Just a few sentences...

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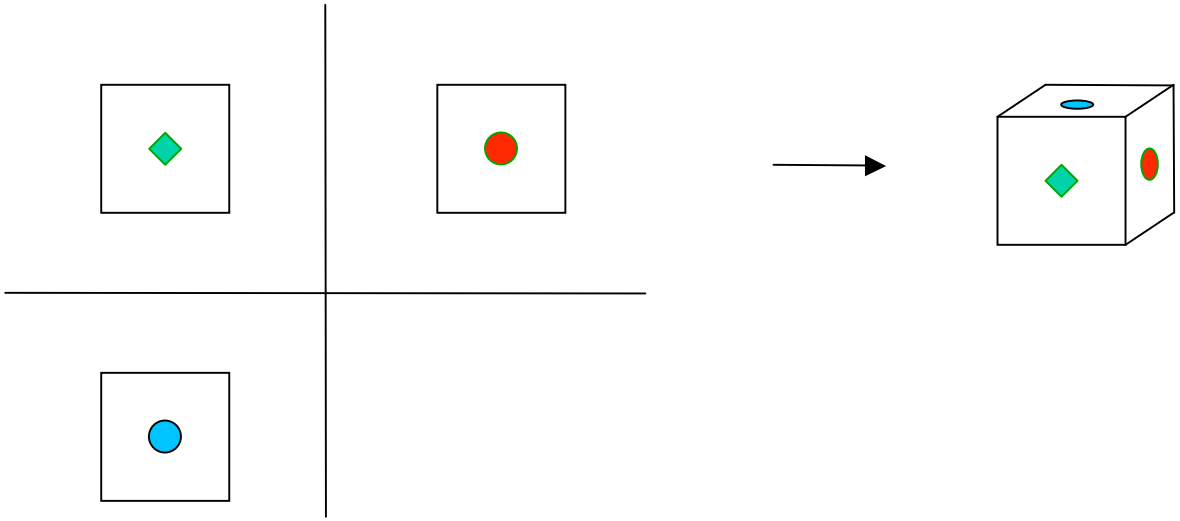
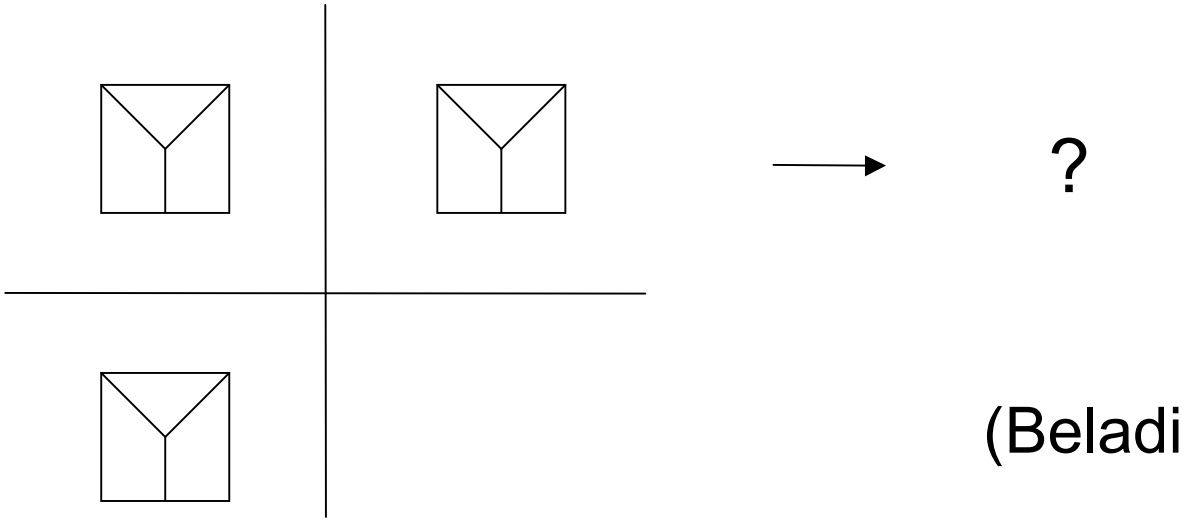
How is the modified cube ?





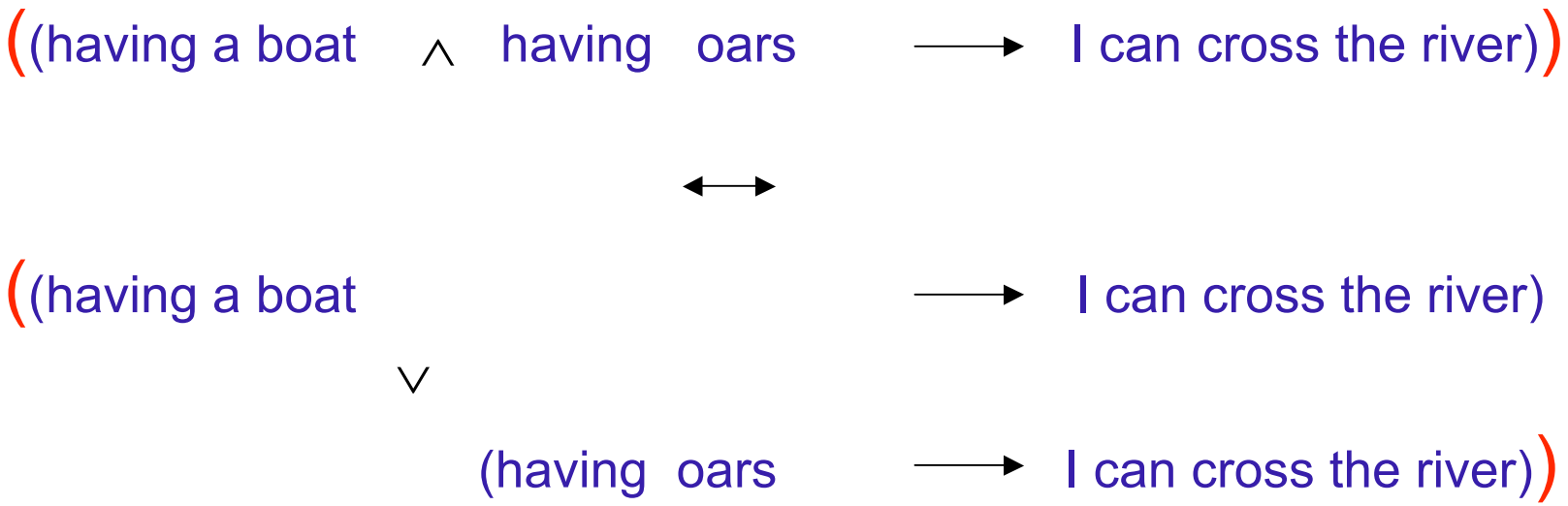
?

(Beladi)

As time goes on...

“ ... a computer scientist becomes
a logician and then
a philosopher ”
(Luca Cardelli)

Modus Ponens, Implication



"... because something funny happens to the left of the arrow"

(John Reynolds)

Generalization

```
gcd(M,M,M).
gcd(M,N,D) :- M>N,  gcd(M-N, N, D).
gcd(M,N,D) :- N>M,  gcd(M, N-M, D).
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Euclid invariant: if M>N then GCD(M,N) = GCD(M-N, M)

if N>M then GCD(M,N) = GCD(M, N-M)

Generalized Euclid invariant:

if M>N then AllComDivs(M,N) = AllComDivs(M-N, M)

if N>M then AllComDivs(M,N) = AllComDivs(M, N-M)

k D = M and h D = N iff (k-h) D = M-N

"... No matter what you study there is always a Greek person involved" (Kostas Stathis)

"... harder theorems may have simpler proofs" (Gordon Plotkin)

Generalization

MORE GCD FORMULAS

1. Marcelo Polezzi

$$GCD(M,N) = 2 \sum_{k=1,\ldots,M-1} \lfloor N / M \rfloor + M + N - M\,N$$

2. Donald Knuth

$$GCD(2^{GCD(M,N)} - 1) = GCD(2^{M} - 1,\ \ 2^{N} - 1)$$

How many deduction rules?

ONE RULE (for machines):

Resolution (Robinson, 1965)

MANY RULES (for humans):

Natural Deduction (Gentzen, 1935)

"... Everybody knew about resolution at that time,
I just wrote it down, together with the unification algorithm."
(Alan Robinson)

What is a proof ?

Proof of the 4-color conjecture by exhaustion:

- after 1000 hours of Fortran program computation examining 1,936 “reduced maps” (Appel & Haken. 1976)

- after a few hours of a general purpose theorem prover and proof checker (Robertson & Sanders & Seymour & Thomas. 2005)

Is the compiler correct ? Is the hardware infallible ?

“ We know only a small fractions of the theorems of Mathematics.
Has $x^n - y^m = 1$ for integers x and y, and positive numbers n and m, others solutions besides $3^2 - 2^3 = 1$?

(Maurice Nivat)

Deduction is limited (1)

- because of the Language:

the solution of the n-body problem cannot be expressed in closed form (Henry Poincaré, 1908)

- because of the Theory:

Incompleteness Theorem of Peano Arithmetics (Kurt Gödel, 1936)

Deduction is limited (in practice) (2)

Computable functions may be very hard to compute.

"... For humans is easy to recognize the face of a friend in a crowd, while for a computer is not."

(Andzrej Skowron)

"... computer science is an experimental science"

(Robin Milner)

Is Mathematical Intuition "Geometrical" ? (*1*)

- Logic Frege (1880), Russell (1900), etc.
- Formal Language (finitary) Hilbert (1900), etc.
- Geometry

"For any given property P(_), there exists a minimal natural number n such that P(n) holds"

is equivalent to Complete Induction

"For any given property P(_), $(\forall n.(\forall k<n.\ P(k)) \rightarrow P(n)\ ) \rightarrow \forall\ n.\ P(n)$"

$\neg\neg p \leftrightarrow p$ $\neg\forall x\ p \leftrightarrow \exists x\ \neg p$ $\neg(p \vee q) \leftrightarrow (\neg p \wedge \neg q)$

Is Mathematical Intuition "Geometrical" ? (*2*)

"... we use symmetries for understanding"

(Giuseppe Longo)

Is Mathematical Intuition "Algorithmic" ?

Brain vs. Mind Problem.

Is Gödel Incompleteness Theorem of PA relevant to the problem ?

yes: Penrose, Searle, ... no: Martin Davis, ...

Gödel (in the Gibbs Lecture,1951)

> " It migth exist a theorem prover machine which is equivalent to mathematical intuition, but you *cannot* prove to be so, nor even be proved to yield only correct theorems of finitary number theory".

"... I wrote only one Prolog program. It was for computing the transitive closure. It did not terminate!"

(Martin Davis)

How to Cope with Circularity ? (1)

$x = -y+2$
$y = x+1$

$x=1/2$
$y=3/2$

invent the Rationals (+,-,×,/)

$fact(n) = \text{if } n=0 \text{ then } 1 \text{ else } n \times fact(n-1)$

invent the well-founded sets

$x = \{y, a\}$
$y = \{x, b\}$

(anti-foundation axiom)
ask the Mayor, while in Udine

$N = \{0\} + \{s\}\times N$
$B = N + B\times N\times B$
$X = X \to X$

invent Scott's domains

How to Cope with Circularity ? (2)

impredicative definitions:

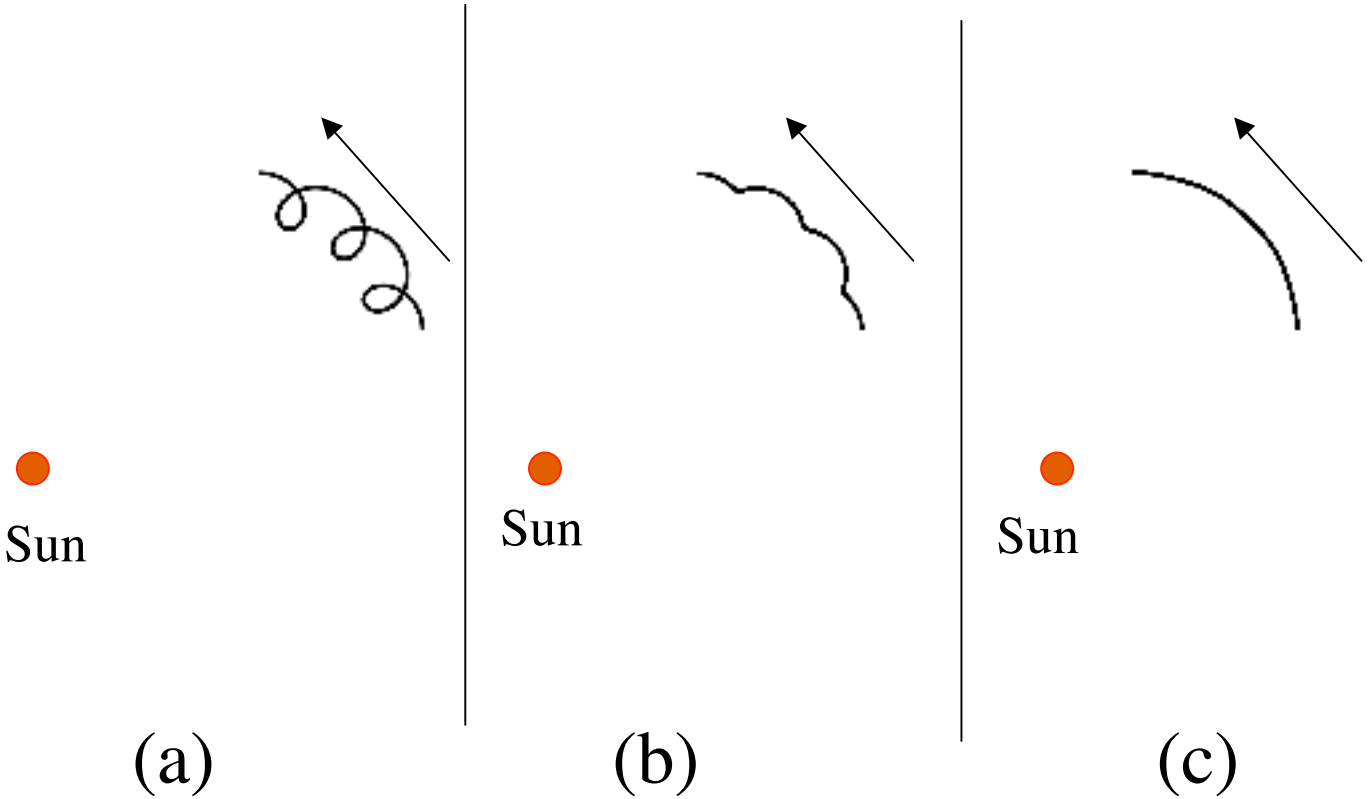
$$N = \bigcap \{ Y \mid N \subseteq Y \text{ and } 0 \in Y \text{ and } \forall y\, (y \in Y \rightarrow s(y) \in Y) \}$$

invent PER categories

"... you have to tie a knot" (Gordon Plotkin)

What is ~~Bart~~ the Moon Movement like ?

What is the Moon Movement like ? (1)

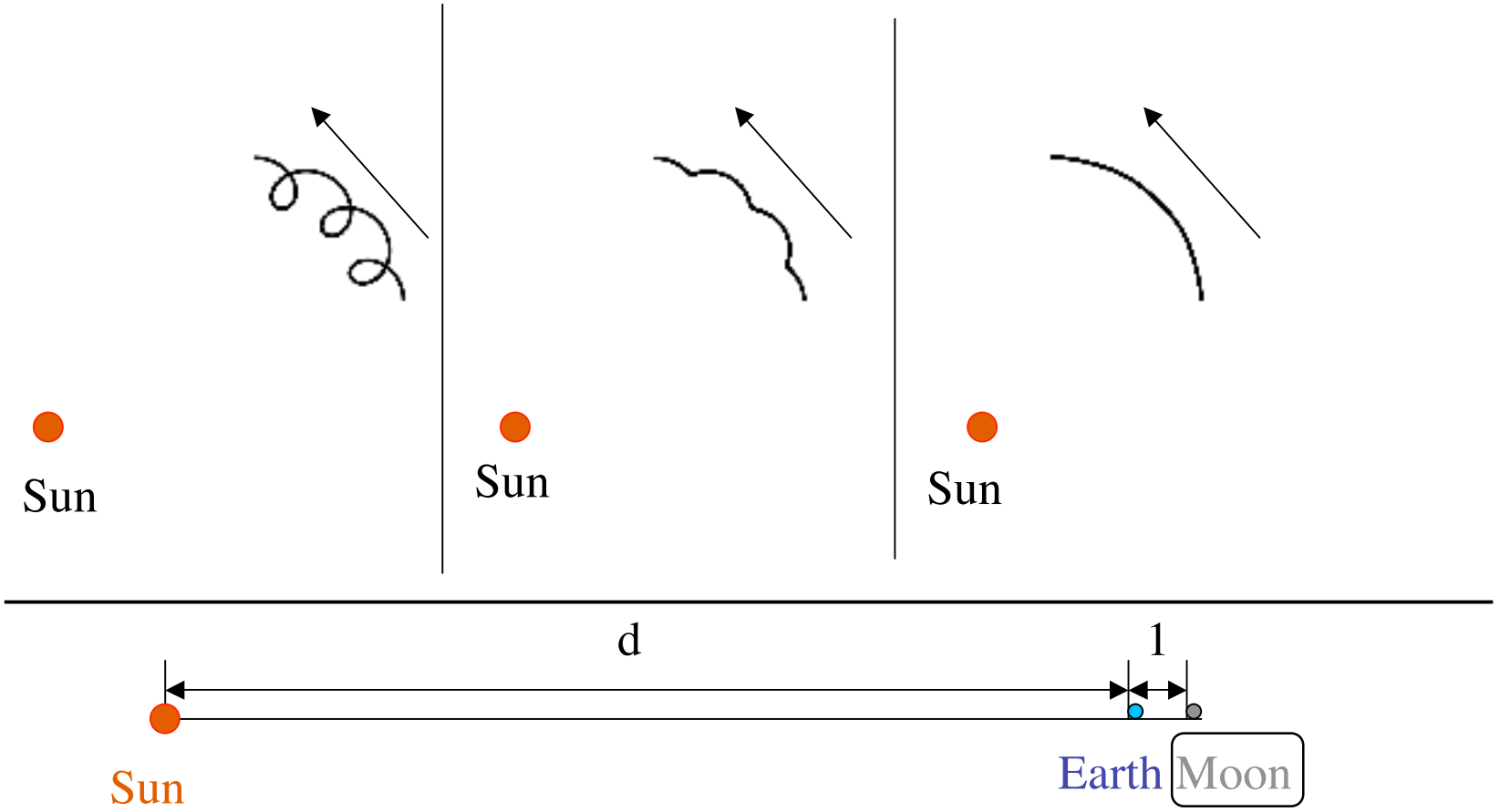


"... be quick, because the Earth is moving"

(Robert Kowalski

preparing his telescope for me to look at Saturn)

What is the Moon Movement like ? (2)

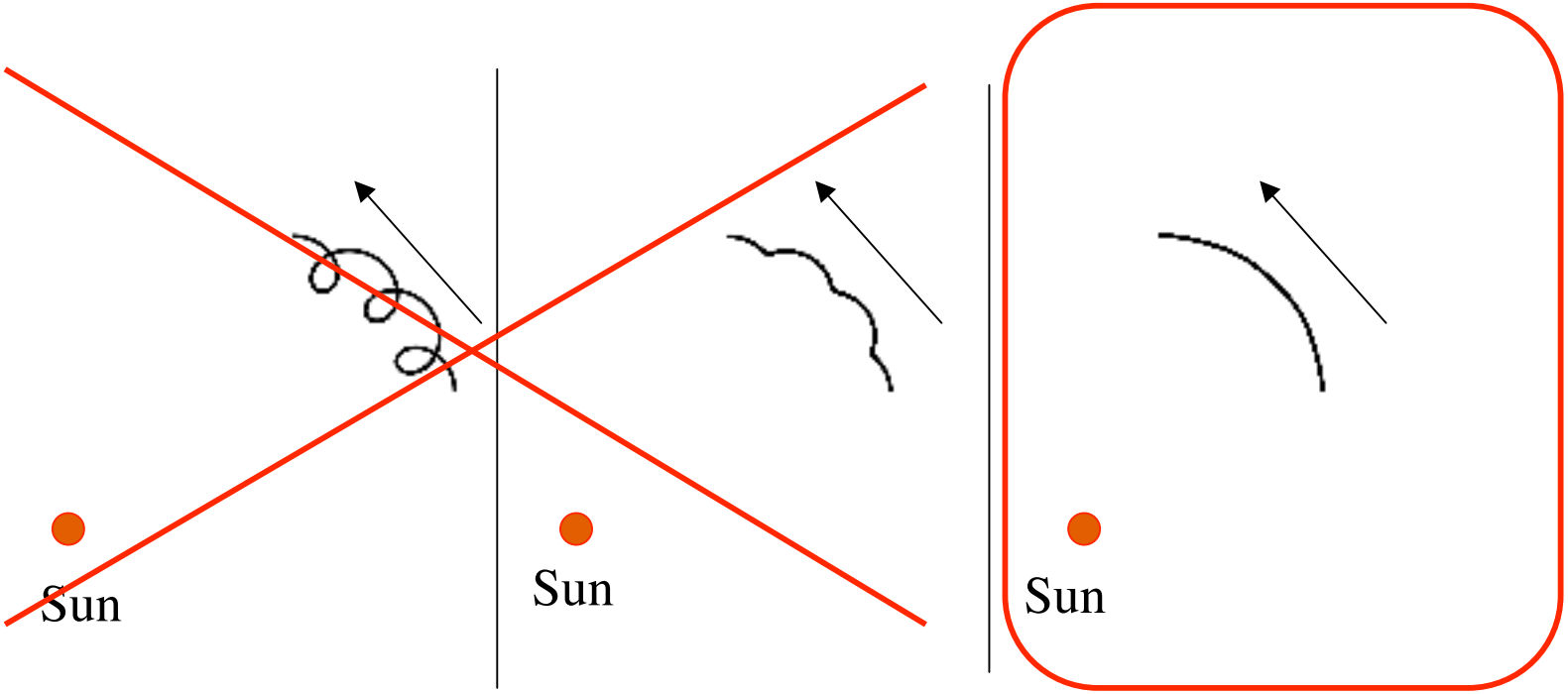


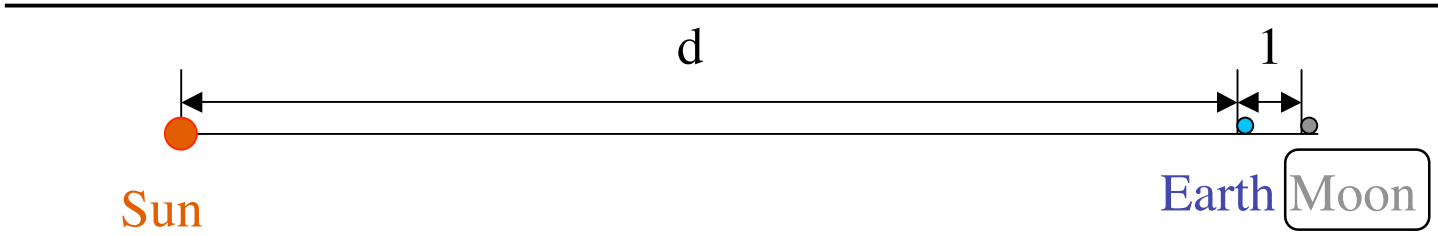
$x(t) = d \cos(t) + 1 \cos(m\,t)$

$y(t) = d \sin(t) + 1 \sin(m\,t)$

$d = 400, \quad m = 13$

What is the Moon Movement like ? (3)



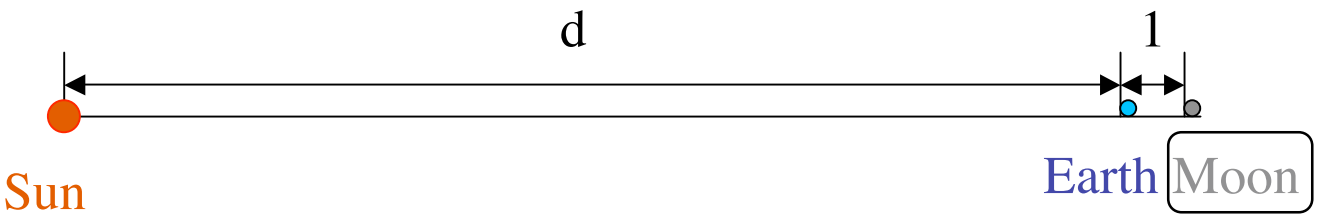


concavity of Moon orbit is always towards the Sun,

i.e., Moon's steering wheel is always turned to the left !

How can the Moon turn around the Earth ?

How big is our Solar System?



Assume d = 10 meters.
How many kilometers away is the nearest star (Proxima Centaury) (4.3 lightyears)?

(a): 100 Km
(b): 1000 Km
(c): more than 2700 Km

Milky Way galaxy: 100,000 lightyears long
20,000 lightyears thick
... and there are “at least” 10^9 galaxies.

As time goes on...

" ... a computer scientist becomes
a logician and then
a philosopher
and finally
an astronomer "

"Dimmi che fai tu Luna in ciel,
dimmi che fai, o graziosa Luna,..." (Giacomo Leopardi)

"Tell me what you are doing in the sky,
tell me, graceful Moon, what you are doing,..."