Bin Packing with Conflicts on interval graphs: some computational results

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Contents

1 Description of the data set

In the present chapter we discuss the results obtained by solving thousands of instances with our heuristic algorithm BN proposed in Bacci and Nicoloso (2017) and other exact approaches.

The test bed was generated as we now describe.

By $TI(n, B, \Delta)$ we denote a set of 100 randomly generated instances of $BPPC$ with n items, weights uniformly distributed in $[20, 100]$ (as in Falkenauer (1996)), bound B, and interval conflict graph with expected edge density Δ. When $\Delta > 0$ we repeatedly run the random interval graph generator described in Bacci and Nicoloso (2017) and we selected 100 sets of n intervals whose intersection graph had edge density $\delta \in [\Delta - 0.02; \Delta + 0.02]$. When $\Delta = 0$ we defined the set $\mathcal{I} = \{I_h = (h, h+1), h = 0, \ldots, n-1\}$ of n mutually non-intersecting intervals (in this case $BPPC$ reduces to BP). In particular, we chose $n \in \{120, 250, 500, 1000\}$, $B \in \{120, 150, 180, 210, 240, 270, 300, 330, 360, 390\}$, and $\Delta \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$. Totally we built 40000 instances of type TI.

By $TT(n, B, \Delta)$ we denote a set of 100 randomly generated instances of $BPPC$ with n items, weights uniformly distributed in $[20, 100]$ (as in Falkenauer (1996)), bound B, and threshold conflict graph with expected edge density Δ. When $0 < \Delta \leq 0.5$ we run the generator described in Gendreau et al. (2004) with $d = \sqrt{\Delta/2}$ and when $\Delta \geq 0.5$ with $d = 1 - \sqrt{(1-\Delta)/2}$. In both cases we selected the graphs with edge density $\delta \in [\Delta - 0.02; \Delta + 0.02]$. When $\Delta = 0$ we defined the set $\mathcal{I} = \{I_h = (h, h+1), h = 0, \ldots, n-1\}$ of n mutually non-intersecting intervals. In particular, we chose $n \in \{120, 250, 500, 1000\}$, $B \in \{120, 150, 180, 210, 240, 270, 300, 330, 360, 390\}$, and $\Delta \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$. We remark that, given $n = \overline{n}$, the weight of the i-th item of the k-th instance of $TT(\overline{n}, B, \Delta)$ is the same for all B and Δ and is exactly the weight of the i-th item of the k-th instance of $TI(\overline{n}, B, \Delta)$, for $k = 1, \ldots, 100$ and $i = 1, \ldots, \overline{n}$. Totally we built 40000 instances of type TT.

By $TM(n, B, f(d))$ we denote a set of ten instances with n items, bound B, and threshold conflict graph with density $f(d)$. In particular $n \in \{120, 250, 500, 1000\}$, and $d \in \{0, 0.1, \ldots, 0.9\}$. The weights and the conflict graphs of all the $TM(n, B, f(d))$ are exactly those in the classes 1,2,3,4 by Fernandes Muritiba et al. (2010). As for B, we considered $B \in \{120, 150, \ldots, 390\} \cup \{400\}$, even if in the cited paper only $B = 150$ is considered. In particular, Fernandes Muritiba et al. (2010) select the first 10 instances of the 20 originally proposed by Falkenauer (1996) for the Bin Packing (without conflicts), and add 10 random threshold conflict graphs generated by means of the generator described in Gendreau et al. (2004), varying d from 0 to 0.9. We recall that the expected edge density δ of the conflict graphs generated in this way is not d as claimed by Fernandes Muritiba et al. (2010).

In order to verify how much the item weights affect the quality of the solution and/or the computing time, we also decided to construct the TS instances: by $TS(n, B, f(d))$ we will denote a set of ten instances with n items, bound B, and threshold conflict graph with density $f(d)$. The conflict graphs of a $TS(n, \cdot, f(d))$ are those of $TM(n, \cdot, f(d))$, and the weights are uniformly distributed in $[500, 2500]$. We choose $B \in \{3000, 3750, \ldots, 9750\} \cup \{10000\}$. We remark that the item weights of $TS(n, B, f(d))$ are generated as the "instances with a larger number of items per bin" by Sadykov and Vanderbeck (2013) (the so-called "d instances"), where, however, only $B = 10000$ is considered.

2 Heuristic algorithms

We compare the computational results obtained by applying the algorithm BN described in Bacci and Nicoloso (2017) and an adaptation to $BPPC$ of the classical heuristic algorithms First-Fit Decreasing, Best-Fit Decreasing, Worst-Fit Decreasing for the classical Bin Packing (Johnson (1974)), as described in Fernandes Muritiba et al. (2010). In particular, these adaptations, $U_{FF(\alpha)}$, $U_{BF(\alpha)}$, and $U_{WF(\alpha)}$ (we shall call them algorithms M), consider an extended conflict graph G_w, obtained by adding to G an edge for each pair of vertices i, j with $w_i + w_j > B$, and consider vertex weights w_i^s defined as follows: $w_i^s = \alpha(w_i/\overline{w}) + (1-\alpha)(\deg(i)/\overline{\deg})$, for $i = 1, 2, \ldots, n$, where $\alpha \in \{0, 0.1, \ldots, 1\}$, $\deg(i)$ is the degree of vertex i in G_w, and $\overline{w}$ and $\overline{\deg}$ are the average weight of the vertices and their average degree in G_w, respectively.

Let S be an instance of $BPPC$ with an interval conflict graph G. Given $\alpha \in \{0, 0.1, \ldots, 1\}$, let $u_{x(\alpha)}(S)$ be the value of the solution output by algorithm $U_{x(\alpha)}$ on S, for $x \in \{FF, BF, WF\}$. By $u^M(S) = \min\{u_{x(\alpha)}(S),\ \ \alpha \in \{0, 0.1, \ldots, 1\}, x \in \{FF, BF, WF\}\}$ we denote the minimum among all the 33 values of the (feasible) solutions output by algorithms M on S. By $u^{BN}(S)$ we denote the value of the (feasible) solution output by algorithm BN on S.

To evaluate the performances of the algorithms we define $LB_{BPPC}(S) = \max\{\lceil \sum_{i \in V} w_i/B \rceil; \chi(G)\}$, a lower bound on the value of an optimum solution of $BPPC$ on instance S.

In each table rows are indexed by Δ and columns by B. In each cell there are six values, each one averaged over the corresponding 100 instances:

- M=LB (BN=LB, respectively) is the percentage of instances S where $u^M(S) = LB_{BPPC}(S)$ ($u^{BN}(S) = LB_{BPPC}(S)$, respectively), i.e. the percentage of instances where $LB_{BPPC}(S)$ allows to certify that the corresponding algorithm found an optimum solution;
- M<BN (BN<M, respectively) is the percentage of instances S where $u^M(S) < u^{BN}(S)$ ($u^{BN}(S) < u^M(S)$, respectively) (notice that the complement to 100% of the sum of the last two values is the percentage of instances where $u^M(S) = u^{BN}(S)$);
- Gap_M (Gap_BN, respectively) is the gap $\frac{u^M(S)-LB_{BPPC}(S)}{LB_{BPPC}(S)}$ ($\frac{u^{BN}(S)-LB_{BPPC}(S)}{LB_{BPPC}(S)}$, respectively). A light grey indicates the algorithm which outperforms the other one w.r.t. the corresponding data. If in a cell the value X=LB is 100%, then all the data of algorithm X are colored in light cyan, for X $\in$ {M,BN}.

The light cyan cells means that an algorithm solves to optimality all the 100 instances. Algorithms BN and M were coded in C++ and ran on an Intel Xeon E5620 2.40GHz with 40 GB RAM under a Linux operating system.

In the following, the computational results of the heuristic procedures for different values of n are shown. Click here to view computational results on TI's instances, here for the ones on the TM's instances, here for the TS's and here for the TT's.

			B									
			120	150	180	210	240	270	300	330	360	390
Δ	0	M=LB	3%	20%	40%	0%	2%	20%	23%	41%	49%	57%
		M<BN	98%	74%	26%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	5.00%	1.80%	1.49%	4.34%	3.56%	3.03%	3.18%	2.69%	2.53%	2.30%
		BN=LB	0%	0%	21%	43%	59%	80%	74%	85%	87%	88%
		BN<M	0%	0%	4%	87%	66%	62%	51%	44%	38%	31%
		Gap_BN	10.02%	4.36%	2.03%	1.65%	1.35%	0.73%	1.09%	0.70%	0.66%	0.64%
	0.1	M=LB	1%	8%	18%	22%	19%	47%	35%	38%	37%	39%
		M<BN	100%	83%	20%	9%	5%	0%	2%	2%	0%	1%
		Gap_M	5.13%	2.07%	2.16%	2.30%	2.71%	2.01%	2.78%	2.83%	3.16%	3.26%
		BN=LB	0%	0%	13%	25%	43%	73%	63%	75%	81%	80%
		BN<M	0%	1%	9%	11%	30%	27%	32%	39%	45%	42%
		Gap_BN	11.02%	5.20%	2.45%	2.23%	1.88%	1.00%	1.55%	1.16%	0.95%	1.07%
	0.2	M=LB	1%	2%	1%	3%	2%	3%	10%	19%	35%	57%
		M<BN	99%	81%	20%	9%	2%	1%	2%	1%	1%	0%
		Gap_M	5.15%	2.70%	3.40%	3.80%	4.66%	4.58%	4.82%	4.58%	3.55%	2.40%
		BN=LB	0%	0%	8%	14%	21%	39%	28%	53%	65%	85%
		BN<M	0%	1%	30%	40%	57%	60%	46%	51%	39%	31%
		Gap_BN	11.13%	5.85%	3.14%	2.88%	2.70%	2.29%	2.97%	2.13%	1.73%	0.78%
	0.3	M=LB	0%	1%	0%	0%	10%	36%	57%	63%	65%	66%
		M<BN	100%	70%	11%	9%	2%	4%	1%	0%	0%	0%
		Gap_M	5.37%	4.01%	5.11%	5.62%	5.63%	3.73%	2.26%	1.89%	1.74%	1.71%
		BN=LB	0%	0%	1%	1%	15%	51%	89%	99%	100%	100%
		BN<M	0%	6%	39%	42%	53%	42%	41%	37%	35%	34%
		Gap_BN	11.79%	6.60%	4.28%	4.62%	3.72%	2.12%	0.46%	0.04%	0.00%	0.00%
	0.4	M=LB	0%	0%	2%	33%	60%	71%	74%	75%	75%	75%
		M<BN	100%	73%	41%	31%	1%	0%	0%	0%	0%	0%
		Gap_M	5.83%	5.91%	6.23%	3.35%	1.61%	1.03%	0.91%	0.82%	0.82%	0.82%
		BN=LB	0%	0%	1%	28%	85%	99%	100%	100%	100%	100%
		BN<M	0%	11%	23%	23%	32%	29%	26%	25%	25%	25%
		Gap_BN	13.97%	8.95%	6.90%	3.67%	0.59%	0.03%	0.00%	0.00%	0.00%	0.00%
	0.5	M=LB	0%	1%	28%	67%	78%	79%	80%	80%	80%	80%
		M<BN	100%	82%	42%	4%	0%	0%	0%	0%	0%	0%
		Gap_M	6.98%	7.38%	3.07%	0.94%	0.57%	0.52%	0.52%	0.52%	0.52%	0.52%
		BN=LB	0%	0%	24%	88%	99%	100%	100%	100%	100%	100%
		BN<M	0%	2%	18%	29%	21%	21%	20%	20%	20%	20%
		Gap_BN	16.06%	11.81%	3.67%	0.33%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.6	M=LB	0%	10%	65%	93%	93%	93%	93%	93%	93%	93%
		M<BN	100%	79%	9%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	9.58%	4.29%	0.81%	0.14%	0.14%	0.14%	0.14%	0.14%	0.14%	0.14%
		BN=LB	0%	2%	73%	99%	100%	100%	100%	100%	100%	100%
		BN<M	0%	4%	14%	6%	7%	7%	7%	7%	7%	7%
		Gap_BN	17.92%	7.95%	0.69%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.7	M=LB	0%	57%	92%	99%	100%	100%	100%	100%	100%	100%
		M<BN	100%	85%	7%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	12.25%	0.88%	0.13%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	6%	91%	99%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	5%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	18.70%	3.81%	0.18%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.8	M=LB	1%	68%	96%	100%	100%	100%	100%	100%	100%	100%
		M<BN	92%	53%	3%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	8.22%	0.69%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	33%	94%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	1%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	11.36%	1.69%	0.08%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.9	M=LB	0%	63%	99%	100%	100%	100%	100%	100%	100%	100%
		M<BN	72%	31%	2%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	4.87%	0.48%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	47%	97%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	6.15%	0.91%	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		tMin_BN	0.0025	0.0008	0.0008	0.0008	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007
		tMax_BN	0.0176	0.0101	0.0064	0.0054	0.0042	0.0037	0.0031	0.0030	0.0026	0.0027
		tavg_BN	0.0060	0.0037	0.0024	0.0018	0.0015	0.0013	0.0012	0.0011	0.0010	0.0010

Table 1: Computational results obtained by algorithms M and algorithm BN on $TI(120, B, \Delta)$

			B									
			120	150	180	210	240	270	300	330	360	390
Δ	0	M=LB	0%	0%	11%	0%	0%	0%	0%	3%	11%	20%
		M<BN	100%	70%	16%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.87%	1.56%	1.18%	3.67%	3.23%	2.86%	2.59%	2.29%	2.13%	2.05%
		BN=LB	0%	0%	12%	35%	61%	75%	64%	88%	88%	86%
		BN<M	0%	1%	13%	100%	100%	100%	89%	91%	78%	66%
		Gap_BN	9.12%	2.92%	1.22%	0.90%	0.61%	0.45%	0.72%	0.26%	0.29%	0.36%
	0.1	M=LB	0%	0%	2%	0%	7%	3%	1%	4%	2%	7%
		M<BN	100%	83%	21%	2%	1%	0%	0%	0%	0%	0%
		Gap_M	3.88%	1.64%	1.56%	1.95%	1.80%	2.04%	2.43%	2.43%	2.87%	2.79%
		BN=LB	0%	0%	5%	19%	52%	64%	53%	80%	80%	81%
		BN<M	0%	2%	14%	51%	63%	76%	73%	84%	88%	81%
		Gap_BN	9.94%	3.41%	1.62%	1.26%	0.76%	0.64%	0.94%	0.44%	0.47%	0.49%
	0.2	M=LB	0%	0%	0%	0%	0%	0%	0%	0%	1%	8%
		M<BN	100%	78%	8%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.95%	2.25%	2.60%	3.22%	4.21%	4.82%	5.22%	5.83%	5.97%	5.06%
		BN=LB	0%	0%	0%	3%	24%	30%	26%	38%	23%	53%
		BN<M	0%	5%	44%	84%	99%	100%	100%	98%	95%	92%
		Gap_BN	9.93%	3.88%	2.08%	1.71%	1.28%	1.25%	1.51%	1.36%	1.85%	1.20%
	0.3	M=LB	0%	0%	0%	0%	0%	7%	16%	25%	28%	31%
		M<BN	100%	63%	2%	1%	0%	1%	2%	0%	0%	0%
		Gap_M	4.03%	3.50%	4.50%	5.30%	6.20%	5.48%	3.67%	2.85%	2.60%	2.50%
		BN=LB	0%	0%	0%	0%	0%	11%	66%	96%	100%	100%
		BN<M	0%	12%	79%	92%	95%	86%	79%	75%	72%	69%
		Gap_BN	10.21%	4.84%	2.98%	2.85%	3.01%	2.69%	1.02%	0.10%	0.00%	0.00%
	0.4	M=LB	0%	0%	0%	2%	17%	31%	35%	38%	37%	37%
		M<BN	100%	70%	19%	19%	4%	0%	0%	0%	0%	0%
		Gap_M	4.24%	5.03%	5.92%	4.73%	2.43%	1.70%	1.67%	1.60%	1.58%	1.58%
		BN=LB	0%	0%	0%	6%	65%	100%	100%	100%	100%	100%
		BN<M	0%	21%	48%	45%	73%	69%	65%	62%	63%	63%
		Gap_BN	11.76%	6.58%	5.35%	4.20%	0.77%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.5	M=LB	0%	0%	5%	31%	48%	47%	48%	48%	48%	48%
		M<BN	100%	90%	51%	7%	0%	0%	0%	0%	0%	0%
		Gap_M	4.83%	6.42%	3.97%	1.13%	0.80%	0.79%	0.79%	0.79%	0.79%	0.79%
		BN=LB	0%	0%	2%	75%	99%	100%	100%	100%	100%	100%
		BN<M	0%	3%	22%	55%	51%	53%	52%	52%	52%	52%
		Gap_BN	13.65%	9.48%	4.54%	0.41%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.6	M=LB	0%	5%	43%	75%	79%	80%	80%	80%	80%	80%
		M<BN	100%	93%	32%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	6.47%	5.00%	0.81%	0.27%	0.23%	0.22%	0.22%	0.22%	0.22%	0.22%
		BN=LB	0%	0%	37%	99%	100%	100%	100%	100%	100%	100%
		BN<M	0%	2%	24%	24%	21%	20%	20%	20%	20%	20%
		Gap_BN	16.28%	8.75%	0.95%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.7	M=LB	0%	22%	87%	99%	99%	99%	99%	99%	99%	99%
		M<BN	100%	93%	14%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	10.68%	1.02%	0.10%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%
		BN=LB	0%	0%	83%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	3%	10%	1%	1%	1%	1%	1%	1%	1%
		Gap_BN	18.29%	3.89%	0.14%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.8	M=LB	0%	44%	95%	100%	100%	100%	100%	100%	100%	100%
		M<BN	100%	83%	6%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	7.69%	0.55%	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	6%	91%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	4%	2%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	11.47%	1.89%	0.06%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.9	M=LB	0%	52%	95%	100%	100%	100%	100%	100%	100%	100%
		M<BN	97%	67%	4%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	4.46%	0.34%	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	20%	93%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	3%	2%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	6.31%	1.00%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		tMin_BN	0.0117	0.0038	0.0034	0.0033	0.0031	0.0030	0.0030	0.0029	0.0029	0.0029
		tMax_BN	0.1201	0.0686	0.0498	0.0365	0.0246	0.0195	0.0235	0.0158	0.0137	0.0124
		tavg_BN	0.0433	0.0234	0.0143	0.0101	0.0076	0.0064	0.0058	0.0051	0.0047	0.0045

Table 2: Computational results obtained by algorithms M and algorithm BN on $TI(250, B, \Delta)$

			B									
			120	150	180	210	240	270	300	330	360	390
Δ	0	M=LB	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		M<BN	100%	56%	3%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	2.42%	1.36%	1.01%	3.44%	3.15%	2.72%	2.48%	2.18%	2.07%	1.86%
		BN=LB	0%	0%	2%	32%	57%	69%	73%	85%	87%	87%
		BN<M	0%	6%	48%	100%	100%	100%	100%	100%	100%	100%
		Gap_BN	8.50%	1.81%	0.72%	0.47%	0.34%	0.28%	0.27%	0.16%	0.16%	0.17%
	0.1	M=LB	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		M<BN	100%	66%	6%	0%	1%	0%	0%	0%	0%	0%
		Gap_M	2.44%	1.51%	1.25%	1.81%	1.51%	1.71%	1.99%	2.14%	2.34%	2.70%
		BN=LB	0%	0%	0%	11%	37%	59%	66%	74%	79%	81%
		BN<M	0%	6%	53%	99%	93%	99%	100%	98%	100%	100%
		Gap_BN	8.65%	2.21%	0.92%	0.69%	0.52%	0.37%	0.34%	0.28%	0.25%	0.25%
	0.2	M=LB	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%
		M<BN	100%	60%	3%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	2.49%	1.99%	2.14%	2.93%	3.84%	4.66%	5.20%	6.27%	6.46%	6.21%
		BN=LB	0%	0%	0%	1%	10%	19%	27%	16%	21%	18%
		BN<M	0%	16%	85%	100%	100%	100%	100%	100%	100%	99%
		Gap_BN	8.87%	2.59%	1.21%	0.99%	0.77%	0.77%	0.74%	0.93%	0.97%	1.15%
	0.3	M=LB	0%	0%	0%	0%	0%	0%	2%	4%	4%	4%
		M<BN	100%	41%	0%	0%	0%	0%	0%	1%	0%	0%
		Gap_M	2.54%	3.17%	4.17%	5.11%	6.52%	5.99%	4.43%	3.83%	3.69%	3.66%
		BN=LB	0%	0%	0%	0%	0%	1%	40%	93%	100%	100%
		BN<M	0%	39%	100%	100%	100%	100%	97%	96%	96%	96%
		Gap_BN	8.97%	3.24%	2.02%	1.89%	1.97%	2.20%	1.30%	0.08%	0.00%	0.00%
	0.4	M=LB	0%	0%	0%	0%	5%	6%	7%	7%	8%	8%
		M<BN	100%	40%	3%	14%	1%	0%	0%	0%	0%	0%
		Gap_M	2.65%	4.80%	5.62%	5.06%	3.28%	2.52%	2.41%	2.38%	2.36%	2.35%
		BN=LB	0%	0%	0%	0%	29%	91%	99%	100%	100%	100%
		BN<M	0%	40%	95%	65%	91%	94%	93%	93%	92%	92%
		Gap_BN	9.61%	4.75%	3.78%	4.21%	1.50%	0.09%	0.01%	0.00%	0.00%	0.00%
	0.5	M=LB	0%	0%	1%	8%	12%	13%	13%	13%	13%	13%
		M<BN	100%	82%	73%	4%	0%	0%	0%	0%	0%	0%
		Gap_M	2.95%	5.60%	4.04%	1.31%	1.18%	1.12%	1.10%	1.09%	1.09%	1.09%
		BN=LB	0%	0%	0%	51%	100%	100%	100%	100%	100%	100%
		BN<M	0%	13%	18%	84%	88%	87%	87%	87%	87%	87%
		Gap_BN	11.83%	7.49%	4.90%	0.38%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.6	M=LB	0%	0%	18%	59%	61%	61%	61%	61%	61%	61%
		M<BN	100%	93%	40%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.88%	4.82%	0.66%	0.26%	0.24%	0.24%	0.24%	0.24%	0.24%	0.24%
		BN=LB	0%	0%	19%	98%	100%	100%	100%	100%	100%	100%
		BN<M	0%	5%	33%	41%	39%	39%	39%	39%	39%	39%
		Gap_BN	14.01%	7.89%	0.74%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.7	M=LB	0%	9%	72%	94%	96%	96%	96%	96%	96%	96%
		M<BN	100%	99%	15%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	7.88%	0.92%	0.13%	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%
		BN=LB	0%	0%	77%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	20%	6%	4%	4%	4%	4%	4%	4%
		Gap_BN	16.68%	3.10%	0.10%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.8	M=LB	0%	26%	87%	100%	100%	100%	100%	100%	100%	100%
		M<BN	100%	85%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	5.18%	0.44%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	3%	86%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	6%	9%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	9.69%	1.31%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.9	M=LB	0%	40%	94%	100%	100%	100%	100%	100%	100%	100%
		M<BN	100%	75%	3%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	2.93%	0.23%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	12%	94%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	7%	3%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	5.13%	0.74%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		tMin_BN	0.0575	0.0419	0.0139	0.0134	0.0129	0.0124	0.0123	0.0117	0.0117	0.0117
		tMax_BN	0.9465	0.5152	0.2794	0.1972	0.1313	0.1096	0.0954	0.0774	0.0672	0.0592
		tavg_BN	0.3018	0.1414	0.0810	0.0533	0.0380	0.0309	0.0263	0.0229	0.0208	0.0201

Table 3: Computational results obtained by algorithms M and algorithm BN on $TI(500, B, \Delta)$

Δ		B										
		120	150	180	210	240	270	300	330	360	390	400
0	M=LB	0%	20%	50%	0%	20%	50%	20%	30%	50%	80%	60%
	M<BN	100%	60%	30%	0%	0%	0%	0%	0%	0%	0%	0%
	Gap_M	3.96%	1.67%	1.26%	4.65%	2.63%	1.84%	3.32%	3.21%	2.50%	1.05%	2.25%
	BN=LB	0%	0%	40%	30%	90%	80%	80%	90%	100%	100%	90%
	BN<M	0%	0%	10%	90%	70%	30%	60%	60%	50%	20%	30%
	Gap_BN	10.17%	2.89%	1.73%	2.03%	0.33%	0.74%	0.83%	0.45%	0.00%	0.00%	0.59%
0.1	M=LB	0%	20%	50%	60%	100%	90%	70%	90%	100%	100%	90%
	M<BN	100%	80%	40%	50%	30%	10%	10%	10%	10%	10%	0%
	Gap_M	3.96%	1.67%	1.26%	1.18%	0.00%	0.37%	1.25%	0.45%	0.00%	0.00%	0.59%
	BN=LB	0%	10%	10%	20%	70%	80%	70%	80%	90%	90%	90%
	BN<M	0%	0%	0%	10%	0%	0%	10%	0%	0%	0%	0%
	Gap_BN	10.79%	4.34%	2.24%	2.31%	0.99%	0.73%	1.27%	0.91%	0.53%	0.53%	0.59%
0.2	M=LB	0%	20%	20%	40%	70%	90%	100%	100%	100%	100%	100%
	M<BN	100%	80%	20%	20%	0%	0%	10%	0%	0%	0%	0%
	Gap_M	3.96%	1.67%	1.98%	1.75%	0.99%	0.37%	0.00%	0.00%	0.00%	0.00%	0.00%
	BN=LB	0%	0%	20%	20%	80%	90%	90%	100%	100%	100%	100%
	BN<M	0%	0%	10%	0%	10%	0%	0%	0%	0%	0%	0%
	Gap_BN	10.12%	3.53%	2.23%	2.32%	0.67%	0.37%	0.42%	0.00%	0.00%	0.00%	0.00%
0.3	M=LB	0%	20%	10%	60%	70%	80%	90%	90%	90%	90%	90%
	M<BN	100%	100%	50%	0%	0%	0%	0%	0%	0%	0%	0%
	Gap_M	4.78%	1.88%	2.19%	1.34%	1.11%	0.79%	0.29%	0.29%	0.29%	0.29%	0.29%
	BN=LB	0%	0%	10%	60%	80%	80%	90%	90%	90%	90%	90%
	BN<M	0%	0%	0%	0%	10%	0%	0%	0%	0%	0%	0%
	Gap_BN	11.76%	6.64%	3.67%	1.34%	0.79%	0.79%	0.29%	0.29%	0.29%	0.29%	0.29%
0.4	M=LB	0%	10%	40%	60%	70%	70%	70%	70%	80%	100%	100%
	M<BN	100%	80%	10%	0%	10%	10%	0%	0%	0%	0%	0%
	Gap_M	7.58%	3.35%	2.10%	1.24%	0.62%	0.62%	0.62%	0.62%	0.43%	0.00%	0.00%
	BN=LB	0%	0%	50%	60%	70%	70%	70%	70%	80%	100%	100%
	BN<M	0%	0%	10%	0%	0%	0%	0%	0%	0%	0%	0%
	Gap_BN	16.25%	8.29%	2.15%	1.24%	0.82%	0.82%	0.62%	0.62%	0.43%	0.00%	0.00%
0.5	M=LB	0%	10%	50%	70%	90%	90%	90%	90%	90%	90%	90%
	M<BN	90%	70%	10%	0%	0%	0%	0%	0%	0%	0%	0%
	Gap_M	13.41%	2.57%	1.09%	0.62%	0.16%	0.16%	0.16%	0.16%	0.16%	0.16%	0.16%
	BN=LB	0%	0%	40%	70%	90%	90%	90%	90%	90%	90%	90%
	BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gap_BN	19.85%	4.52%	1.26%	0.62%	0.16%	0.16%	0.16%	0.16%	0.16%	0.16%	0.16%
0.6	M=LB	0%	10%	50%	70%	80%	80%	90%	90%	90%	90%	90%
	M<BN	100%	80%	50%	0%	0%	10%	0%	0%	0%	0%	0%
	Gap_M	12.75%	3.79%	1.48%	0.92%	0.46%	0.46%	0.16%	0.16%	0.16%	0.16%	0.16%
	BN=LB	0%	10%	30%	70%	80%	80%	90%	90%	90%	90%	90%
	BN<M	0%	0%	10%	0%	0%	0%	0%	0%	0%	0%	0%
	Gap_BN	16.71%	5.46%	2.06%	0.92%	0.46%	0.62%	0.16%	0.16%	0.16%	0.16%	0.16%
0.7	M=LB	0%	30%	60%	90%	90%	100%	100%	100%	100%	100%	100%
	M<BN	100%	40%	20%	10%	0%	10%	0%	0%	0%	0%	0%
	Gap_M	6.07%	1.18%	0.47%	0.12%	0.12%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	BN=LB	0%	30%	50%	80%	90%	90%	100%	100%	100%	100%	100%
	BN<M	0%	10%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gap_BN	8.96%	1.67%	0.71%	0.24%	0.12%	0.12%	0.00%	0.00%	0.00%	0.00%	0.00%
0.8	M=LB	0%	10%	50%	80%	90%	90%	90%	90%	90%	100%	100%
	M<BN	80%	50%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gap_M	5.80%	1.76%	0.63%	0.21%	0.11%	0.11%	0.11%	0.11%	0.11%	0.00%	0.00%
	BN=LB	0%	0%	60%	90%	90%	90%	90%	90%	90%	100%	100%
	BN<M	0%	0%	10%	10%	0%	0%	0%	0%	0%	0%	0%
	Gap_BN	7.15%	2.28%	0.53%	0.11%	0.11%	0.11%	0.11%	0.11%	0.11%	0.00%	0.00%
0.9	M=LB	0%	30%	60%	90%	100%	100%	100%	100%	100%	100%	100%
	M<BN	40%	30%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gap_M	2.69%	0.93%	0.37%	0.09%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	BN=LB	0%	20%	60%	90%	100%	100%	100%	100%	100%	100%	100%
	BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	Gap_BN	3.06%	1.21%	0.37%	0.09%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	tMin_BN	0.0010	0.0008	0.0007	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005
	tMax_BN	0.0069	0.0043	0.0032	0.0028	0.0019	0.0018	0.0017	0.0016	0.0012	0.0013	0.0013
	tavg_BN	0.0031	0.0019	0.0014	0.0012	0.0010	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008

Table 4: Computational results obtained by algorithms M and algorithm BN on $TM(120, B, \Delta)$

			B										
			120	150	180	210	240	270	300	330	360	390	400
Δ	0	M=LB	0%	0%	20%	0%	0%	0%	0%	10%	10%	20%	20%
		M<BN	100%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.29%	1.47%	0.95%	3.72%	3.15%	2.66%	2.35%	2.16%	2.11%	2.04%	2.09%
		BN=LB	0%	0%	40%	20%	60%	80%	100%	80%	90%	80%	80%
		BN<M	0%	0%	20%	100%	100%	100%	100%	70%	80%	60%	60%
		Gap_BN	9.05%	2.56%	0.71%	1.10%	0.64%	0.36%	0.00%	0.43%	0.24%	0.52%	0.53%
	0.1	M=LB	0%	0%	20%	50%	50%	70%	90%	40%	70%	70%	80%
		M<BN	100%	80%	20%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.29%	1.47%	0.95%	0.69%	0.79%	0.54%	0.20%	1.30%	0.71%	0.78%	0.53%
		BN=LB	0%	0%	20%	50%	60%	80%	100%	90%	80%	80%	80%
		BN<M	0%	0%	10%	0%	10%	10%	10%	50%	10%	10%	0%
		Gap_BN	9.30%	3.05%	1.06%	0.69%	0.64%	0.36%	0.00%	0.22%	0.47%	0.52%	0.53%
	0.2	M=LB	0%	0%	0%	30%	30%	70%	90%	70%	80%	100%	100%
		M<BN	100%	90%	20%	10%	10%	0%	0%	0%	0%	0%	0%
		Gap_M	3.29%	1.58%	1.30%	1.10%	1.10%	0.54%	0.20%	0.65%	0.47%	0.00%	0.00%
		BN=LB	0%	0%	0%	40%	50%	80%	90%	100%	90%	100%	100%
		BN<M	0%	0%	0%	10%	30%	10%	0%	30%	10%	0%	0%
		Gap_BN	9.78%	3.15%	1.53%	1.09%	0.79%	0.36%	0.20%	0.00%	0.24%	0.00%	0.00%
	0.3	M=LB	0%	0%	10%	60%	70%	90%	90%	90%	100%	100%	100%
		M<BN	100%	80%	40%	20%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.37%	1.77%	1.66%	0.54%	0.45%	0.13%	0.13%	0.13%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	0%	60%	70%	90%	90%	90%	100%	100%	100%
		BN<M	0%	0%	20%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	9.68%	3.93%	2.24%	0.95%	0.45%	0.13%	0.13%	0.13%	0.00%	0.00%	0.00%
	0.4	M=LB	0%	10%	30%	80%	80%	90%	90%	100%	100%	100%	100%
		M<BN	100%	100%	20%	20%	10%	0%	0%	0%	0%	0%	0%
		Gap_M	4.16%	2.22%	1.17%	0.23%	0.23%	0.11%	0.11%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	30%	60%	70%	90%	90%	100%	100%	100%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	11.41%	6.58%	1.51%	0.43%	0.33%	0.11%	0.11%	0.00%	0.00%	0.00%	0.00%
	0.5	M=LB	0%	10%	40%	70%	80%	90%	90%	100%	100%	100%	100%
		M<BN	100%	100%	20%	0%	0%	10%	0%	0%	0%	0%	0%
		Gap_M	11.37%	2.05%	0.73%	0.40%	0.16%	0.08%	0.08%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	40%	70%	80%	80%	90%	100%	100%	100%	100%
		BN<M	0%	0%	0%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	18.72%	5.27%	0.97%	0.32%	0.16%	0.16%	0.08%	0.00%	0.00%	0.00%	0.00%
	0.6	M=LB	0%	20%	40%	70%	80%	80%	80%	80%	90%	100%	100%
		M<BN	100%	90%	10%	10%	10%	0%	0%	0%	0%	0%	0%
		Gap_M	10.89%	1.71%	0.61%	0.19%	0.13%	0.13%	0.13%	0.13%	0.07%	0.00%	0.00%
		BN=LB	0%	0%	50%	60%	70%	80%	80%	80%	90%	100%	100%
		BN<M	0%	0%	10%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	15.14%	3.44%	0.62%	0.27%	0.19%	0.13%	0.13%	0.13%	0.07%	0.00%	0.00%
	0.7	M=LB	0%	0%	40%	60%	80%	90%	90%	90%	100%	100%	100%
		M<BN	100%	70%	30%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	8.34%	2.30%	0.82%	0.41%	0.12%	0.06%	0.06%	0.06%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	20%	60%	80%	90%	90%	90%	100%	100%	100%
		BN<M	0%	10%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	11.64%	3.60%	0.99%	0.47%	0.12%	0.06%	0.06%	0.06%	0.00%	0.00%	0.00%
	0.8	M=LB	0%	30%	60%	70%	90%	100%	100%	100%	100%	100%	100%
		M<BN	100%	50%	20%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.94%	0.86%	0.25%	0.15%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	20%	50%	70%	90%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	5.83%	1.26%	0.36%	0.15%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.9	M=LB	10%	60%	60%	80%	90%	90%	90%	90%	100%	100%	100%
		M<BN	90%	60%	10%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	2.50%	0.74%	0.41%	0.14%	0.05%	0.05%	0.05%	0.05%	0.00%	0.00%	0.00%
		BN=LB	0%	40%	60%	80%	90%	90%	90%	90%	100%	100%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	3.30%	1.11%	0.46%	0.14%	0.05%	0.05%	0.05%	0.05%	0.00%	0.00%	0.00%
		tMin_BN	0.0042	0.0035	0.0032	0.0028	0.0027	0.0025	0.0024	0.0022	0.0023	0.0022	0.0022
		tMax_BN	0.0601	0.0276	0.0205	0.0155	0.0115	0.0097	0.0070	0.0076	0.0075	0.0061	0.0062
		tavg_BN	0.0220	0.0115	0.0073	0.0058	0.0048	0.0042	0.0038	0.0038	0.0036	0.0035	0.0034

Table 5: Computational results obtained by algorithms M and algorithm BN on $TM(250, B, \Delta)$

			B										
			120	150	180	210	240	270	300	330	360	390	400
Δ	0	M=LB	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		M<BN	100%	60%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.02%	1.33%	1.06%	3.66%	3.23%	2.84%	2.76%	2.28%	2.13%	1.92%	1.57%
		BN=LB	0%	0%	0%	10%	50%	50%	60%	80%	70%	70%	80%
		BN<M	0%	20%	30%	100%	100%	100%	100%	100%	100%	100%	90%
		Gap_BN	8.75%	1.87%	0.89%	0.62%	0.39%	0.44%	0.40%	0.22%	0.35%	0.38%	0.26%
	0.1	M=LB	0%	0%	0%	0%	0%	0%	0%	30%	30%	50%	70%
		M<BN	100%	50%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.02%	1.38%	1.12%	0.97%	0.79%	0.98%	0.99%	0.76%	0.83%	0.64%	0.39%
		BN=LB	0%	0%	10%	0%	70%	40%	60%	80%	70%	70%	80%
		BN<M	0%	0%	40%	40%	70%	50%	60%	50%	40%	20%	10%
		Gap_BN	8.77%	1.82%	0.88%	0.69%	0.24%	0.53%	0.40%	0.22%	0.35%	0.38%	0.26%
	0.2	M=LB	0%	0%	0%	0%	0%	10%	60%	70%	80%	90%	90%
		M<BN	100%	90%	0%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.02%	1.38%	1.48%	1.24%	1.03%	1.24%	0.49%	0.32%	0.22%	0.10%	0.10%
		BN=LB	0%	0%	0%	0%	30%	30%	80%	90%	80%	90%	90%
		BN<M	0%	0%	70%	50%	60%	70%	30%	20%	0%	0%	0%
		Gap_BN	8.53%	2.46%	1.07%	0.90%	0.55%	0.63%	0.20%	0.10%	0.22%	0.10%	0.10%
	0.3	M=LB	0%	0%	0%	70%	100%	100%	100%	100%	100%	100%	100%
		M<BN	100%	100%	20%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.02%	1.53%	1.71%	0.55%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	0%	60%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	40%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	8.36%	3.40%	1.54%	0.61%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.4	M=LB	0%	10%	40%	70%	90%	90%	90%	90%	90%	90%	90%
		M<BN	100%	100%	10%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.18%	1.79%	0.49%	0.20%	0.05%	0.05%	0.05%	0.05%	0.05%	0.05%	0.05%
		BN=LB	0%	0%	70%	70%	90%	90%	90%	90%	90%	90%	90%
		BN<M	0%	0%	30%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	10.90%	5.18%	0.39%	0.20%	0.05%	0.05%	0.05%	0.05%	0.05%	0.05%	0.05%
	0.5	M=LB	0%	0%	40%	80%	90%	100%	100%	100%	100%	100%	100%
		M<BN	100%	90%	10%	10%	10%	0%	0%	0%	0%	0%	0%
		Gap_M	10.27%	1.52%	0.32%	0.08%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	60%	70%	80%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	30%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	17.17%	2.94%	0.24%	0.12%	0.08%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.6	M=LB	0%	0%	40%	80%	90%	90%	90%	90%	90%	100%	100%
		M<BN	100%	90%	20%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	7.83%	0.93%	0.30%	0.10%	0.07%	0.03%	0.03%	0.03%	0.03%	0.00%	0.00%
		BN=LB	0%	0%	40%	80%	90%	90%	90%	90%	90%	100%	100%
		BN<M	0%	10%	10%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	13.13%	2.16%	0.33%	0.10%	0.07%	0.03%	0.03%	0.03%	0.03%	0.00%	0.00%
	0.7	M=LB	0%	0%	60%	80%	90%	90%	90%	100%	100%	100%	100%
		M<BN	100%	100%	30%	0%	0%	10%	0%	10%	0%	0%	0%
		Gap_M	7.37%	1.16%	0.26%	0.09%	0.06%	0.03%	0.03%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	30%	80%	90%	90%	90%	90%	100%	100%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	10.28%	2.28%	0.35%	0.09%	0.06%	0.06%	0.03%	0.03%	0.00%	0.00%	0.00%
	0.8	M=LB	0%	0%	30%	50%	70%	90%	90%	90%	90%	90%	90%
		M<BN	100%	90%	20%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	4.85%	0.84%	0.33%	0.18%	0.10%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%
		BN=LB	0%	0%	30%	50%	70%	90%	90%	90%	90%	90%	90%
		BN<M	0%	10%	10%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	6.67%	1.57%	0.36%	0.18%	0.10%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%
	0.9	M=LB	0%	50%	70%	70%	70%	100%	100%	100%	100%	100%	100%
		M<BN	90%	80%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	1.95%	0.22%	0.07%	0.07%	0.07%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	70%	70%	70%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	2.46%	0.60%	0.07%	0.07%	0.07%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		tMin_BN	0.0174	0.0159	0.0120	0.0113	0.0106	0.0102	0.0100	0.0100	0.0091	0.0089	0.0088
		tMax_BN	0.4111	0.1915	0.1243	0.0928	0.0662	0.0601	0.0502	0.0393	0.0399	0.0327	0.0305
		tavg_BN	0.1606	0.0712	0.0424	0.0306	0.0212	0.0202	0.0179	0.0159	0.0155	0.0145	0.0140

Table 6: Computational results obtained by algorithms M and algorithm BN on $TM(500, B, \Delta)$

			B										
			120	150	180	210	240	270	300	330	360	390	400
Δ	0	M=LB	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		M<BN	100%	60%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	1.98%	1.19%	0.84%	3.31%	3.02%	2.64%	2.39%	2.08%	2.03%	1.81%	1.92%
		BN=LB	0%	0%	10%	10%	70%	80%	100%	100%	90%	100%	80%
		BN<M	0%	20%	100%	100%	100%	100%	100%	100%	100%	100%	100%
		Gap_BN	7.70%	1.37%	0.45%	0.31%	0.12%	0.09%	0.00%	0.00%	0.06%	0.00%	0.13%
	0.1	M=LB	0%	0%	0%	0%	0%	0%	0%	0%	30%	20%	10%
		M<BN	100%	20%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	1.98%	1.17%	0.87%	0.94%	0.84%	0.81%	0.50%	0.60%	0.48%	0.52%	0.60%
		BN=LB	0%	0%	10%	10%	70%	80%	100%	100%	70%	100%	90%
		BN<M	0%	50%	100%	100%	100%	100%	100%	100%	50%	80%	80%
		Gap_BN	7.92%	1.17%	0.48%	0.31%	0.12%	0.09%	0.00%	0.00%	0.18%	0.00%	0.07%
	0.2	M=LB	0%	0%	0%	0%	0%	0%	60%	90%	100%	100%	100%
		M<BN	100%	50%	0%	0%	0%	10%	0%	0%	0%	0%	0%
		Gap_M	1.98%	1.30%	1.28%	1.19%	1.03%	0.98%	0.40%	0.11%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	0%	0%	30%	60%	100%	100%	100%	100%	100%
		BN<M	0%	0%	100%	100%	90%	90%	40%	10%	0%	0%	0%
		Gap_BN	8.26%	1.59%	0.60%	0.42%	0.28%	0.22%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.3	M=LB	0%	0%	0%	70%	70%	90%	90%	100%	100%	100%	100%
		M<BN	100%	90%	10%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	1.98%	1.42%	1.70%	0.21%	0.10%	0.03%	0.03%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	0%	60%	70%	90%	90%	100%	100%	100%	100%
		BN<M	0%	0%	60%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	7.84%	2.54%	1.43%	0.20%	0.10%	0.03%	0.03%	0.00%	0.00%	0.00%	0.00%
	0.4	M=LB	0%	0%	0%	30%	60%	60%	80%	80%	90%	90%	90%
		M<BN	100%	100%	0%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	2.14%	1.84%	0.65%	0.20%	0.12%	0.10%	0.05%	0.05%	0.02%	0.02%	0.02%
		BN=LB	0%	0%	20%	30%	60%	60%	80%	80%	90%	90%	90%
		BN<M	0%	0%	90%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	9.23%	4.84%	0.32%	0.22%	0.12%	0.10%	0.05%	0.05%	0.02%	0.02%	0.02%
	0.5	M=LB	0%	0%	20%	90%	100%	100%	100%	100%	100%	100%	100%
		M<BN	100%	90%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	7.35%	1.07%	0.24%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	60%	90%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	50%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	14.88%	2.01%	0.12%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.6	M=LB	0%	0%	20%	80%	80%	80%	80%	80%	80%	90%	90%
		M<BN	100%	80%	20%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	5.97%	0.75%	0.20%	0.05%	0.03%	0.03%	0.03%	0.03%	0.03%	0.02%	0.02%
		BN=LB	0%	0%	40%	80%	80%	80%	80%	80%	80%	90%	90%
		BN<M	0%	0%	40%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	11.00%	1.53%	0.17%	0.05%	0.03%	0.03%	0.03%	0.03%	0.03%	0.02%	0.02%
	0.7	M=LB	0%	0%	50%	80%	90%	90%	90%	90%	100%	100%	100%
		M<BN	100%	80%	10%	10%	0%	10%	0%	0%	0%	0%	0%
		Gap_M	4.23%	0.70%	0.14%	0.06%	0.03%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	60%	80%	90%	90%	90%	100%	100%	100%	100%
		BN<M	0%	0%	10%	0%	0%	0%	0%	10%	0%	0%	0%
		Gap_BN	7.58%	1.26%	0.14%	0.07%	0.03%	0.03%	0.01%	0.00%	0.00%	0.00%	0.00%
	0.8	M=LB	0%	0%	30%	70%	70%	90%	90%	90%	90%	90%	100%
		M<BN	100%	100%	10%	10%	20%	0%	0%	0%	0%	0%	0%
		Gap_M	2.94%	0.48%	0.16%	0.07%	0.05%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%
		BN=LB	0%	0%	30%	60%	60%	90%	90%	90%	90%	100%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	10%	0%
		Gap_BN	4.73%	0.97%	0.19%	0.09%	0.07%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%
	0.9	M=LB	0%	10%	60%	100%	100%	100%	100%	100%	100%	100%	100%
		M<BN	100%	90%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	1.61%	0.22%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	60%	100%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	2.25%	0.55%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		tMin_BN	0.0739	0.0603	0.0474	0.0455	0.0433	0.0397	0.0397	0.0397	0.0356	0.0356	0.0355
		tMax_BN	3.1896	1.3708	0.8684	0.6439	0.4185	0.3408	0.1923	0.1705	0.1979	0.1369	0.1832
		tavg_BN	1.2327	0.4651	0.2458	0.1666	0.1085	0.0890	0.0736	0.0659	0.0631	0.0591	0.0603

Table 7: Computational results obtained by algorithms M and algorithm BN on $TM(1000, B, \Delta)$

			B										
			120	150	180	210	240	270	300	330	360	390	400
Δ	0	M=LB	0%	0%	20%	0%	0%	10%	20%	20%	40%	80%	30%
		M<BN	100%	70%	10%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	4.12%	2.08%	2.00%	3.47%	3.65%	3.31%	3.35%	3.68%	3.03%	1.08%	3.89%
		BN=LB	0%	0%	20%	40%	40%	100%	60%	90%	80%	90%	70%
		BN<M	0%	0%	10%	60%	50%	90%	40%	70%	40%	10%	40%
		Gap_BN	9.24%	4.33%	1.99%	1.73%	2.00%	0.00%	1.68%	0.48%	1.00%	0.53%	1.67%
	0.1	M=LB	0%	0%	30%	80%	70%	60%	70%	100%	80%	90%	90%
		M<BN	100%	80%	30%	20%	10%	10%	0%	0%	0%	10%	0%
		Gap_M	5.21%	2.28%	1.74%	0.57%	1.00%	1.52%	1.22%	0.00%	0.98%	0.56%	0.56%
		BN=LB	0%	0%	10%	70%	60%	60%	70%	100%	90%	80%	90%
		BN<M	0%	0%	0%	10%	0%	10%	0%	0%	10%	0%	0%
		Gap_BN	11.51%	5.34%	2.47%	0.84%	1.32%	1.50%	1.22%	0.00%	0.50%	1.11%	0.56%
	0.2	M=LB	0%	0%	20%	40%	60%	90%	90%	90%	100%	90%	100%
		M<BN	100%	90%	50%	20%	30%	0%	10%	0%	0%	0%	0%
		Gap_M	6.26%	2.23%	1.96%	1.70%	1.28%	0.38%	0.40%	0.43%	0.00%	0.56%	0.00%
		BN=LB	0%	0%	0%	30%	30%	90%	80%	90%	100%	90%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	11.82%	4.88%	3.63%	2.26%	2.25%	0.38%	0.78%	0.43%	0.00%	0.56%	0.00%
	0.3	M=LB	0%	0%	30%	50%	90%	90%	90%	90%	90%	90%	90%
		M<BN	100%	90%	50%	30%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	4.47%	3.12%	1.94%	1.93%	0.86%	0.57%	0.57%	0.57%	0.57%	0.29%	0.29%
		BN=LB	0%	0%	20%	50%	90%	90%	90%	90%	90%	90%	90%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	12.14%	7.45%	3.18%	2.79%	0.86%	0.57%	0.57%	0.57%	0.57%	0.29%	0.29%
	0.4	M=LB	0%	10%	50%	70%	70%	80%	80%	90%	90%	90%	90%
		M<BN	100%	80%	30%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	9.44%	3.95%	1.48%	1.04%	0.85%	0.42%	0.42%	0.23%	0.23%	0.23%	0.23%
		BN=LB	0%	0%	50%	70%	70%	80%	80%	90%	90%	90%	90%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	16.15%	7.75%	2.14%	1.04%	0.85%	0.42%	0.42%	0.23%	0.23%	0.23%	0.23%
	0.5	M=LB	0%	10%	40%	70%	90%	90%	90%	90%	90%	90%	90%
		M<BN	100%	70%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	14.40%	2.08%	1.09%	0.64%	0.16%	0.16%	0.16%	0.16%	0.16%	0.16%	0.16%
		BN=LB	0%	0%	40%	70%	90%	90%	90%	90%	90%	90%	90%
		BN<M	0%	10%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	19.90%	4.99%	1.09%	0.64%	0.16%	0.16%	0.16%	0.16%	0.16%	0.16%	0.16%
	0.6	M=LB	0%	20%	40%	60%	80%	80%	80%	80%	90%	90%	90%
		M<BN	100%	90%	30%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	12.30%	3.31%	1.46%	0.89%	0.46%	0.46%	0.30%	0.30%	0.16%	0.16%	0.16%
		BN=LB	0%	0%	40%	60%	80%	80%	80%	80%	90%	90%	90%
		BN<M	0%	0%	10%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	16.27%	5.39%	1.76%	1.02%	0.46%	0.46%	0.30%	0.30%	0.16%	0.16%	0.16%
	0.7	M=LB	0%	60%	60%	70%	100%	100%	100%	100%	100%	100%	100%
		M<BN	80%	50%	0%	0%	10%	0%	0%	0%	0%	0%	0%
		Gap_M	7.00%	1.31%	0.60%	0.36%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	30%	60%	70%	90%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	8.76%	2.36%	0.60%	0.36%	0.12%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.8	M=LB	0%	30%	60%	90%	90%	90%	90%	90%	90%	100%	100%
		M<BN	80%	60%	30%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	6.09%	1.56%	0.52%	0.11%	0.11%	0.11%	0.11%	0.11%	0.11%	0.00%	0.00%
		BN=LB	0%	10%	50%	80%	90%	90%	90%	90%	90%	100%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	7.03%	2.38%	0.84%	0.21%	0.11%	0.11%	0.11%	0.11%	0.11%	0.00%	0.00%
	0.9	M=LB	0%	30%	70%	70%	100%	100%	100%	100%	100%	100%	100%
		M<BN	0%	30%	20%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	4.17%	1.40%	0.37%	0.28%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	20%	60%	70%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	4.17%	1.68%	0.56%	0.28%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		tMin_BN	0.0010	0.0008	0.0007	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005
		tMax_BN	0.0072	0.0046	0.0031	0.0025	0.0024	0.0017	0.0018	0.0015	0.0015	0.0013	0.0014
		tavg_BN	0.0032	0.0020	0.0014	0.0012	0.0010	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008

Table 8: Computational results obtained by algorithms M and algorithm BN on $TS(120, B, \Delta)$

			B										
			120	150	180	210	240	270	300	330	360	390	400
Δ	0	M=LB	0%	0%	0%	0%	0%	0%	0%	0%	10%	10%	20%
		M<BN	100%	80%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	4.01%	1.70%	1.44%	4.05%	3.18%	2.67%	2.79%	2.18%	2.37%	2.32%	2.11%
		BN=LB	0%	0%	10%	40%	60%	90%	70%	80%	80%	90%	80%
		BN<M	0%	0%	20%	100%	100%	100%	90%	80%	80%	80%	60%
		Gap_BN	9.04%	2.99%	1.19%	0.83%	0.64%	0.18%	0.61%	0.43%	0.48%	0.26%	0.55%
	0.1	M=LB	0%	0%	0%	10%	40%	60%	70%	80%	60%	80%	80%
		M<BN	100%	70%	30%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.27%	1.71%	1.45%	1.27%	0.96%	0.73%	0.60%	0.44%	0.96%	0.52%	0.53%
		BN=LB	0%	0%	10%	0%	60%	70%	80%	90%	80%	80%	90%
		BN<M	0%	0%	20%	0%	20%	10%	10%	10%	20%	0%	10%
		Gap_BN	9.56%	2.41%	1.56%	1.41%	0.64%	0.54%	0.41%	0.22%	0.48%	0.52%	0.26%
	0.2	M=LB	0%	0%	0%	0%	20%	50%	70%	80%	90%	100%	100%
		M<BN	100%	70%	30%	20%	0%	0%	10%	0%	0%	0%	0%
		Gap_M	4.05%	1.90%	1.91%	1.53%	1.27%	0.90%	0.60%	0.43%	0.23%	0.00%	0.00%
		BN=LB	0%	0%	0%	0%	30%	50%	80%	90%	90%	100%	100%
		BN<M	0%	0%	40%	10%	10%	0%	20%	10%	0%	0%	0%
		Gap_BN	9.49%	3.08%	1.79%	1.67%	1.12%	0.90%	0.39%	0.22%	0.23%	0.00%	0.00%
	0.3	M=LB	0%	0%	0%	50%	80%	90%	100%	100%	100%	100%	100%
		M<BN	100%	70%	50%	0%	10%	0%	0%	0%	0%	0%	0%
		Gap_M	3.04%	2.21%	1.92%	0.68%	0.29%	0.13%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	10%	50%	80%	90%	100%	100%	100%	100%	100%
		BN<M	0%	0%	10%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	8.00%	4.31%	2.64%	0.68%	0.45%	0.13%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.4	M=LB	0%	0%	30%	80%	90%	90%	90%	90%	90%	90%	90%
		M<BN	100%	100%	30%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	4.39%	2.43%	1.28%	0.21%	0.11%	0.11%	0.11%	0.11%	0.11%	0.11%	0.11%
		BN=LB	0%	0%	30%	80%	90%	90%	90%	90%	90%	90%	90%
		BN<M	0%	0%	20%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	13.01%	6.67%	2.21%	0.21%	0.11%	0.11%	0.11%	0.11%	0.11%	0.11%	0.11%
	0.5	M=LB	0%	30%	40%	60%	70%	80%	100%	100%	100%	100%	100%
		M<BN	100%	80%	10%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	9.83%	2.09%	0.81%	0.41%	0.24%	0.16%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	10%	50%	60%	70%	80%	100%	100%	100%	100%	100%
		BN<M	0%	0%	20%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	16.43%	4.02%	0.75%	0.49%	0.24%	0.16%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.6	M=LB	0%	10%	30%	60%	80%	90%	90%	90%	90%	90%	90%
		M<BN	100%	80%	20%	20%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	9.94%	1.60%	0.67%	0.26%	0.13%	0.07%	0.07%	0.07%	0.07%	0.07%	0.07%
		BN=LB	0%	0%	30%	50%	80%	90%	90%	90%	90%	90%	90%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	13.80%	2.84%	0.82%	0.40%	0.13%	0.07%	0.07%	0.07%	0.07%	0.07%	0.07%
	0.7	M=LB	0%	0%	30%	60%	80%	90%	90%	90%	90%	100%	100%
		M<BN	100%	100%	50%	10%	0%	0%	0%	0%	0%	10%	0%
		Gap_M	8.63%	2.50%	0.69%	0.29%	0.17%	0.06%	0.06%	0.06%	0.06%	0.00%	0.00%
		BN=LB	0%	0%	10%	60%	80%	90%	90%	90%	90%	90%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	11.36%	3.91%	1.05%	0.35%	0.17%	0.06%	0.06%	0.06%	0.06%	0.06%	0.00%
	0.8	M=LB	0%	30%	70%	100%	100%	100%	100%	100%	100%	100%	100%
		M<BN	100%	90%	10%	20%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	4.29%	0.82%	0.15%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	70%	80%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	5.70%	1.58%	0.20%	0.10%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.9	M=LB	0%	30%	60%	80%	100%	100%	100%	100%	100%	100%	100%
		M<BN	90%	50%	10%	0%	10%	0%	0%	0%	0%	0%	0%
		Gap_M	3.15%	0.87%	0.33%	0.14%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	20%	60%	80%	90%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	3.78%	1.27%	0.37%	0.14%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		tMin_BN	0.0043	0.0041	0.0031	0.0027	0.0026	0.0025	0.0024	0.0024	0.0022	0.0022	0.0022
		tMax_BN	0.0596	0.0301	0.0207	0.0168	0.0125	0.0100	0.0095	0.0085	0.0074	0.0064	0.0071
		tavg_BN	0.0222	0.0118	0.0076	0.0060	0.0048	0.0042	0.0040	0.0037	0.0036	0.0035	0.0034

Table 9: Computational results obtained by algorithms M and algorithm BN on $TS(250, B, \Delta)$

			B										
			120	150	180	210	240	270	300	330	360	390	400
Δ	0	M=LB	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		M<BN	100%	70%	10%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	2.50%	1.56%	1.09%	3.71%	3.37%	2.71%	2.71%	2.20%	2.05%	2.22%	2.40%
		BN=LB	0%	0%	0%	60%	50%	60%	50%	80%	70%	70%	70%
		BN<M	0%	10%	60%	100%	100%	100%	100%	100%	100%	100%	100%
		Gap_BN	8.26%	2.44%	0.84%	0.41%	0.41%	0.36%	0.50%	0.22%	0.37%	0.39%	0.41%
	0.1	M=LB	0%	0%	0%	0%	0%	0%	50%	50%	50%	30%	50%
		M<BN	100%	80%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	2.09%	1.51%	1.39%	0.99%	1.05%	1.00%	0.50%	0.55%	0.61%	0.92%	0.67%
		BN=LB	0%	0%	0%	20%	30%	70%	100%	80%	70%	70%	80%
		BN<M	0%	0%	70%	60%	60%	80%	50%	30%	20%	40%	30%
		Gap_BN	8.03%	2.26%	0.85%	0.56%	0.56%	0.28%	0.00%	0.22%	0.36%	0.40%	0.27%
	0.2	M=LB	0%	0%	0%	0%	0%	10%	30%	80%	90%	90%	90%
		M<BN	100%	70%	0%	0%	0%	10%	0%	0%	0%	0%	0%
		Gap_M	3.68%	1.74%	1.73%	1.38%	1.11%	1.16%	0.68%	0.21%	0.10%	0.10%	0.10%
		BN=LB	0%	0%	0%	10%	50%	50%	70%	80%	90%	90%	90%
		BN<M	0%	0%	70%	80%	90%	80%	40%	0%	0%	0%	0%
		Gap_BN	9.55%	2.28%	1.19%	0.69%	0.39%	0.54%	0.29%	0.21%	0.10%	0.10%	0.10%
	0.3	M=LB	0%	0%	0%	70%	100%	100%	100%	100%	100%	100%	100%
		M<BN	100%	100%	20%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	2.26%	1.76%	1.87%	0.48%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	0%	70%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	50%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	8.41%	2.86%	1.74%	0.48%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.4	M=LB	0%	0%	30%	90%	90%	90%	90%	90%	90%	90%	90%
		M<BN	100%	100%	10%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.96%	2.18%	0.45%	0.05%	0.05%	0.05%	0.05%	0.05%	0.05%	0.05%	0.05%
		BN=LB	0%	0%	70%	90%	90%	90%	90%	90%	90%	90%	90%
		BN<M	0%	0%	40%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	10.68%	5.54%	0.35%	0.09%	0.05%	0.05%	0.05%	0.05%	0.05%	0.05%	0.05%
	0.5	M=LB	0%	0%	60%	90%	90%	100%	100%	100%	100%	100%	100%
		M<BN	100%	100%	10%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	9.40%	1.21%	0.21%	0.04%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	70%	80%	90%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	30%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	16.82%	2.58%	0.12%	0.08%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.6	M=LB	0%	0%	40%	80%	90%	90%	90%	90%	100%	100%	100%
		M<BN	100%	90%	10%	0%	10%	0%	0%	0%	0%	0%	0%
		Gap_M	7.01%	0.86%	0.30%	0.10%	0.03%	0.03%	0.03%	0.03%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	40%	80%	90%	90%	90%	90%	100%	100%	100%
		BN<M	0%	10%	20%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	12.31%	1.69%	0.27%	0.10%	0.07%	0.03%	0.03%	0.03%	0.00%	0.00%	0.00%
	0.7	M=LB	0%	10%	90%	90%	90%	90%	90%	90%	100%	100%	100%
		M<BN	100%	100%	20%	10%	0%	10%	0%	0%	0%	0%	0%
		Gap_M	6.57%	0.81%	0.12%	0.06%	0.06%	0.03%	0.03%	0.03%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	70%	90%	90%	90%	90%	90%	100%	100%	100%
		BN<M	0%	0%	10%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	9.42%	1.62%	0.18%	0.09%	0.06%	0.06%	0.03%	0.03%	0.00%	0.00%	0.00%
	0.8	M=LB	0%	20%	30%	40%	70%	70%	80%	80%	90%	100%	100%
		M<BN	100%	90%	30%	0%	10%	0%	0%	0%	0%	0%	0%
		Gap_M	4.12%	0.86%	0.38%	0.23%	0.08%	0.08%	0.05%	0.05%	0.02%	0.00%	0.00%
		BN=LB	0%	0%	40%	40%	70%	70%	80%	80%	90%	100%	100%
		BN<M	0%	0%	20%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	6.33%	1.57%	0.43%	0.20%	0.10%	0.08%	0.05%	0.05%	0.02%	0.00%	0.00%
	0.9	M=LB	0%	30%	60%	80%	90%	100%	100%	100%	100%	100%	100%
		M<BN	100%	60%	10%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	1.73%	0.33%	0.09%	0.04%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	10%	50%	80%	90%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	2.42%	0.56%	0.11%	0.04%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		tMin_BN	0.0188	0.0172	0.0135	0.0112	0.0106	0.0102	0.0100	0.0100	0.0091	0.0089	0.0089
		tMax_BN	0.4178	0.2111	0.1287	0.0986	0.0696	0.0675	0.0517	0.0451	0.0396	0.0390	0.0370
		tavg_BN	0.1600	0.0689	0.0422	0.0290	0.0234	0.0203	0.0177	0.0162	0.0153	0.0150	0.0147

Table 10: Computational results obtained by algorithms M and algorithm BN on $TS(500, B, \Delta)$

			B										
			120	150	180	210	240	270	300	330	360	390	400
Δ	0	M=LB	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		M<BN	100%	80%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	2.30%	1.40%	0.96%	3.49%	3.19%	2.78%	2.65%	2.41%	2.03%	1.75%	1.79%
		BN=LB	0%	0%	0%	0%	20%	70%	30%	70%	70%	90%	70%
		BN<M	0%	10%	100%	100%	100%	100%	100%	100%	100%	100%	100%
		Gap_BN	8.33%	1.79%	0.54%	0.35%	0.32%	0.13%	0.35%	0.17%	0.18%	0.07%	0.20%
	0.1	M=LB	0%	0%	0%	0%	0%	0%	0%	0%	10%	10%	0%
		M<BN	100%	60%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	2.43%	1.41%	1.13%	0.97%	1.07%	0.98%	0.79%	0.87%	0.60%	0.58%	0.66%
		BN=LB	0%	0%	0%	0%	40%	40%	60%	70%	80%	90%	100%
		BN<M	0%	10%	100%	100%	100%	100%	100%	100%	80%	80%	100%
		Gap_BN	8.68%	1.78%	0.60%	0.42%	0.24%	0.27%	0.20%	0.16%	0.12%	0.06%	0.00%
	0.2	M=LB	0%	0%	0%	0%	0%	0%	60%	90%	100%	100%	100%
		M<BN	100%	80%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	2.06%	1.57%	1.58%	1.39%	1.15%	1.16%	0.40%	0.16%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	0%	0%	10%	30%	80%	90%	100%	100%	100%
		BN<M	0%	10%	100%	90%	80%	90%	30%	10%	0%	0%	0%
		Gap_BN	7.97%	2.14%	0.81%	0.52%	0.48%	0.40%	0.15%	0.05%	0.00%	0.00%	0.00%
	0.3	M=LB	0%	0%	0%	80%	100%	100%	100%	100%	100%	100%	100%
		M<BN	100%	100%	20%	20%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	1.50%	1.73%	1.80%	0.21%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	0%	60%	100%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	60%	10%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	7.62%	3.03%	1.54%	0.20%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.4	M=LB	0%	0%	10%	30%	50%	50%	70%	80%	80%	80%	80%
		M<BN	100%	100%	0%	0%	0%	10%	0%	0%	0%	0%	0%
		Gap_M	2.58%	1.94%	0.69%	0.25%	0.17%	0.12%	0.07%	0.05%	0.05%	0.05%	0.05%
		BN=LB	0%	0%	10%	30%	50%	50%	70%	80%	80%	80%	80%
		BN<M	0%	0%	70%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	9.49%	5.62%	0.45%	0.25%	0.17%	0.15%	0.07%	0.05%	0.05%	0.05%	0.05%
	0.5	M=LB	0%	0%	30%	90%	90%	90%	90%	100%	100%	100%	100%
		M<BN	100%	100%	0%	10%	10%	0%	0%	0%	0%	0%	0%
		Gap_M	7.86%	1.05%	0.28%	0.06%	0.04%	0.02%	0.02%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	40%	90%	90%	90%	90%	100%	100%	100%	100%
		BN<M	0%	0%	30%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	15.36%	2.54%	0.20%	0.08%	0.06%	0.02%	0.02%	0.00%	0.00%	0.00%	0.00%
	0.6	M=LB	0%	0%	50%	70%	80%	80%	80%	80%	80%	90%	90%
		M<BN	100%	100%	10%	0%	10%	0%	0%	0%	0%	0%	0%
		Gap_M	5.64%	0.65%	0.20%	0.07%	0.03%	0.03%	0.03%	0.03%	0.03%	0.02%	0.02%
		BN=LB	0%	0%	50%	70%	70%	80%	80%	80%	80%	90%	90%
		BN<M	0%	0%	40%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	10.98%	1.53%	0.13%	0.07%	0.05%	0.03%	0.03%	0.03%	0.03%	0.02%	0.02%
	0.7	M=LB	0%	0%	40%	90%	90%	90%	90%	90%	100%	100%	100%
		M<BN	100%	70%	20%	0%	10%	0%	0%	0%	0%	0%	0%
		Gap_M	3.70%	0.49%	0.16%	0.06%	0.03%	0.03%	0.01%	0.01%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	60%	90%	90%	90%	90%	90%	100%	100%	100%
		BN<M	0%	0%	30%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	6.94%	0.89%	0.16%	0.06%	0.04%	0.03%	0.01%	0.01%	0.00%	0.00%	0.00%
	0.8	M=LB	0%	0%	50%	70%	90%	90%	90%	90%	90%	100%	100%
		M<BN	100%	80%	10%	0%	0%	0%	0%	0%	0%	10%	0%
		Gap_M	2.57%	0.41%	0.12%	0.05%	0.02%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%
		BN=LB	0%	0%	50%	70%	90%	90%	90%	90%	90%	90%	100%
		BN<M	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	4.26%	0.74%	0.14%	0.05%	0.02%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%
	0.9	M=LB	0%	20%	70%	90%	90%	100%	100%	100%	100%	100%	100%
		M<BN	100%	100%	10%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	1.65%	0.19%	0.06%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		BN=LB	0%	0%	70%	90%	90%	100%	100%	100%	100%	100%	100%
		BN<M	0%	0%	10%	0%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	2.41%	0.41%	0.06%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
		tMin_BN	0.0852	0.0707	0.0524	0.0467	0.0440	0.0402	0.0402	0.0402	0.0359	0.0359	0.0357
		tMax_BN	3.3840	1.6556	1.2061	0.8726	0.4875	0.3589	0.3273	0.2463	0.2329	0.1734	0.1916
		tavg_BN	1.2766	0.5274	0.2934	0.1928	0.1281	0.1053	0.0900	0.0771	0.0691	0.0635	0.0630

Table 11: Computational results obtained by algorithms M and algorithm BN on $TS(1000, B, \Delta)$

	Δ		120	150	180	210	240	270	300	330	360	390
			B									
Δ	0	M=LB	3%	20%	40%	0%	2%	20%	23%	41%	49%	57%
		M<BN	98%	74%	26%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	5.00%	1.80%	1.49%	4.34%	3.56%	3.03%	3.18%	2.69%	2.53%	2.30%
		BN=LB	0%	0%	21%	43%	59%	80%	74%	85%	87%	88%
		BN<M	0%	0%	4%	87%	66%	62%	51%	44%	38%	31%
		Gap_BN	10.02%	4.36%	2.03%	1.65%	1.35%	0.73%	1.09%	0.70%	0.66%	0.64%
	0.1	M=LB	3%	16%	24%	47%	57%	86%	93%	94%	97%	99%
		M<BN	98%	72%	31%	23%	13%	4%	2%	0%	0%	0%
		Gap_M	5.00%	1.83%	1.89%	1.55%	1.42%	0.52%	0.27%	0.24%	0.12%	0.04%
		BN=LB	0%	1%	11%	27%	49%	83%	91%	94%	97%	99%
		BN<M	0%	1%	3%	2%	3%	1%	0%	0%	0%	0%
		Gap_BN	10.83%	4.52%	2.60%	2.14%	1.75%	0.63%	0.35%	0.24%	0.12%	0.04%
	0.2	M=LB	0%	2%	21%	75%	87%	92%	95%	97%	100%	100%
		M<BN	100%	88%	55%	19%	6%	0%	1%	0%	0%	0%
		Gap_M	5.31%	2.34%	2.09%	0.74%	0.38%	0.22%	0.14%	0.08%	0.00%	0.00%
		BN=LB	0%	0%	15%	63%	83%	92%	94%	97%	100%	100%
		BN<M	0%	0%	2%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	11.61%	6.23%	3.70%	1.31%	0.56%	0.22%	0.17%	0.08%	0.00%	0.00%
	0.3	M=LB	0%	9%	54%	79%	84%	90%	97%	99%	99%	100%
		M<BN	98%	94%	22%	5%	1%	0%	1%	0%	0%	0%
		Gap_M	6.13%	2.82%	1.37%	0.53%	0.38%	0.23%	0.07%	0.02%	0.02%	0.00%
		BN=LB	0%	2%	51%	77%	83%	90%	96%	99%	99%	100%
		BN<M	0%	2%	3%	1%	0%	0%	0%	0%	0%	0%
		Gap_BN	13.64%	7.66%	1.92%	0.63%	0.40%	0.23%	0.09%	0.02%	0.02%	0.00%
	0.4	M=LB	0%	14%	58%	81%	90%	95%	97%	98%	100%	100%
		M<BN	100%	78%	9%	1%	0%	1%	1%	0%	1%	0%
		Gap_M	9.04%	3.22%	0.95%	0.42%	0.19%	0.10%	0.06%	0.04%	0.00%	0.00%
		BN=LB	0%	2%	57%	80%	90%	94%	96%	98%	99%	100%
		BN<M	0%	2%	3%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	17.37%	6.63%	1.06%	0.43%	0.19%	0.12%	0.08%	0.04%	0.02%	0.00%
	0.5	M=LB	0%	19%	56%	72%	80%	83%	90%	94%	96%	97%
		M<BN	100%	82%	17%	8%	1%	0%	1%	0%	0%	0%
		Gap_M	14.79%	3.49%	1.21%	0.65%	0.41%	0.33%	0.17%	0.10%	0.07%	0.05%
		BN=LB	0%	2%	49%	68%	80%	83%	89%	94%	96%	97%
		BN<M	0%	1%	2%	0%	0%	1%	0%	0%	0%	0%
		Gap_BN	21.23%	6.19%	1.47%	0.79%	0.43%	0.31%	0.19%	0.10%	0.07%	0.05%
	0.6	M=LB	0%	28%	62%	76%	86%	94%	95%	98%	99%	99%
		M<BN	98%	77%	9%	5%	1%	1%	0%	0%	0%	0%
		Gap_M	14.00%	2.76%	0.91%	0.42%	0.23%	0.09%	0.08%	0.03%	0.02%	0.02%
		BN=LB	0%	7%	58%	75%	85%	94%	95%	98%	99%	99%
		BN<M	0%	0%	2%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	19.10%	5.15%	1.03%	0.50%	0.25%	0.11%	0.08%	0.03%	0.02%	0.02%
	0.7	M=LB	0%	26%	53%	72%	82%	92%	94%	95%	96%	97%
		M<BN	99%	76%	16%	7%	2%	0%	0%	0%	0%	0%
		Gap_M	12.04%	2.66%	0.87%	0.44%	0.26%	0.13%	0.08%	0.07%	0.06%	0.04%
		BN=LB	0%	9%	49%	66%	81%	92%	94%	95%	96%	97%
		BN<M	0%	2%	1%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	15.88%	4.60%	1.10%	0.54%	0.29%	0.13%	0.08%	0.07%	0.06%	0.04%
	0.8	M=LB	0%	28%	65%	80%	92%	95%	98%	98%	99%	99%
		M<BN	96%	64%	15%	4%	0%	2%	0%	0%	0%	0%
		Gap_M	9.01%	1.83%	0.56%	0.31%	0.11%	0.06%	0.03%	0.03%	0.01%	0.01%
		BN=LB	0%	9%	60%	77%	92%	94%	98%	98%	99%	99%
		BN<M	0%	1%	1%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	11.60%	3.04%	0.74%	0.36%	0.11%	0.09%	0.03%	0.03%	0.01%	0.01%
	0.9	M=LB	1%	31%	65%	80%	89%	93%	98%	98%	98%	98%
		M<BN	84%	51%	8%	3%	1%	0%	1%	0%	0%	0%
		Gap_M	6.10%	1.49%	0.54%	0.27%	0.12%	0.08%	0.02%	0.02%	0.02%	0.02%
		BN=LB	0%	20%	66%	78%	88%	94%	97%	98%	98%	98%
		BN<M	0%	1%	4%	0%	0%	1%	0%	0%	0%	0%
		Gap_BN	7.63%	2.22%	0.59%	0.30%	0.13%	0.06%	0.03%	0.02%	0.02%	0.02%
		tMin_BN	0.0011	0.0008	0.0007	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005
		tMax_BN	0.0085	0.0052	0.0032	0.0031	0.0023	0.0021	0.0019	0.0016	0.0015	0.0014
		tavg_BN	0.0029	0.0017	0.0012	0.0010	0.0009	0.0008	0.0008	0.0008	0.0007	0.0007

Table 12: Computational results obtained by algorithms M and algorithm BN on $TT(120, B, \Delta)$

			B									
			120	150	180	210	240	270	300	330	360	390
Δ	0	M=LB	0%	0%	11%	0%	0%	0%	0%	3%	11%	20%
		M<BN	100%	70%	16%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	3.87%	1.56%	1.18%	3.67%	3.23%	2.86%	2.59%	2.29%	2.13%	2.05%
		BN=LB	0%	0%	12%	35%	61%	75%	64%	88%	88%	86%
		BN<M	0%	1%	13%	100%	100%	100%	89%	91%	78%	66%
		Gap_BN	9.12%	2.92%	1.22%	0.90%	0.61%	0.45%	0.72%	0.26%	0.29%	0.36%
	0.1	M=LB	0%	0%	1%	8%	33%	64%	88%	97%	98%	98%
		M<BN	100%	82%	15%	20%	4%	2%	0%	0%	0%	0%
		Gap_M	3.88%	1.63%	1.66%	1.33%	1.06%	0.63%	0.26%	0.07%	0.05%	0.05%
		BN=LB	0%	1%	4%	12%	49%	74%	88%	97%	98%	98%
		BN<M	0%	1%	22%	12%	18%	12%	0%	0%	0%	0%
		Gap_BN	9.39%	3.63%	1.57%	1.45%	0.84%	0.45%	0.26%	0.07%	0.05%	0.05%
	0.2	M=LB	0%	0%	4%	66%	84%	89%	93%	94%	96%	98%
		M<BN	100%	95%	63%	23%	3%	3%	0%	0%	0%	0%
		Gap_M	4.03%	1.92%	1.88%	0.54%	0.27%	0.16%	0.09%	0.08%	0.05%	0.03%
		BN=LB	0%	0%	2%	57%	81%	87%	93%	94%	96%	98%
		BN<M	0%	2%	4%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	9.82%	4.66%	2.86%	0.88%	0.31%	0.20%	0.09%	0.08%	0.05%	0.03%
	0.3	M=LB	0%	3%	52%	75%	86%	92%	95%	95%	96%	98%
		M<BN	100%	97%	31%	5%	0%	1%	0%	0%	0%	0%
		Gap_M	4.45%	2.28%	0.76%	0.32%	0.19%	0.10%	0.07%	0.05%	0.04%	0.02%
		BN=LB	0%	0%	45%	72%	86%	92%	95%	95%	96%	98%
		BN<M	0%	1%	6%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	11.40%	6.90%	1.19%	0.37%	0.19%	0.11%	0.07%	0.05%	0.04%	0.02%
	0.4	M=LB	0%	12%	54%	78%	88%	93%	94%	96%	96%	97%
		M<BN	100%	89%	15%	2%	1%	0%	0%	0%	0%	0%
		Gap_M	6.16%	2.25%	0.59%	0.24%	0.12%	0.06%	0.05%	0.04%	0.04%	0.03%
		BN=LB	0%	2%	51%	76%	88%	93%	94%	96%	96%	97%
		BN<M	0%	3%	8%	1%	1%	0%	0%	0%	0%	0%
		Gap_BN	14.55%	5.16%	0.66%	0.25%	0.12%	0.06%	0.05%	0.04%	0.04%	0.03%
	0.5	M=LB	0%	12%	42%	69%	84%	86%	89%	92%	93%	96%
		M<BN	100%	87%	19%	4%	2%	0%	2%	0%	0%	0%
		Gap_M	11.27%	2.13%	0.68%	0.31%	0.16%	0.12%	0.10%	0.07%	0.06%	0.03%
		BN=LB	0%	1%	43%	67%	83%	86%	87%	92%	93%	96%
		BN<M	0%	2%	7%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	18.40%	4.28%	0.82%	0.34%	0.17%	0.12%	0.11%	0.07%	0.06%	0.03%
	0.6	M=LB	0%	14%	61%	77%	85%	90%	93%	96%	96%	96%
		M<BN	100%	90%	19%	3%	2%	0%	0%	0%	0%	0%
		Gap_M	12.09%	1.70%	0.46%	0.24%	0.14%	0.09%	0.06%	0.03%	0.03%	0.03%
		BN=LB	0%	1%	59%	75%	84%	90%	93%	96%	96%	96%
		BN<M	0%	3%	6%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	17.92%	3.64%	0.59%	0.26%	0.16%	0.09%	0.06%	0.03%	0.03%	0.03%
	0.7	M=LB	0%	10%	59%	82%	88%	92%	94%	97%	98%	99%
		M<BN	100%	84%	18%	2%	0%	0%	1%	1%	0%	0%
		Gap_M	9.14%	1.38%	0.35%	0.14%	0.08%	0.05%	0.04%	0.02%	0.01%	0.01%
		BN=LB	0%	4%	52%	82%	88%	92%	93%	96%	98%	99%
		BN<M	0%	5%	5%	1%	0%	0%	0%	0%	0%	0%
		Gap_BN	13.72%	3.03%	0.47%	0.14%	0.08%	0.05%	0.05%	0.03%	0.01%	0.01%
	0.8	M=LB	0%	14%	50%	64%	75%	81%	90%	93%	93%	95%
		M<BN	100%	85%	16%	6%	3%	0%	1%	0%	0%	0%
		Gap_M	7.91%	1.42%	0.51%	0.28%	0.17%	0.12%	0.07%	0.05%	0.04%	0.03%
		BN=LB	0%	1%	46%	62%	74%	81%	89%	93%	93%	95%
		BN<M	0%	1%	1%	1%	0%	0%	0%	0%	0%	0%
		Gap_BN	11.13%	2.74%	0.61%	0.31%	0.19%	0.12%	0.07%	0.05%	0.04%	0.03%
	0.9	M=LB	0%	15%	59%	80%	91%	95%	96%	98%	99%	99%
		M<BN	99%	71%	16%	2%	0%	0%	0%	1%	0%	0%
		Gap_M	5.21%	1.03%	0.30%	0.13%	0.05%	0.03%	0.02%	0.01%	0.01%	0.01%
		BN=LB	0%	9%	54%	79%	91%	95%	96%	97%	99%	99%
		BN<M	0%	2%	2%	1%	0%	0%	0%	0%	0%	0%
		Gap_BN	7.14%	1.81%	0.37%	0.14%	0.05%	0.03%	0.02%	0.02%	0.01%	0.01%
		tMin_BN	0.0049	0.0042	0.0031	0.0028	0.0026	0.0025	0.0023	0.0023	0.0023	0.0023
		tMax_BN	0.0673	0.0360	0.0225	0.0183	0.0132	0.0112	0.0099	0.0085	0.0083	0.0071
		tavg_BN	0.0201	0.0101	0.0065	0.0050	0.0042	0.0038	0.0036	0.0034	0.0033	0.0032

Table 13: Computational results obtained by algorithms M and algorithm BN on $TT(250, B, \Delta)$

			B									
			120	150	180	210	240	270	300	330	360	390
Δ	0	M=LB	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
		M<BN	100%	56%	3%	0%	0%	0%	0%	0%	0%	0%
		Gap_M	2.42%	1.36%	1.01%	3.44%	3.15%	2.72%	2.48%	2.18%	2.07%	1.86%
		BN=LB	0%	0%	2%	32%	57%	69%	73%	85%	87%	87%
		BN<M	0%	6%	48%	100%	100%	100%	100%	100%	100%	100%
		Gap_BN	8.50%	1.81%	0.72%	0.47%	0.34%	0.28%	0.27%	0.16%	0.16%	0.17%
	0.1	M=LB	0%	0%	0%	0%	1%	45%	94%	99%	100%	100%
		M<BN	100%	84%	4%	8%	0%	2%	0%	0%	0%	0%
		Gap_M	2.43%	1.44%	1.48%	1.28%	1.15%	0.60%	0.06%	0.01%	0.00%	0.00%
		BN=LB	0%	0%	1%	5%	32%	73%	97%	99%	100%	100%
		BN<M	0%	4%	64%	60%	71%	40%	3%	0%	0%	0%
		Gap_BN	8.57%	2.41%	1.03%	0.87%	0.57%	0.26%	0.03%	0.01%	0.00%	0.00%
	0.2	M=LB	0%	0%	1%	76%	87%	93%	95%	97%	98%	98%
		M<BN	100%	99%	66%	14%	0%	1%	0%	0%	0%	0%
		Gap_M	2.47%	1.60%	1.72%	0.20%	0.09%	0.05%	0.03%	0.02%	0.01%	0.01%
		BN=LB	0%	0%	0%	66%	87%	92%	96%	97%	98%	98%
		BN<M	0%	0%	10%	2%	0%	0%	1%	0%	0%	0%
		Gap_BN	8.40%	3.52%	2.28%	0.31%	0.09%	0.05%	0.03%	0.02%	0.01%	0.01%
	0.3	M=LB	0%	0%	27%	70%	80%	87%	89%	93%	94%	97%
		M<BN	100%	100%	26%	4%	1%	1%	0%	0%	0%	0%
		Gap_M	2.63%	1.99%	0.62%	0.18%	0.11%	0.07%	0.06%	0.04%	0.03%	0.02%
		BN=LB	0%	0%	43%	69%	79%	86%	89%	93%	94%	97%
		BN<M	0%	0%	31%	1%	0%	0%	0%	0%	0%	0%
		Gap_BN	9.74%	5.94%	0.62%	0.20%	0.12%	0.08%	0.06%	0.04%	0.03%	0.02%
	0.4	M=LB	0%	2%	40%	81%	87%	93%	96%	96%	97%	97%
		M<BN	100%	93%	14%	4%	1%	1%	0%	0%	0%	0%
		Gap_M	3.69%	1.56%	0.38%	0.10%	0.06%	0.03%	0.02%	0.02%	0.01%	0.01%
		BN=LB	0%	0%	48%	77%	87%	92%	96%	96%	97%	97%
		BN<M	0%	6%	22%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	12.78%	3.49%	0.34%	0.12%	0.07%	0.04%	0.02%	0.02%	0.01%	0.01%
	0.5	M=LB	0%	4%	41%	74%	85%	91%	96%	97%	98%	98%
		M<BN	100%	98%	16%	2%	1%	0%	0%	0%	0%	0%
		Gap_M	8.28%	1.41%	0.36%	0.13%	0.07%	0.04%	0.02%	0.02%	0.01%	0.01%
		BN=LB	0%	0%	43%	74%	84%	91%	96%	97%	98%	98%
		BN<M	0%	1%	17%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	16.19%	3.05%	0.35%	0.14%	0.08%	0.04%	0.02%	0.02%	0.01%	0.01%
	0.6	M=LB	0%	9%	43%	72%	81%	89%	92%	94%	96%	96%
		M<BN	100%	88%	16%	1%	1%	1%	0%	0%	0%	0%
		Gap_M	8.44%	1.09%	0.30%	0.12%	0.08%	0.04%	0.03%	0.02%	0.01%	0.01%
		BN=LB	0%	0%	44%	72%	81%	88%	92%	94%	96%	96%
		BN<M	0%	2%	17%	0%	1%	0%	0%	0%	0%	0%
		Gap_BN	14.72%	2.42%	0.30%	0.13%	0.08%	0.04%	0.03%	0.02%	0.01%	0.01%
	0.7	M=LB	0%	8%	51%	80%	83%	91%	92%	94%	95%	97%
		M<BN	100%	88%	13%	3%	0%	1%	0%	0%	0%	1%
		Gap_M	6.83%	0.94%	0.24%	0.08%	0.07%	0.03%	0.03%	0.02%	0.02%	0.01%
		BN=LB	0%	0%	54%	79%	83%	90%	92%	94%	95%	97%
		BN<M	0%	3%	15%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	11.61%	2.01%	0.24%	0.10%	0.07%	0.04%	0.03%	0.02%	0.02%	0.01%
	0.8	M=LB	0%	8%	51%	73%	80%	85%	93%	97%	98%	98%
		M<BN	100%	95%	10%	3%	2%	0%	1%	0%	0%	0%
		Gap_M	5.76%	0.76%	0.22%	0.09%	0.06%	0.05%	0.02%	0.01%	0.01%	0.01%
		BN=LB	0%	0%	55%	72%	80%	85%	92%	97%	98%	98%
		BN<M	0%	0%	10%	1%	0%	0%	0%	0%	0%	0%
		Gap_BN	9.24%	1.70%	0.22%	0.10%	0.07%	0.05%	0.02%	0.01%	0.01%	0.01%
	0.9	M=LB	0%	16%	54%	80%	89%	93%	96%	97%	98%	99%
		M<BN	100%	81%	15%	5%	0%	0%	0%	0%	0%	0%
		Gap_M	3.95%	0.62%	0.17%	0.05%	0.03%	0.02%	0.01%	0.01%	0.01%	0.00%
		BN=LB	0%	3%	51%	77%	89%	93%	96%	97%	98%	99%
		BN<M	0%	6%	8%	0%	0%	0%	0%	0%	0%	0%
		Gap_BN	5.97%	1.21%	0.18%	0.07%	0.03%	0.02%	0.01%	0.01%	0.01%	0.00%
		tMin_BN	0.0232	0.0166	0.0127	0.0117	0.0108	0.0105	0.0097	0.0095	0.0092	0.0092
		tMax_BN	0.4295	0.2102	0.1598	0.0994	0.0722	0.0626	0.0520	0.0450	0.0400	0.0344
		tavg_BN	0.1380	0.0569	0.0320	0.0230	0.0190	0.0166	0.0152	0.0141	0.0136	0.0134

Table 14: Computational results obtained by algorithms M and algorithm BN on $TT(500, B, \Delta)$

3 Exact algorithms

In this section we tested two exact algorithms for $BPPC$ to see if we could get optimum solutions in a reasonable time on the instances of our test bed. Precisely, we compare the results obtained by solving the arc-flow formulation generated by the Vector Packing Solver (VPS for short) by Brandão and Pedroso (2016), available at Brandão (2016), and the formulation $\mathcal{F}_2$ presented in Appendix A. Brandão and Pedroso (2016) apply VPS to instances with $B = 150$, only. When we applied it to instances with values of $B \neq 150$, we noticed that its computing time and its percentage of unsolved instances rapidly increase for increasing B. For this reason we decided to solve $\mathcal{F}_2$, a classical $BPPC$ formulation, and compare its results with those of VPS. Both formulations, coded in C++, are solved with Cplex 12.6 on an Intel Xeon E5620 2.40GHz with 40 GB RAM under a Linux operating system. In particular, for the formulation $\mathcal{F}_2$, we set $UB = n$ and $LB = 0$. Due to the high running times, we had to limit ourselves to consider only the first ten instances of $TI(n, B, \Delta)$, $TM(n, B, \Delta)$, $TS(n, B, \Delta)$, $TT(n, B, \Delta)$, and to set a time limit of 600 seconds for each formulation on each instance.

In each table rows are indexed by Δ and columns by B. In each cell there are up to four values:

- TimeX denotes the time (in seconds, rounded to the first digit) required to generate and solve to optimality the formulation X, with X $\in \{VPS, \mathcal{F}_2\}$ on one instance, averaged over the solved instances, only;
- ¬OptX denotes the percentage of instances which X was unable to solve within the time limit, with X $\in \{VPS, \mathcal{F}_2\}$. A " - " in a cell indicates that none of the ten instances was solved to optimality because the time limit was exceeded. An empty space in a cell indicates that all the ten instances were solved to optimality (an entire row of empty spaces was removed from the table).

Denote by t_X, q_X the computing time of formulation X, for X $\in \{VPS, \mathcal{F}_2\}$, averaged over the solved instances, and its percentage of unsolved instances within the time limit, and let $t_X\ (1 - q_X) + 600\ q_X$ be a lower bound on the computing time averaged over solved and unsolved instances. We say that formulation A outperforms formulation B when *i*) $t_A \leq t_B$ and $q_A \leq q_B$ (see $TI(120, 210, 0.2)$), or when *ii*) $t_A\ (1 - q_A) + 600\ q_A\ \leq\ t_B\ (1 - q_B) + 600\ q_B$ (see $TI(120, 180, 0.1)$). In each cell the approach which outperforms the other one is highlighted in grey.

In the following, the computational results of the exact procedures for different values of n are shown. Click here to view computational results on TI's instances, here for the ones on the TM's instances, here for the TS's and here for the TT's.

			B									
			120	150	180	210	240	270	300	330	360	390
Δ	0	TimeVPS	2.04	6.28	12.38	18.77	31.03	48.90	68.71	126.11	190.38	221.24
		¬OptVPS										10%
		Time$\mathcal{F}_2$	90.81	-	-	-	-	-	-	-	418.90	-
		¬Opt$\mathcal{F}_2$	20%	100%	100%	100%	100%	100%	100%	100%	90%	100%
	0.1	TimeVPS	-	-	-	-	-	-	-	-	-	-
		¬OptVPS	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	119.73	-	-	-	-	-	447.70	535.76	472.37	405.40
		¬Opt$\mathcal{F}_2$	10%	100%	100%	100%	100%	100%	90%	70%	70%	40%
	0.2	TimeVPS	-	-	-	-	-	-	-	-	-	-
		¬OptVPS	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	106.70	-	-	-	-	-	-	499.24	448.13	379.24
		¬Opt$\mathcal{F}_2$	10%	100%	100%	100%	100%	100%	100%	90%	60%	40%
	0.3	TimeVPS	305.41	-	-	-	-	-	-	-	-	-
		¬OptVPS	50%	100%	100%	100%	100%	100%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	130.79	-	-	-	-	-	37.80	16.45	16.55	16.60
		¬Opt$\mathcal{F}_2$	10%	100%	100%	100%	100%	100%	30%			
	0.4	TimeVPS	151.30	-	-	-	-	-	-	-	-	-
		¬OptVPS		100%	100%	100%	100%	100%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	120.65	-	-	110.99	20.30	20.45	20.37	20.38	20.35	20.67
		¬Opt$\mathcal{F}_2$	10%	100%	100%	80%						
	0.5	TimeVPS	61.74	423.58	-	-	-	-	-	-	-	-
		¬OptVPS		90%	100%	100%	100%	100%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	121.16	-	41.77	23.01	23.14	23.48	23.30	23.20	23.47	23.43
		¬Opt$\mathcal{F}_2$	10%	100%	30%							
	0.6	TimeVPS	17.50	72.88	134.58	234.46	317.47	369.83	368.22	334.63	308.13	259.03
		Time$\mathcal{F}_2$	126.75	28.79	27.39	26.80	26.98	26.97	26.80	26.97	27.07	26.93
		¬Opt$\mathcal{F}_2$	20%	30%								
	0.7	TimeVPS	8.02	15.31	23.66	30.41	33.63	34.39	33.76	32.88	32.00	30.98
		Time$\mathcal{F}_2$	38.64	29.87	30.06	29.48	29.40	29.92	29.38	29.19	29.57	29.51
		¬Opt$\mathcal{F}_2$	10%									
	0.8	TimeVPS	5.03	7.82	9.89	10.68	10.87	10.60	10.39	10.28	9.99	9.85
		Time$\mathcal{F}_2$	31.34	30.98	30.86	30.42	30.32	30.69	30.55	30.41	30.60	30.75
	0.9	TimeVPS	3.11	4.66	5.65	5.86	5.77	5.62	5.53	5.44	5.42	5.23
		Time$\mathcal{F}_2$	29.00	29.68	29.95	29.51	29.90	30.06	29.53	29.34	29.52	29.52

Table 15: Computational results obtained by VPS and $\mathcal{F}_2$ on $TI(500, B, \Delta)$

d	f(d)							B					
			120	150	180	210	240	270	300	330	360	390	400
0	0	TimeVPS	0.16	0.42	0.95	1.68	2.55	4.39	5.46	7.25	11.16	11.84	16.42
		Time$\mathcal{F}_2$	0.41	78.18	111.32	65.26	11.98	6.80	7.91	4.10	4.41	2.69	2.69
		$\neg$Opt$\mathcal{F}_2$			30%	20%							
0.1	0.02	TimeVPS	0.24	0.65	1.64	2.95	5.01	6.91	10.97	15.92	21.26	25.65	27.09
		Time$\mathcal{F}_2$	0.38	20.66	24.69	129.33	10.03	35.45	5.41	3.49	3.10	1.51	2.09
		$\neg$Opt$\mathcal{F}_2$		20%	20%	30%			10%				
0.2	0.08	TimeVPS	0.28	0.90	2.38	5.90	9.00	12.97	19.94	24.39	24.86	28.34	30.86
		Time$\mathcal{F}_2$	0.43	47.14	26.69	66.40	5.33	2.78	1.73	0.39	1.07	0.81	0.18
		$\neg$Opt$\mathcal{F}_2$		10%	20%	40%							
0.3	0.18	TimeVPS	0.35	1.11	2.97	4.56	7.71	11.96	12.19	15.16	19.91	23.66	21.30
		Time$\mathcal{F}_2$	0.41	73.85	36.55	1.41	1.92	0.18	0.13	0.13	0.13	0.13	0.12
		$\neg$Opt$\mathcal{F}_2$		10%	10%								
0.4	0.32	TimeVPS	0.35	1.07	2.47	4.06	6.44	9.61	13.46	16.87	20.37	25.33	28.54
		Time$\mathcal{F}_2$	0.50	3.05	0.47	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
0.5	0.5	TimeVPS	0.28	0.76	1.62	3.14	5.32	8.52	12.02	15.49	19.66	23.19	23.64
		Time$\mathcal{F}_2$	0.21	0.10	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
0.6	0.68	TimeVPS	0.18	0.50	1.00	1.93	3.38	5.24	7.54	10.65	13.15	15.92	15.85
		Time$\mathcal{F}_2$	0.15	0.10	0.10	0.10	0.10	0.10	0.09	0.09	0.10	0.09	0.10
0.7	0.82	TimeVPS	0.10	0.23	0.49	0.89	1.39	2.20	3.15	4.37	5.30	6.26	6.81
		Time$\mathcal{F}_2$	0.07	0.08	0.09	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
0.8	0.92	TimeVPS	0.06	0.12	0.20	0.35	0.58	0.79	1.04	1.32	1.62	1.87	2.08
		Time$\mathcal{F}_2$	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
0.9	0.98	TimeVPS	0.04	0.06	0.06	0.08	0.10	0.13	0.15	0.17	0.20	0.23	0.26
		Time$\mathcal{F}_2$	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05

Table 16: Computational results obtained by VPS and $\mathcal{F}_2$ on $TM(120, B, \Delta)$

d	f(d)							B					
			120	150	180	210	240	270	300	330	360	390	400
0	0	TimeVPS	0.28	1.95	9.39	79.37	278.45	376.92	-	-	-	-	-
		$\neg$OptVPS					10%	40%	100%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	0.74	61.39	166.84	23.26	78.21	8.84	11.91	6.45	6.79	1.04	5.93
		$\neg$Opt$\mathcal{F}_2$		50%	60%	20%	30%				10%	10%	20%
0.1	0.02	TimeVPS	0.39	2.03	10.16	76.46	328.84	419.05	593.53	-	-	-	-
		$\neg$OptVPS					20%	60%	90%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	0.23	26.64	125.42	192.31	57.24	7.08	40.43	3.15	3.20	2.18	3.77
		$\neg$Opt$\mathcal{F}_2$	10%	40%	70%	40%	20%	20%	20%			10%	
0.2	0.08	TimeVPS	0.36	2.27	13.91	53.13	194.91	396.67	292.04	441.25	411.00	-	-
		$\neg$OptVPS						20%	60%	90%	80%	100%	100%
		Time$\mathcal{F}_2$	0.38	32.49	188.01	99.74	4.42	3.40	10.47	13.56	0.29	0.18	0.14
		$\neg$Opt$\mathcal{F}_2$		40%	70%	40%	50%	10%				10%	
0.3	0.18	TimeVPS	0.54	3.87	21.67	58.72	98.53	193.73	372.88	402.83	380.79	-	-
		$\neg$OptVPS					10%	10%	20%	50%	80%	100%	100%
		Time$\mathcal{F}_2$	52.68	6.03	161.72	2.13	0.34	0.13	0.13	0.12	0.12	0.13	0.12
		$\neg$Opt$\mathcal{F}_2$		50%	30%	20%							
0.4	0.32	TimeVPS	0.41	1.55	5.41	15.07	44.17	100.07	203.84	316.29	421.42	516.67	522.12
		$\neg$OptVPS							10%	30%	60%	80%	80%
		Time$\mathcal{F}_2$	0.15	2.41	0.31	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
		$\neg$Opt$\mathcal{F}_2$		30%									
0.5	0.5	TimeVPS	0.27	0.79	2.21	6.44	17.83	44.36	119.44	246.39	369.41	530.68	534.25
		$\neg$OptVPS									30%	70%	80%
		Time$\mathcal{F}_2$	0.20	0.10	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
0.6	0.68	TimeVPS	0.19	0.55	1.35	3.74	9.49	22.78	50.01	96.82	137.81	251.82	253.93
		$\neg$OptVPS									10%	10%	20%
		Time$\mathcal{F}_2$	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.09	0.10	0.10
0.7	0.82	TimeVPS	0.09	0.23	0.51	1.14	2.10	4.48	8.50	16.42	29.56	45.17	54.03
		Time$\mathcal{F}_2$	0.08	0.08	0.09	0.09	0.08	0.08	0.08	0.08	0.08	0.08	0.08
0.8	0.92	TimeVPS	0.06	0.11	0.23	0.43	0.70	1.19	1.85	2.97	4.35	6.16	6.99
		Time$\mathcal{F}_2$	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
0.9	0.98	TimeVPS	0.04	0.07	0.09	0.13	0.19	0.29	0.43	0.61	0.81	1.01	1.20
		Time$\mathcal{F}_2$	0.05	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05

Table 17: Computational results obtained by VPS and $\mathcal{F}_2$ on $TS(120, B, \Delta)$

			B									
			120	150	180	210	240	270	300	330	360	390
Δ	0	TimeVPS	2.04	6.16	12.10	18.35	30.68	48.14	138.93	125.99	345.48	353.63
		¬OptVPS									10%	10%
		Time$\mathcal{F}_2$	95.40	-	-	-	-	-	-	-	419.12	-
		¬Opt$\mathcal{F}_2$	20%	100%	100%	100%	100%	100%	100%	100%	90%	100%
	0.1	TimeVPS	18.69	96.42	266.36	494.62	-	-	-	-	-	-
		¬OptVPS				70%	100%	100%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	48.80	-	-	-	-	41.98	2.73	2.67	2.63	2.66
		¬Opt$\mathcal{F}_2$	10%	100%	100%	100%	100%	40%				
	0.2	TimeVPS	30.05	159.04	464.00	548.15	-	-	-	-	-	-
		¬OptVPS			40%	60%	100%	100%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	64.49	-	-	2.39	2.44	2.62	2.64	2.64	2.63	2.65
		¬Opt$\mathcal{F}_2$	20%	100%	100%							
	0.3	TimeVPS	33.13	179.72	302.98	506.35	-	-	-	-	-	-
		¬OptVPS				70%	100%	100%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	164.41	154.79	2.48	2.49	2.50	2.60	2.63	2.64	2.60	2.65
		¬Opt$\mathcal{F}_2$	10%	70%								
	0.4	TimeVPS	33.61	118.63	319.06	529.44	-	-	-	-	-	-
		¬OptVPS				90%	100%	100%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	88.87	5.38	2.75	2.71	2.75	2.80	2.81	2.83	2.81	2.82
		¬Opt$\mathcal{F}_2$	20%									
	0.5	TimeVPS	27.23	108.90	325.88	587.70	-	-	-	-	-	-
		¬OptVPS				90%	100%	100%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	23.42	2.87	3.06	3.05	3.07	3.09	3.13	3.08	3.11	3.13
		¬Opt$\mathcal{F}_2$	10%									
	0.6	TimeVPS	20.14	74.86	230.80	532.32	-	-	-	-	-	-
		¬OptVPS				40%	100%	100%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	5.22	3.14	3.31	3.37	3.37	3.40	3.40	3.40	3.41	3.46
	0.7	TimeVPS	13.49	50.79	149.61	408.60	-	-	-	-	-	-
		¬OptVPS					100%	100%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	3.81	3.39	3.56	3.65	3.62	3.63	3.61	3.72	3.63	3.66
	0.8	TimeVPS	8.41	30.88	88.39	246.07	514.53	599.05	-	-	-	-
		¬OptVPS					50%	90%	100%	100%	100%	100%
		Time$\mathcal{F}_2$	3.28	3.48	3.65	3.72	3.69	3.68	3.73	3.73	3.80	3.71
	0.9	TimeVPS	3.70	12.11	33.02	87.39	262.41	446.03	407.47	-	-	-
		¬OptVPS							60%	100%	100%	100%
		Time$\mathcal{F}_2$	3.01	3.25	3.31	3.30	3.31	3.31	3.30	3.31	3.30	3.41

Table 18: Computational results obtained by VPS and $\mathcal{F}_2$ on $TT(500, B, \Delta)$

A General ILP formulation for $BPPC$

$BPPC$ on arbitrary conflict graphs $G = (V, E)$ is usually formulated in this way (Gendreau et al. (2004); Fernandes Muritiba et al. (2010); Sadykov and Vanderbeck (2013)), where $x_{ij} = 1$ if item i belongs to subset V_j and 0 otherwise, for $i = 1, \dots, n$ and $j = 1, \dots, n$, and $y_j = 1$ if subset $V_j \neq \emptyset$ and 0 otherwise, for $j = 1, \dots, n$.

$$
\begin{array}{lll}
\mathcal{F}_1 : \min & \sum_{j=1}^{n} y_j & \\
& \sum_{j=1}^{n} x_{ij} = 1 & i = 1, \dots, n \qquad (1) \\
& \sum_{i=1}^{n} w_i x_{ij} \leq B y_j & j = 1, \dots, n \qquad (2) \\
& x_{ij} + x_{kj} \leq 1 & \forall (i,k) \in E \text{ and } j = 1, \dots, n \qquad (3) \\
& x_{ij} \in \{0,1\} & i = 1, \dots, n \text{ and } j = 1, \dots, n \\
& y_j \in \{0,1\} & j = 1, \dots, n
\end{array}
$$

Assignment constraints (1) can be also written as $\sum_{j=1}^{n} x_{ij} \geq 1$ for all i. Constraints (2) are the classical capacity constraints and force variable $y_j = 1$ when the corresponding bin is non-empty, and constraints (3) are the classical conflict constraints.

The formulation can be strengthened in several ways:

i) Constraints (3) can be replaced by the *clique constraints*: $\sum_{i \in C} x_{ij} \leq 1$ for all the maximal subsets C of vertices inducing a complete subgraph. This is particularly convenient when the graph allows to easily compute all the maximal subsets C of vertices inducing a complete subgraph. We remark that when G is an interval graph all the maximal subsets C of vertices inducing a complete subgraph can be computed in linear time;

ii) Both versions of constraints (3) can be strengthened by replacing the right hand side by y_j;

iii) To break the symmetries, one can add the constraints $y_{j+1} \leq y_j$ for $j = 1, \dots, n-1$;

iv) To reduce the number of the x variables, one can define x_{ij} for $i = 1, \dots, n$ and $j = 1, \dots, \min\{i, n\}$, only.

v) Let W be a subset of vertices inducing a complete subgraph of maximum size and H be the subset of vertices with weights greater than $B/2$. If $|W| \geq |H|$ then set $A := W$, otherwise set $A := H$. W.l.o.g. rename the items in such a way that the items in A are the first ones in the new ordering. Then, to break symmetries, one can add to the formulation the constraints $x_{ii} = 1$ for $i = 1, \dots, |A|$.

vi) If one knows a lower bound ($LB \geq |A|$) on the value of the objective function for a given instance, then w.l.o.g. one can fix $y_j = 1$ for $j = 1, \ldots, LB$, and if one knows an upper bound (UB), then w.l.o.g. one can fix $y_j = 0$ for $j = UB + 1, \ldots, n$.

The resulting formulation $\mathcal{F}_2$ is the following:

$$
\begin{array}{lll}
\mathcal{F}_2 : \min & \sum_{j=LB+1}^{UB} y_j + LB & \\
 & x_{ii} = 1 & i = 1, \ldots, |A| \\
 & x_{ij} = 0 & i = 1, \ldots, |A|, \quad j = 1, \ldots, UB, \quad i \neq j \\
 & \sum_{j=1}^{\min\{i;UB\}} x_{ij} = 1 & i = |A| + 1, \ldots, n \\
 & \sum_{i=|A|+1}^{n} w_i x_{ij} \leq B - w_j & j = 1, \ldots, |A| \\
 & \sum_{i=j}^{n} w_i x_{ij} \leq B & j = |A| + 1, \ldots, LB \\
 & \sum_{i=j}^{n} w_i x_{ij} \leq B y_j & j = LB + 1, \ldots, UB \\
 & \sum_{i \in C \, \wedge \, i \geq j} x_{ij} \leq 1 & \text{for all maximal subsets } C \text{ inducing a complete subgraph, } j = 1, \ldots, UB \\
 & y_{j+1} \leq y_j & j = LB + 1, \ldots, UB - 1 \\
 & x_{ij} \in \{0, 1\} & i = 1, \ldots, n, \quad j = 1, \ldots, UB, i \geq j \\
 & y_j \in \{0, 1\} & j = LB + 1, \ldots, UB
\end{array}
$$

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