

# Verification of Time-Aware Business Processes using Constrained Horn Clauses

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# Talk Outline

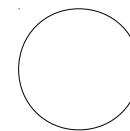
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- Business Process Model and Notation (BPMN)
- Semantics of time-aware BPMN
- Verification method
  - CHC encoding of the interpreter  $I$ 
    - BPMN model, semantics, property
  - CHC specialization of  $I$  (unfold/fold)
  - Predicate equivalence
  - CHC satisfiability checking using SMT solvers
- Experimental evaluation

# Business Process Modeling and Notation

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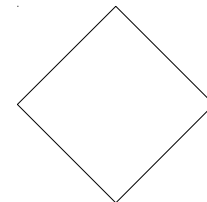
- Graphical language for modeling organizational processes: activities, events, and their composition (OMG standard)
- Tasks
  - atomic units of work
- Events
  - something that ‘happens’
- Gateways
  - model flow branching / merging
- Sequence flow
  - specifies the order of execution



start



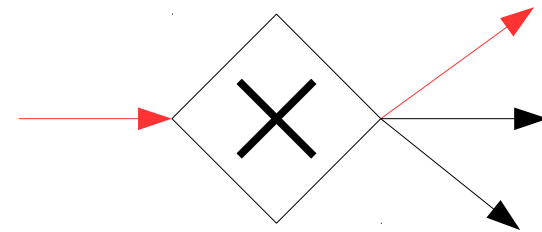
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# Branch Gateways

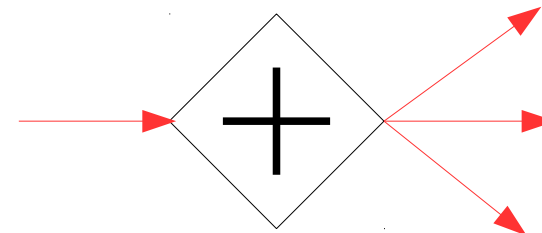
- single incoming flow / multiple outgoing flows
- **exclusive** branch gateway (XOR)

- upon activation of the incoming flow exactly one outgoing flow is activated



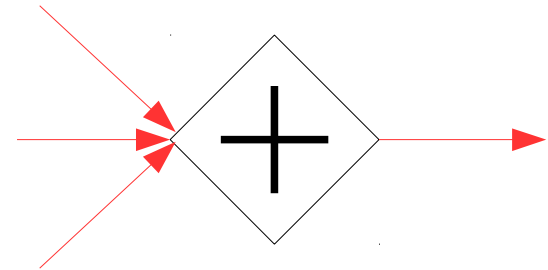
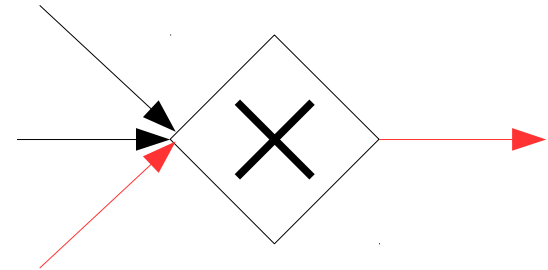
- **parallel** branch gateway (AND)

- upon activation of the incoming flow all outgoing flows are activated



# Merge Gateways

- multiple incoming flows / single outgoing flow
- **exclusive** merge gateway (XOR)
  - the outgoing flow is activated upon activation of one of the incoming flows
- **parallel** merge gateway (AND)
  - the outgoing flow is activated upon activation of all the incoming flows



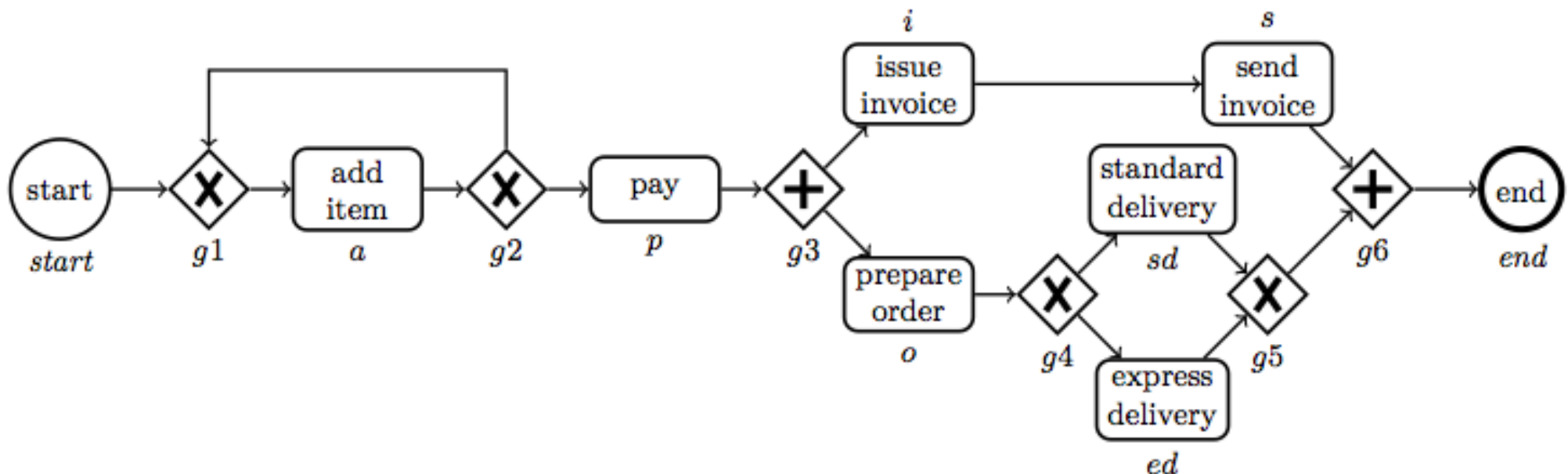
# Well-formed BPMN models

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- the *start* and *end* events are unique
- all flow objects are on some path from *start* to *end*
- $| \text{Preds}(\text{start}) | = 0$  and  $| \text{SucCs}(\text{start}) | = 1$ 
  - similar constraints for the *end* event, tasks and gateways
- on every cyclic path there is at least one occurrence of a task
  - no cycles through gateways only

# Purchase Order process (PO)

- A customer adds one or more items to the shopping cart and pays.
- Then, the invoice is sent and the order is delivered.



# Time-aware BPMN model

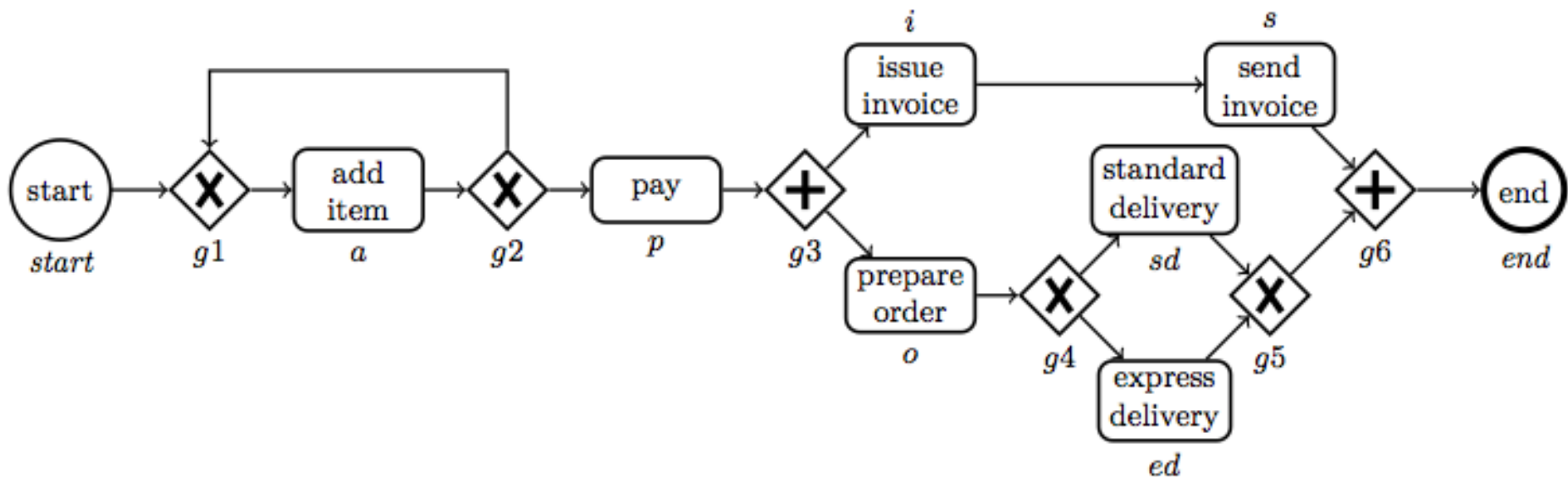
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- Tasks have a duration
  - a positive integer, with upper and lower bounds
  - for simplicity, events and gateways are instantaneous
- CHC specification
  - $duration(x,d) \leftarrow d_{min} \leq d \leq d_{max}$ .  
 $x$  is a task of duration  $d$
  - $task(x)$ .  
 $x$  is a task
  - $par\_branch(y)$ .  
 $y$  is a parallel branch gw
  - $seq(x,y)$ .  
sequence flow from  $x$  to  $y$
- BPMN Meta-model
  - Well-formedness and disjointness of elements



# CHC encoding of the Purchase Order process

Events	<code>start(start). end(end).</code>
Gateways	<code>exc_merge(g1). exc_branch(g2). ...</code>
Tasks	<code>task(a). task(p). ...</code>
Sequence flow	<code>seq(start,g1). seq(g1,a). ...</code>
Task durations	<code>duration(a, D):- D&gt;=1, D=&lt;6. % add item</code> <code>duration(p, D):- D&gt;=1, D=&lt;2. % pay</code> <code>...</code> <code>duration(X, D):- not_task(X), D=0.</code> <code>% gateways and events</code>



# Semantics of time-aware BPMN

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- Transition relation  $\rightarrow$  between states  $\langle F, t \rangle$
- $t$  time point integer
- $F$  set of fluents properties that hold at time point  $t$ 
  - *begins*( $x$ ):  $x$  begins its execution (enactment)
  - *enacting*( $x, r$ ):  $x$  is enacting,  
 $r$  residual time to completion
  - *completes*( $x$ ):  $x$  has completed its execution
  - *enables*( $x, y$ ):  $x$  has completed its execution  
and enables its successor  $y$ 
    - $x, y$  denote flow objects (tasks, or events, or gateways)

# Semantics of time-aware BPMN

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$$(S_1) \frac{\textit{begins}(x) \in F \quad \textit{duration}(x, d)}{\langle F, t \rangle \longrightarrow \langle (F \setminus \{\textit{begins}(x)\}) \cup \{\textit{enacting}(x, d)\}, t \rangle}$$

$$(S_2) \frac{\textit{completes}(x) \in F \quad \textit{par\_branch}(x)}{\langle F, t \rangle \longrightarrow \langle (F \setminus \{\textit{completes}(x)\}) \cup \{\textit{enables}(x, s) \mid \textit{seq}(x, s)\}, t \rangle}$$

$$(S_3) \frac{\textit{completes}(x) \in F \quad \textit{not\_par\_branch}(x) \quad \textit{seq}(x, s)}{\langle F, t \rangle \longrightarrow \langle (F \setminus \{\textit{completes}(x)\}) \cup \{\textit{enables}(x, s)\}, t \rangle}$$

# Semantics of time-aware BPMN

$$(S_4) \frac{\forall p \text{ seq}(p, x) \rightarrow \text{enables}(p, x) \in F \quad \text{par\_merge}(x)}{\langle F, t \rangle \longrightarrow \langle (F \setminus \{\text{enables}(p, x) \mid \text{enables}(p, x) \in F\}) \cup \{\text{begins}(x)\}, t \rangle}$$

$$(S_5) \frac{\text{enables}(p, x) \in F \quad \text{not\_par\_merge}(x)}{\langle F, t \rangle \longrightarrow \langle (F \setminus \{\text{enables}(p, x)\}) \cup \{\text{begins}(x)\}, t \rangle}$$

$$(S_6) \frac{\text{enacting}(x, 0) \in F}{\langle F, t \rangle \longrightarrow \langle (F \setminus \{\text{enacting}(x, 0)\}) \cup \{\text{completes}(x)\}, t \rangle}$$

# Semantics of time-aware BPMN

- Time elapses

$$(S_7) \frac{\text{no\_other\_premises}(F) \quad \exists x \exists r \text{enacting}(x, r) \in F \quad m > 0}{\langle F, t \rangle \longrightarrow \langle (F \setminus \{\text{enacting}(x, r) \mid \text{enacting}(x, r) \in F\}) \cup \{\text{enacting}(x, r-m) \mid \text{enacting}(x, r) \in F\}, t+m \rangle}$$

where: (i)  $\text{no\_other\_premises}(F)$  holds iff none of the rules  $S_1$ – $S_6$  has its premise true, and (ii)  $m = \min\{r \mid \text{enacting}(x, r) \in F\}$ .

# CHC Encoding of the Semantics

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- The predicate `tr` encodes the operational semantics 

```
S1. tr(s(F,T), s(FU,T)):- member(begins(X),F), duration(X,D),
                           update(F,[begins(X)],[enacting(X,D)],FU).
```

```
S2. tr(s(F,T), s(FU,T)):- member(completes(X),F), par_branch(X),
                           findall(enables(X,S),(seq(X,S)),Enbls),
                           update(F,[completes(X)],Enbls,FU).
```

```
S3. tr(s(F,T), s(FU,T)):- member(completes(X),F),
                           not_par_branch(X), seq(X,S),
                           update(F,[completes(X)],[enables(X,S)],FU).
```

...

- The predicate `reach` encodes reachability 

```
R1. reach(S,S).
```

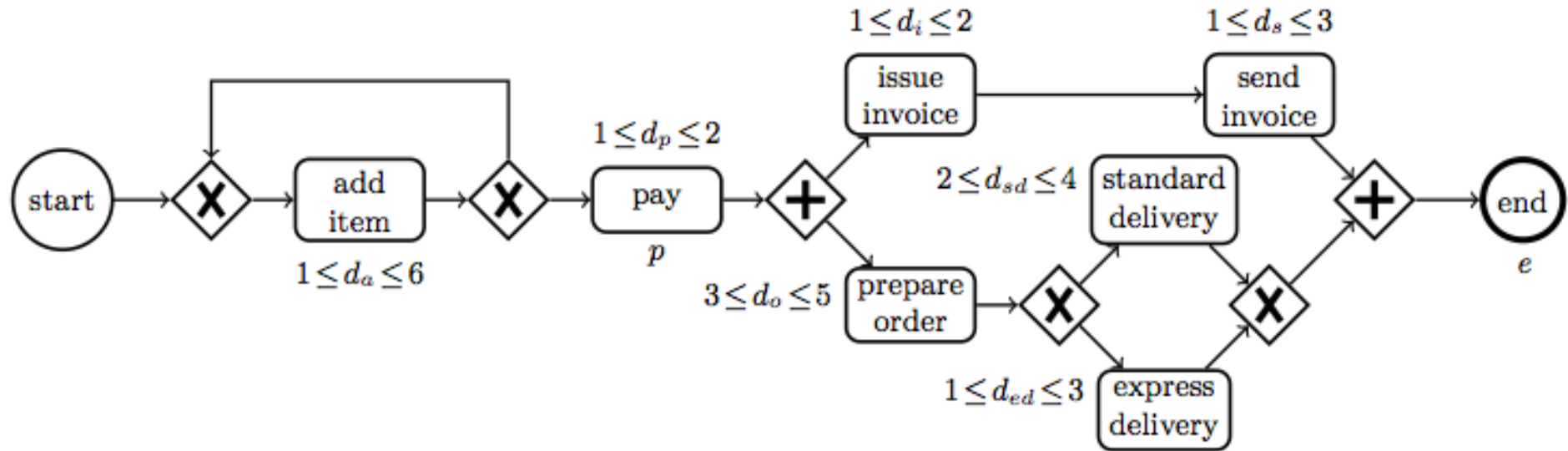
```
R2. reach(S,S2) :- tr(S,S1), reach(S1,S2).
```

# CHC Encoding of the Semantics

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- S4. `tr(s(F,T), s(FU,T)) :- member(enables(_,_),F), par_merge(X),  
findall(enables(P,X),(seq(P,X)),Enbls),  
sublist(Enbls,F),  
update(F,Enbls,[begins(X)],FU).`
- S5. `tr(s(F,T), s(FU,T)):- member(enables(P,X),F), not_par_merge(X),  
update(F,[enables(P,X)],[begins(X)],FU).`
- S6. `tr(s(F,T), s(FU,T)):- member(enacting(X,R),F), R=0,  
update(F,[enacting(X,R)],[completes(X)],FU).`
- S7. `tr(s(F,T), s(FU,TU)) :- no_other_premises(F),  
member(enacting(_,_),F),  
findall(Y,(Y=enacting(X,R),member(Y,F)),Enacts),  
mintime(Enacts,M), M>0,  
  
decrease_residual_times(Enacts,M,EnactsU),  
update(F,Enacts,EnactsU,FU), TU=T+M.`

# Verification of the PO process



- Property to be verified

*prop*: if  $init \rightarrow^* \langle \{ completes(p) \}, t_p \rangle \rightarrow^* \langle \{ completes(e) \}, t_e \rangle$ , then  $t_e \leq t_p + 9$ .

- CHC encoding of the (negation of the) property

NP.     false :-     Ts=0, Te>Tp+9,  
                       reach(s([begins(start)],Ts), s([completes(p)],Tp)),  
                       reach(s([completes(p) ],Tp), s([completes(e)],Te)).

- The property holds     iff  
 the set  $I = \{S1-S7, R1-R2, PO, NP\}$  of clauses is satisfiable.



# Program transformation

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- Cannot check satisfiability of  $I$  using SMT solvers
  - lists, terms, findall predicate
- CLP systems do not terminate
  - recursive reach predicate
- Removal of the interpreter
  - Rule-based transformation strategies  
(unfolding, definition, folding,  
useless/subsumed clauses removal)
  - Enables the use of CHC/SMT solvers
- Predicate equivalence
  - Possibly reduces the number of predicates

# Removal of the interpreter

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- From  $I$  derive an equisatisfiable set of clauses  $I_{sp}$ 
  - $I$  satisfiable iff  $I_{sp}$  satisfiable
- $I_{sp}$  contains
  - no references to the CHC encoding of the business process and of the semantics
  - no lists, no terms, no `findall`
  - Clauses of the form
$$\text{new21}(A,B,C) \text{ :- } A=0, D=<3, D>=1, \text{new10}(D,B,C).$$
- We can apply CHC/SMT solvers for checking the satisfiability of  $I_{sp}$

# Removal of the Interpreter

---

```
false :- A=0, B>=1, B<=2, C>=1, C<=6,  
         D>=3, D<=5, E>0, F-E>9,  
         new1(C,A,E), new2(B,D,E,F).
```

```
new1(A,B,C) :- A=0, D<=6, D>=1, new1(D,B,C).
```

```
new1(A,B,C) :- A=0, D<=2, D>=1, new44(D,B,C).
```

```
new1(A,B,C) :- D=0, E=A+B, A>0, new1(D,E,C).
```

```
new44(A,B,C) :- A=0, B=C.
```

```
new44(A,B,C) :- D=0, E=A+B, A>0, new44(D,E,C).
```

...

# Predicate Equivalence

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- Decidable predicate equivalence test
  - based on predicate renaming and constraint equivalence
- Discovers classes of equivalent predicates
  - $\{\{\text{new17}, \text{new11}\}, \{\text{new6}, \text{new7}\}\}$

```
new17(A,B,C) :- A=0, B=C.  
new17(A,B,C) :- D=0, E=A+B, A>0, new17(D,E,C).  
new6(A,B,C,D) :- B=0, new17(A,C,D).
```

```
new11(A,B,C) :- A=0, B=C.  
new11(A,B,C) :- D=0, E=A+B, A>0, new11(D,E,C).  
new7(A,B,C,D) :- B=0, new11(A,C,D).
```

# Experimental evaluation

Process / Property	RI	Preds	Z3	Answer	PE	Preds %	Z3 %
<b>1. PO</b>							
Q1.1	0.42	14	0.87	<i>true</i>	0.00	64%	68%
Q1.2	0.23	14	0.75	<i>true</i>	0.01	71%	71%
Q1.3	0.26	5	0.10	<i>false</i>	0.00	100%	100%
<b>2. RDOA</b>							
Q1.1	0.07	9	0.31	<i>false</i>	0.00	78%	74%
<b>3. STEMI_ED</b>							
Q3.1	0.25	12	1.05	<i>true</i>	0.00	75%	91%
Q3.2	0.27	4	0.09	<i>false</i>	0.00	100%	100%
Q3.3	0.29	19	1.80	<i>true</i>	0.00	79%	91%
<b>4. STEMI_ED_CCU</b>							
Q4.1	1.39	36	11.11	<i>true</i>	0.03	75%	87%
Q4.2	0.10	14	34.21	<i>true</i>	0.00	79%	37%
Q4.3	0.06	8	2.11	<i>false</i>	0.00	100%	100%

- Similar results using EldaRICA

# Conclusions

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- Flexible framework for reasoning about BP
  - Parametric w.r.t. the semantics and property
  - Satisfiability-preserving program transformations
    - enables the use of state-of-the-art SMT solvers
- Future developments
  - different semantics of time, different properties
  - data (Montali, Deutsch, ...)
  - ontologies (Proietti&Smith)
  - VeriMAP system
    - <http://www.map.uniroma2.it/VeriMAP/>

The end

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Thank you!





# 1. PO

---

- % Q1.1 (paper running example)

% after payment is completed, the process takes at most 9 time units to complete

$r\_clause(\text{incorrect1}, [\text{reachable}(s([\text{begins}(\text{start})], T1), s([\text{completes}(p)], T2)), T1=0, \text{reachable}(s([\text{completes}(p)], T2), s([\text{completes}(\text{end})], T3)), T2>T1, T3-T2>9 ])$ .

- % Q1.2 the process takes at least 4 time units to complete

$r\_clause(\text{incorrect2}, [\text{reachable}(s([\text{begins}(\text{start})], T1), s([\text{completes}(\text{end})], T2)), T2-T1=\leq 3])$ .

- % Q1.3 - FALSE

% the task prepare order (o) and the task send invoice (s) cannot be enacting simultaneously

$r\_clause(\text{incorrect3}, [\text{reachable}(s([\text{begins}(\text{start})], T1), s([\text{enacting}(o, O), \text{enacting}(s, S)], T2)), S\geq 1, O\geq 1])$ .

# 2. RDOA

- Request Day-Off Approval
  - Huai et al. Towards Trustworthy Composite Service Through Business Process Model Verification UIC-ATC 2010

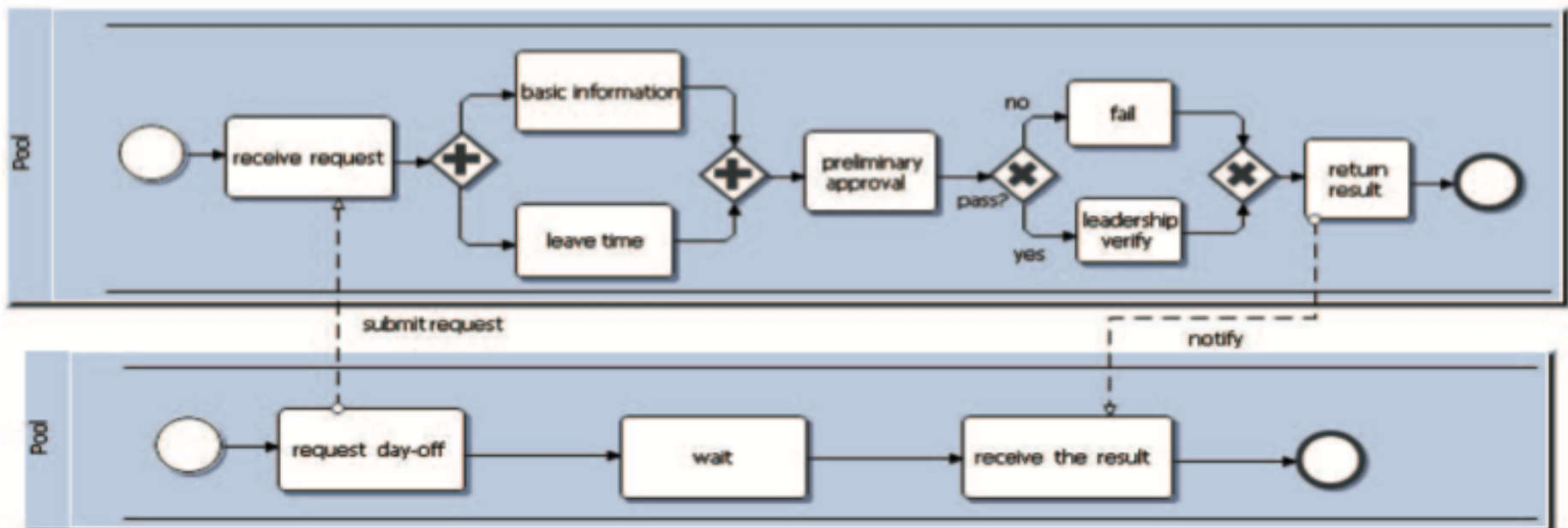
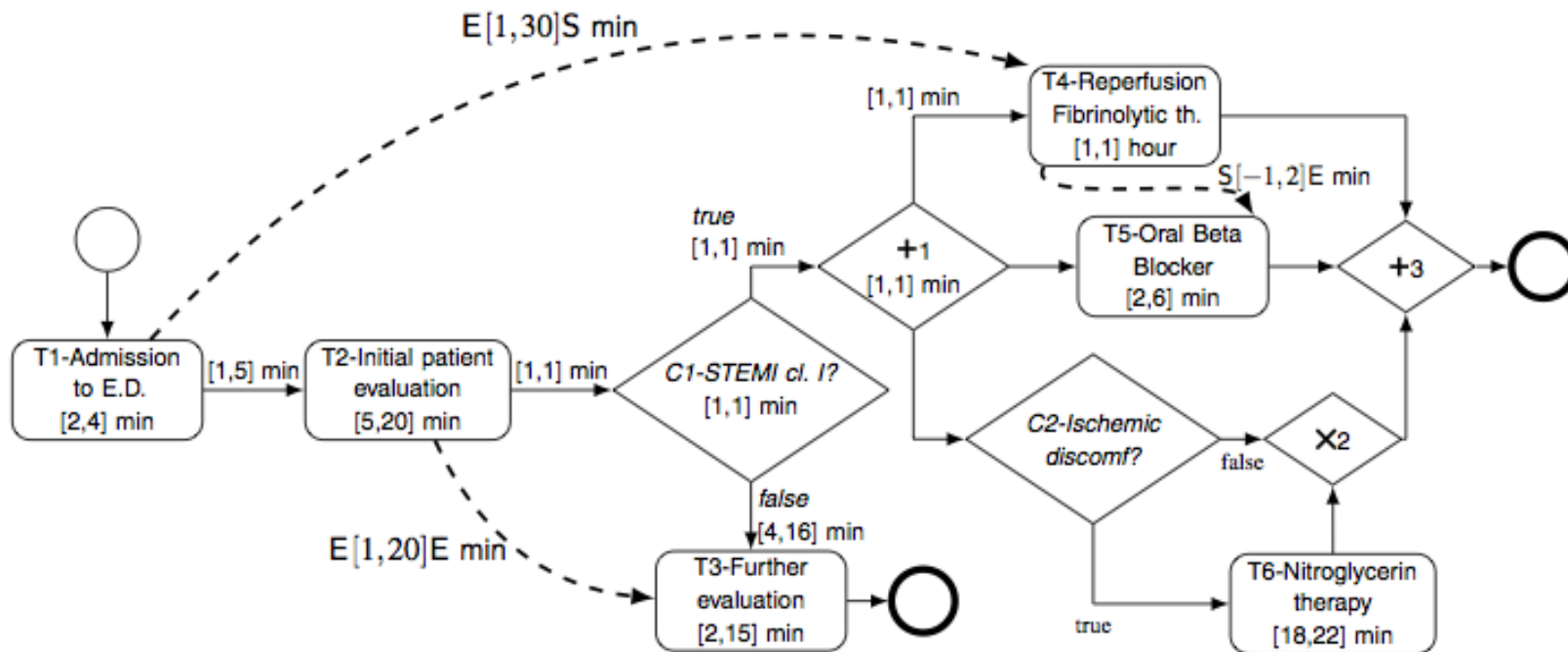


Figure 1. Request day off approval process

- 
- % Q2.1 - FALSE
  - % After the employee submits the day-off request, a result is returned in less than 120 time units.
  - r\_clause(incorrect1,  
[reachable(s([begins(s)],T1),s([completes(return\_result)],T2)), T2-T1>=120]).

# 3. STEMI: ST-segment Evaluation Myocardial Infarction

- ED (Emergency Department) Admission
  - Controllability in temporal Conceptual workflow Schemata Combi et. al. BPM 2009



**Fig. 6.** Workflow graph example of patient admission to an hospital. The dashed edges represent relative constraints.

- 
- % Q3.1

% since the completion of g1, the process takes at most 22 minutes to be completed

r\_clause(incorrect1, [reachable(s([completes(g1)],T1),s([completes(g3)],T2)), T2-T1>=23]).

- % Q3.2 - FALSE

% since the completion of T2, it takes at most 20 minutes to begin T3

r\_clause(incorrect2, [reachable(s([completes(t2)],T1),s([completes(t3)],T2)), T2-T1>=21]).

- % Q3.3 % the process takes at least 11 minutes to be completed

r\_clause(incorrect3, [reachable(s([begins(start)],T1),s([completes(end)],T2)), T2-T1=< 10]).

# 4. STEMI-CCU: ST-segment Evaluation Myocardial Infarction

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- ED (Emergency Department) Admission  
+ CCU (Coronary Care Unit) Admission
  - Combi et al.. Conceptual Modeling of Flexible Temporal Workflows. ACM AAS 2012

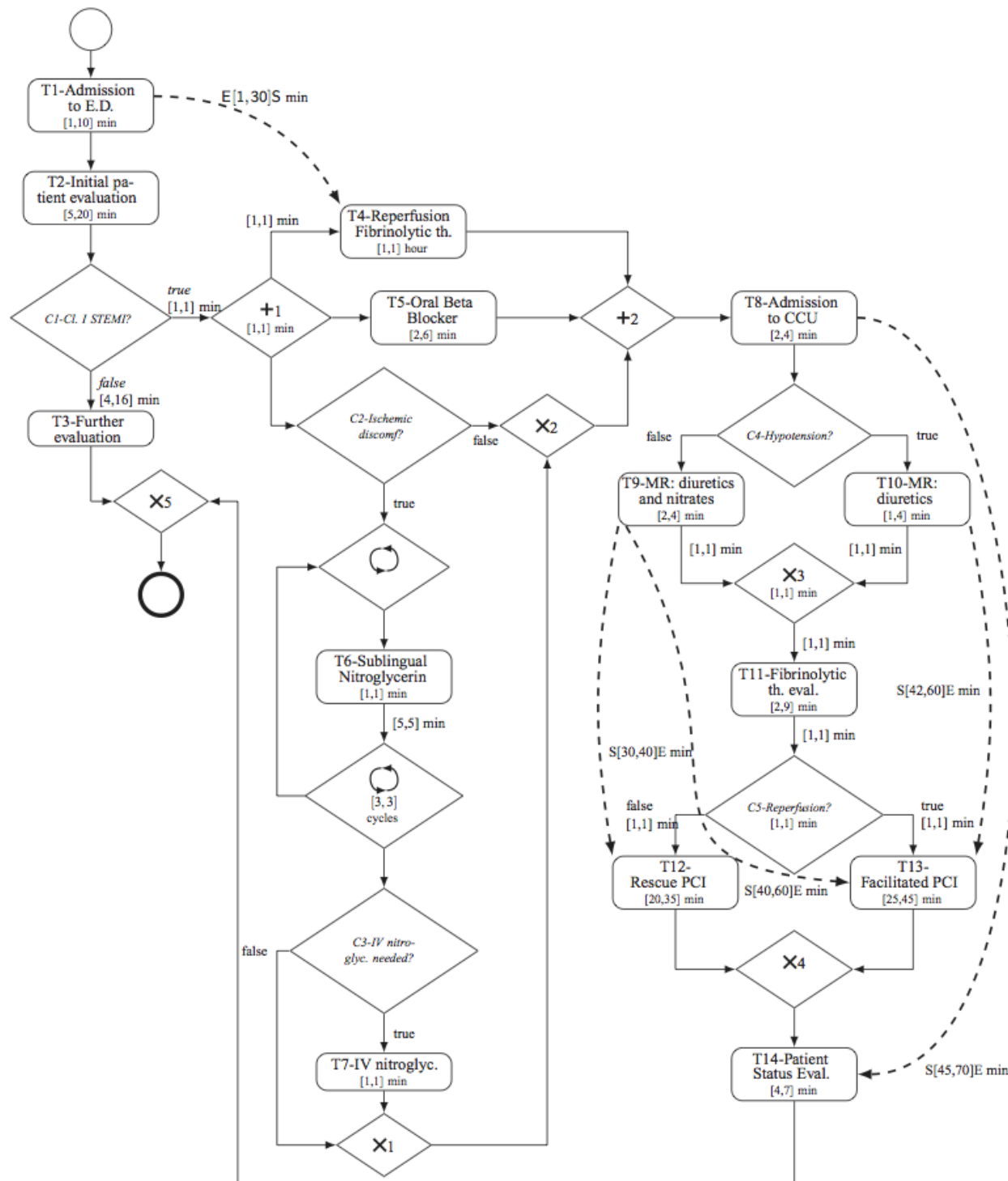


Fig. 5. Workflow graph example of patient admission to an hospital. The dashed edges represent relative constraints. It is worth noting that the last Or-join  $\times 5$  could be avoided, maintaining the same expressivity, if we allow more Ends, one for each alternative path

- 
- % Q4.1 % since the beginning of the admission to E.D. procedure, it takes at most 96 minutes for a patient to be admitted to the CCU

r\_clause(incorrect1, [reachable(s([begins(t1)],T1),s([completes(t8)],T2)), T2-T1>=97]).

- % Q4.2 % since the beginning of T8, it takes at least 35 minutes to complete T4

% Therefore, it satisfies (the lowerbound of) the constraint S\_T8[45,70]E\_T14

r\_clause(incorrect2, [reachable(s([begins(t8)],T1),s([completes(t14)],T2)), T2-T1=<=34]).

- % Q4.3 - FALSE

% since the beginning of T10, it takes at most 60 minutes to complete T13

% Therefore, it violates (the upperbound of) the constraint S\_T10[42,60]E\_T13

r\_clause(incorrect3, [reachable(s([begins(t10)],T1),s([completes(t13)],T2)), T2-T1>=61]).



# Unfold/Fold program specialization

incorrect :- initConf(C), reach(C,C1), **errorConf(C1)**.

- **UNFOLDING** (replace initConf(C) with the body of its definition)

incorrect :- X>=1, Y>=1, reach(cf(cmd(3,ite(neq(x,y)),4,h), [(x,X),(y,Y)],[]), C1),  
errorConf(C1).

- **UNFOLDING** (wrt errorConf(C1) )

incorrect :- X>=1, Y>=1, X1=<-1, reach(cf(cmd(3,ite(neq(x,y)),4,h),[(x,X),  
(y,Y)],[]),  
cf(cmd(h,halt),[(x,X1),(y,Y1)],[]))).

- **DEFINITION-INTRODUCTION** (with constraint generalization)

new3(X,Y, X1,Y1) :- reach(cf(cmd(3,ite(neq(x,y)),4,h),[(x,X),(y,Y)],[]),  
cf(cmd(h,halt),[(x,X1),(y,Y1)],[])).

- **FOLDING** (replace an instance of the body of a definition by its head)

incorrect :- X>=1, Y>=1, X1=<-1, new3(X,Y, X1,Y1).