program **from the partial correctness triple and the formal definition of the semantics**:

V: 1*. p(X,Y,N) :- $X=0,\ Y=0,\ N \geq 1$. (a constrained fact)

2*. p(X1,Y1,N) :- $X<N,\ X1=X+1,\ Y1=Y+2$, p(X, Y, N).

3*. incorrect :- $X \geq N, Y \leq X$, p(X, Y, N).

THM: The VC's are **satisfiable** iff incorrect $\notin$ **the least model** $M(V)$.

(B) Apply transformation rules that **preserve the least model** $M(V)$.

V': 4. q(X1, Y1, N) :- $X<N, X>Y, Y \geq 0, X1=X+1, Y1=Y+2$, q(X, Y, N).

5. incorrect :- $X \geq N, Y \leq X, Y \geq 0\ N \geq 1$, q(X, Y, N).

least model preserved:	incorrect $\notin M(V)$ iff incorrect $\notin M(V')$
no constrained facts for q:	incorrect $\notin M(V')$
Thus,	$\{x=0 \wedge y=0 \wedge n \geq 1\}\ prog\ \{y>x\}$ holds.

Language independent verification via CLP transformations

- The verification method based on CLP transformation is parametric with respect to:
  - programming language and its operational semantics
  - properties and proof rules
  - theory of data structures.
- The input and the output of transformation are semantically equivalent CLP programs. This allows:
  - composition of verification tasks
  - iteration for refining verification
  - easy interoperation with other verifiers that use CLP (Horn clause) format.

Encoding partial correctness of array programs into CLP

Encoding Partial Correctness into CLP

Consider the triple $\{\varphi_{init}\}$ *prog* $\{\neg\varphi_{error}\}$.

A program *prog* is incorrect w.r.t. φ_{init} and φ_{error}
if a final configuration satisfying φ_{error}
is reachable from an initial configuration satisfying φ_{init}.

Definition (the interpreter *Int* with the transition predicate `tr(X,Y)`)

```
reach(X) :- initConf(X).
reach(Y) :- tr(X,Y), reach(X).
incorrect :- errorConf(X), reach(X).
```

+ clauses for `tr` (i.e., the operat. semantics of the programming language)

Theorem

prog is incorrect iff `incorrect` $\in M(Int)$

A program *prog* is correct iff it is not incorrect.

tr(X,Y): the operational semantics

`L: Id = Expr`	`tr( cf(cmd(L,asgn(Id,Expr)),S), cf(cmd(L1,C1),S1)) :-` `aeval(Expr,S,V),` *evaluate expression* `update(Id,V,S,S1),` *update store* `nextlabel(L,L1),` *next label* `at(L1,C1).` *next command*

`L: if(Expr) {`
`L1: ...`
`}`
`else`
`L2: ...`
`}`
	`tr( cf(cmd(L,ite(Expr,L1,L2)),S), cf(C,S)) :-`
	`beval(Expr,S),` *expression is true*
	`at(L1,C).` *next command*
	`tr( cf(cmd(L,ite(Expr,L1,L2)),S), cf(C,S)) :-`
	`beval(not(Expr),S),` *expression is false*
	`at(L2,C).` *next command*

`L: goto L1`	`tr( cf(cmd(L,goto(L1)),S), cf(C,S)) :-` `at(L1,C).` *next command*

tr(X,Y): the operational semantics for array assignment

array assignment: L : a[ie] = e

`tr( cf(cmd(L,asgn(elem(A,IE),E)),S),`	*source configuration* `cf`
`    cf(cmd(L1,C),S1) ) :-`	*target configuration* `cf`
`      eval(IE,S,I),`	*evaluate index expr* `IE`
`      eval(E,S,V),`	*evaluate expression* `E`
`      lookup(S,array(A),FA),`	*get array* `FA` *from store*
`      write(FA,I,V,FA1),`	*update array* `FA`, *getting* `FA1`
`      update(S,array(A),FA1,S1),`	*update store* `S`, *getting* `S1`
`      nextlab(L,L1),`	*next label* `L1`
`      at(L1,C).`	*command* `C` *at next label*

Running Example: Ascending Array Initialization

Given the **program** *SeqInit* and the **partial correctness triple**

```
i=1;
while(i<n) {
 a[i]=a[i-1]+1;
 i=i+1;
}
```

$\{i \geq 0 \land n \geq 1 \land n = dim(\mathtt{a})\}$

SeqInit

$\{\forall j \ (0 \leq j \land j+1 < n \ \rightarrow \ \mathtt{a}[j] < \mathtt{a}[j+1])\}$

CLP encoding of program *SeqInit*

- A set of at(label, command) facts.
- while = ite + goto.
- elem(a,i) stands for a[i].

$\mathtt{at}(\ell_0, \mathtt{asgn}(\mathtt{i}, 1)).$

$\mathtt{at}(\ell_1, \mathtt{ite}(\mathtt{less}(\mathtt{i}, \mathtt{n}), \ell_2, \ell_\mathtt{h})).$

$\mathtt{at}(\ell_2, \mathtt{asgn}(\mathtt{elem}(\mathtt{a}, \mathtt{i}),$
$\quad \mathtt{plus}(\mathtt{elem}(\mathtt{a}, \mathtt{minus}(\mathtt{i}, 1)), 1))).$

$\mathtt{at}(\ell_3, \mathtt{asgn}(\mathtt{i}, \mathtt{plus}(\mathtt{i}, 1))).$

$\mathtt{at}(\ell_4, \mathtt{goto}(\ell_1)).$

$\mathtt{at}(\ell_\mathtt{h}, \mathtt{halt}).$

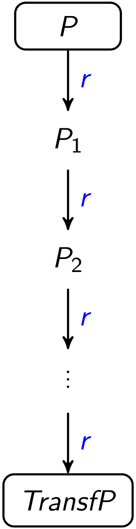
CLP encoding of φ_{init} and φ_{error}

$\mathtt{initConf}(\ell_0, \mathtt{I}, \mathtt{N}, \mathtt{A})$:-
$\mathtt{I} \geq 0,\ \mathtt{N} \geq 1.$

$\mathtt{errorConf}(\ell_\mathtt{h}, \mathtt{N}, \mathtt{A})$:-
$\mathtt{W} \geq 0,\ \mathtt{W}+1 < \mathtt{N},\ \boxed{\mathtt{Z} = \mathtt{W}+1,\ \mathtt{U} \geq \mathtt{V},}$
$\mathtt{read}(\mathtt{A}, \boxed{\mathtt{W}, \mathtt{U}}), \mathtt{read}(\mathtt{A}, \boxed{\mathtt{Z}, \mathtt{V}}).$

Generating Verification Conditions via CLP transformation

'Rule + Strategies' Program Transformation



- The transformation **rules**:

  $r \in \{$ Definition, Unfolding, Folding, Clause Removal $\}$

- The rules **preserve** the least model:

Theorem (Least model preservation)

`incorrect` $\in M(P)$ iff `incorrect` $\in M(TransfP)$

- The rules must be guided by **strategies**.

[Burstall-Darlington 77,Tamaki-Sato 84, Etalle-Gabbrielli 96]

Rules for Transforming CLP Programs

R1. **Definition**. Introducing a new predicate (e.g., a loop invariant)

```
newp(X) :- c,A.
```

R2. **Unfolding**. A symbolic evaluation step (e.g., a resolution step)

given $\quad$ `H :- c,A,G.`

$\quad\quad$ `A :- d`$_1$`,G`$_1$`. , ..., A :- d`$_m$`,G`$_m$`.`

derive $\quad$ `H :- c,d`$_1$`,G`$_1$`,G. , ..., H :- c,d`$_m$`,G`$_m$`,G.`

R3. **Folding**. Using a predicate definition

given $\quad$ `H :- d,A,G.`

$\quad\quad$ `newp(X) :- c,A.` $\quad$ and $\quad$ $d \rightarrow c$

derive $\quad$ `H :- d,newp(X),G.`

R4. **Clause Removal**. Delete clauses with
(i) unsatisfiable constraint or (ii) subsumed by other clauses

The Strategy for Generation (Specialization of Int)

Specialize(P)

$TransfP = \emptyset$;

Defs = {`incorrect :- errorConf(X), reach(X)`};

while $\exists q \in$ Defs **do**

```
%execute a symbolic evaluation step (i.e., resolution)
```

Cls = Unfolding(q);

```
%remove unsatisfiable and subsumed clauses
```

Cls = ClauseRemoval(Cls);

```
%introduce new predicates (i.e., a loop invariant)
```

Defs = (Defs $-$ {q}) $\cup$ Definition(Cls);

```
%match a predicate definition
```

$TransfP = TransfP \cup$ Folding(Cls, Defs);

od

Generation of Verification Conditions

The specialization of *Int* w.r.t. *prog* removes all references to:

- `tr` and
- `at`

VC: The Verification Conditions for *SeqInit*

```
incorrect :- Z=W+1, W≥0, W+1<N, U≥V, N≤I,
             read(A,W,U), read(A,Z,V), new1(I,N,A).
new1(I1,N,B) :- 1≤I, I<N, D=I-1, I1=I+1, V=U+1,
             read(A,D,U), write(A,I,V,B), new1(I,N,A).
new1(I,N,A) :- I=1, N≥1.
```

- A constrained fact is present:
  we cannot conclude that the program is correct.
- The fact `incorrect` is not present:
  we cannot conclude that the program is incorrect either.

Checking satisfiability of VC's via CLP transformation

The Strategy for Satisfiability

Transform(P)

$TransfP = \emptyset$;
Defs $= \{$`incorrect :- errorConf(X), reach(X)`$\}$;
while $\exists q \in$ Defs **do**
  Cls $=$ Unfolding(q);
  Cls $=$ ConstraintReplacement(Cls);
  Cls $=$ ClauseRemoval(Cls);
  Defs $=$ (Defs $- \{q\}$) $\cup$ Definition$_{\text{array}}$(Cls);
  $TransfP = TransfP \cup$ Folding(Cls, Defs);
od

Constraint manipulation in the theory of arrays

Constraint Replacement Rule

If $\mathcal{A} \models \forall(\mathtt{c}_0 \leftrightarrow (\mathtt{c}_1 \vee \ldots \vee \mathtt{c}_n))$, where $\mathcal{A}$ is the Theory of Arrays

Then replace $\quad$ `H :- c0, d, G`

by $\quad$ `H :- c1, d, G,` ..., `H :- cn, d, G`

Constraint Handling Rules [Frühwirth et al.] for Constraint Replacement:

AC1. Array-Congruence-1: if i=j then a[i]=a[j]

$$\mathtt{read(A,I,X)} \setminus \mathtt{read(A1,J,Y)} \Leftrightarrow \mathtt{A} == \mathtt{A1}, \mathtt{I} = \mathtt{J} \mid \mathtt{X} = \mathtt{Y}.$$

AC2. Array-Congruence-2: if a[i]≠a[j] then i≠j

$$\mathtt{read(A,I,X)}, \mathtt{read(A1,J,Y)} \Rightarrow \mathtt{A} == \mathtt{A1}, \mathtt{X} <> \mathtt{Y} \mid \mathtt{I} <> \mathtt{J}.$$

ROW. Read-Over-Write: {a[i]=x; y=a[j]} if i=j then x=y

$$\mathtt{write(A,I,X,A1)} \setminus \mathtt{read(A2,J,Y)} \Leftrightarrow \mathtt{A1} == \mathtt{A2} \mid (\mathtt{I} = \mathtt{J}, \mathtt{X} = \mathtt{Y}) \; ; \; (\mathtt{I} <> \mathtt{J}, \mathtt{read(A,J,Y)}).$$

Ascending Array Initialization

```
new3(A,B,C) :-  A=2+H, B-H≤3, E-H≤1, E≥1, B-H≥2,... ,
  read(N,H,M), read(C,D,F), write(N,J,K,C), read(C,E,G),
  reach(J,B,N).
```

- by applying the ROW rule:

```
new3(A,B,C) :- J=1+D, A=2+D, K=1+I, I<F, ... ,  J=E, K=G,
  read(C,D,F), read(N,D,I), write(N,J,K,C), read(C,E,G),
write(N,J,K,C), read(C,E,G),
  reach(J,B,N).

new3(A,B,C) :- J=1+D, A=2+D, K=1+I, I<F, ... , J<>E,
  read(C,D,F), read(N,D,I), write(N,J,K,C), read(C,E,G),
write(N,J,K,C), read(C,E,G),
  reach(J,B,N).
```

- by applying the ROW, AC1, and AC2 rules:

```
new3(A,B,C) :- A=1+H, E=1+D, J=-1+H, K=1+L, D-H≤-2, H<B,...
  read(N,E,G), read(N,D,F), read(N,J,L), write(N,H,K,C),
  reach(J,B,M).
```

Definition Introduction

Introduction of suitable new predicate **definitions** (they correspond to **program invariants**).

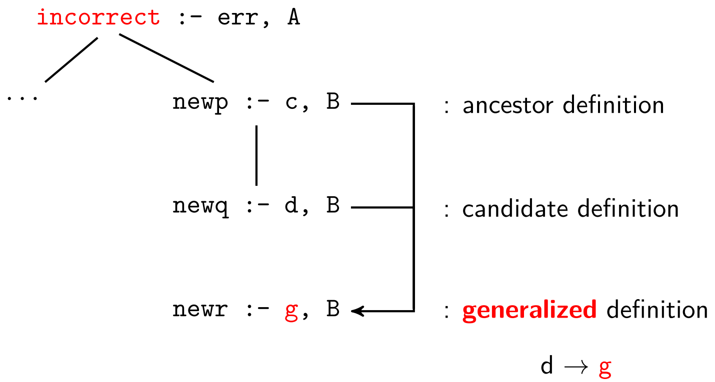
Difficulty: Introduction of an unbounded number of new predicate definitions.

Solution: Use of **generalization** operators:

- to ensure the termination of the transformation,
- to generate program invariants.

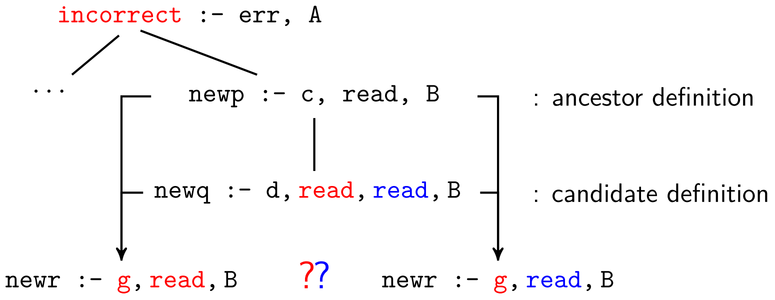
Constraint Generalizations

Definitions are arranged as a tree:



Generalization operators based on widening and convex-hull [Cousot-Cousot 77, Cousot-Halbwachs 78].

Array Constraint Generalizations



- We decorate CLP variables with the variable identifiers of the imperative program.

VC: The Verification Conditions for *SeqInit* (decorated)

incorrect :- $Z=W+1$, $W \geq 0$, $W+1<N$, $U \geq V$, $N \leq I$,
  read(A, W^{j}, $U^{a[j]}$), read(A, Z^{j1}, $V^{a[j1]}$), new1(I, N, A).

new1(I1, N, B) :- $1 \leq I$, $I<N$, $D=I-1$, $I1=I+1$, $V=U+1$,
  read(A, D^{i}, $U^{a[i]}$), write(A, I, V, B), new1(I, N, A).

new1(I, N, A) :- $I=1$, $N \geq 1$.

Ascending Array Initialization

: ancestor definition

```
new3(I,N,A) :- E+1=F, E≥0, I>F, G≥H, N>F, N≤I+1,
read(A,E^j,G^a[j]), read(A,F^j1,H^a[j1]), reach(I,N,A).
```

: candidate definition

```
new4(I,N,A) :- E+1=F, E≥0, I>F, G≥H, I=1+I1, I1+2≤C, N≤I1+3,
read(A,E^j,G^a[j]), read(A,F^j1,H^a[j1]), read(A,P^i,Q^a[i]),
reach(I,N,A).
```

: **generalized** definition

```
new5(I,N,A) :- E+1=F, E≥0, I>F, G≥H, N>F,
read(A,E^j,G^a[j]), read(A,F^j1,H^a[j1]), reach(I,N,A).
```

In the paper: a variable of the form G^{v} is encoded by `val(v,G)`.

Result of Transformation

By applying the transformation strategy *Transform* to the verification conditions for *SeqInit*:

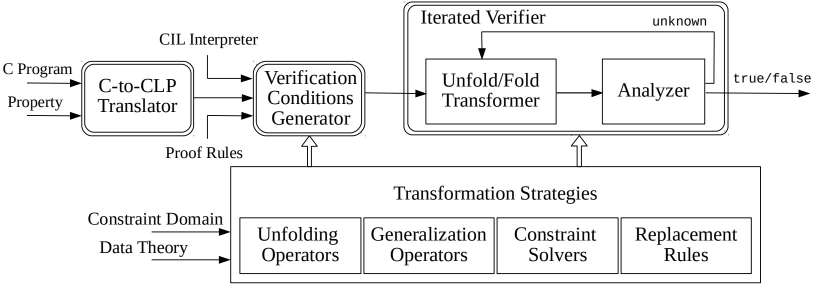
VC': Transformed verification conditions for *SeqInit*

```
incorrect :- J1=J+1, J≥0, J1<I, AJ≥AJ1, D=I−1, N=I+1, Y=X+1,
     read(A,J,AJ), read(A,J1,AJ1), read(A,D,X), write(A,I,Y,B),
     new1(I,N,A).
new1(I1,N,B) :- I1=I+1, Z=W+1, Y=X+1, D=I−1, N≤I+2,
     I≥1, Z<I, Z≥1, N>I, U≥V, read(A,W,U), read(A,Z,V),
     read(A,D,X), write(A,I,Y,B), new5(I,N,A).
new5(I1,N,B) :- I1=I+1, Z=W+1, Y=X+1, D=I−1, I≥1,
     Z<I, Z≥1, N>I, U≥V, read(A,W,U), read(A,Z,V),
     read(A,D,X), write(A,I,Y,B), new5(I,N,A).
```

No constrained facts in VC': incorrect $\notin M(VC')$.
The program *SeqInit* is correct.

Experimental results

- The VeriMAP tool http://map.uniroma2.it/VeriMAP



Experimental evaluation

Program	$Gen_{W,\mathcal{I},\Cap}$	$Gen_{H,\mathcal{V},\subseteq}$	$Gen_{H,\mathcal{V},\Cap}$	$Gen_{H,\mathcal{I},\subseteq}$	$Gen_{H,\mathcal{I},\Cap}$
bubblesort-inner	0.9	*unknown*	*unknown*	*unknown*	1.52
copy-partial	*unknown*	*unknown*	3.52	3.51	3.54
copy-reverse	*unknown*	*unknown*	5.25	*unknown*	5.23
copy	*unknown*	*unknown*	5.00	4.88	4.90
find-first-non-null	0.14	0.66	0.64	0.28	0.27
find	1.04	6.53	2.35	2.33	2.29
first-not-null	0.11	0.22	0.22	0.22	0.22
init-backward	*unknown*	1.04	1.04	1.03	1.04
init-non-constant	*unknown*	2.51	2.51	2.47	2.47
init-partial	*unknown*	0.9	0.89	0.9	0.89
init-sequence	*unknown*	4.38	4.33	4.41	4.29
init	*unknown*	1.00	0.97	0.98	0.98
insertionsort-inner	0.58	2.41	2.4	2.38	2.37
max	*unknown*	*unknown*	0.8	0.81	0.82
partition	0.84	1.77	1.78	1.76	1.76
rearrange-in-situ	*unknown*	*unknown*	3.06	3.01	3.03
selectionsort-inner	*unknown*	*time-out*	*unknown*	2.84	2.83
verified	6	10	15	15	17
total time	3.61	21.42	34.76	31.81	38.45
average time	0.60	2.14	2.31	2.12	2.26

Conclusions

Our verification framework:

- CLP as a metalanguage for a formal definition of the programming language semantics and program properties
- Semantics preserving transformations of CLP as programming language independent proof rules.