Optimal Planning of Waste Sorting Operations

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A mixed integer linear program is used to schedule the selection operations of a two-phase waste selection process.
MILP - parameters

\( j = \{1, \ldots, J\} \): index of the \( J \) sorting stages

\( p = \{1, \ldots, P\} \): index of the \( P \) time-shifts

\( T \): time horizon partitioned in time shifts with \( t \in \{1, \ldots, T\} = T_1 \cup \ldots \cup T_P \)

\( C \): hourly cost of each operator

\( \sigma_t \): working hours for time \( t \) determined by the corresponding shift \( p \)

\( C_t = C \times \sigma_t \): cost of each operator at time \( t \)

\( f_j \): set-up cost of sorting stage \( j \)

\( a_t \): quantity of material in kg unloaded from trucks at time \( t \)

\( \alpha_j \): percentage of waste processed in stage \( j - 1 \), received in input by buffer \( j \)

\( S_j \): maximum inventory capacity of the sorting stage buffer \( j \)

\( LC_j \): critical stock level threshold of buffer \( j \)

\( \rho_j \): fraction of material allowed to be left at buffer \( j \) at the end of time horizon

\( K_j \): single operator hourly production capacity [kg/h] of sorting stage \( j \)

\( SK_{j,t} = K_j \times \sigma_t \): operator sorting capacity in sorting stage \( j \), at time \( t \)

\( M \): maximum number of operators available in each time shift

\( E_j \): minimum number of operators to be employed in each time shift of stage \( j \)

\( \partial h_{ij} \): slope of the \( i \)-th part of linearization of the buffer \( j \) stock cost curve
MILP - parameters

- Time partioned in working shift

\[ j = \{1, \ldots, J\} : \text{index of the } J \text{ sorting stages} \]
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\[ T : \text{time horizon partitioned in time shifts with } t \in \{1, \ldots, T\} = T_1 \cup \ldots \cup T_P \]
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\[ f_j : \text{set-up cost of sorting stage } j \]
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\[ K_j : \text{single operator hourly production capacity [kg/h] of sorting stage } j \]
\[ SK_{j,t} = K_j \cdot \sigma_t : \text{operator sorting capacity in sorting stage } j, \text{ at time } t \]
\[ M : \text{maximum number of operators available in each time shift} \]
\[ E_j : \text{minimum number of operators to be employed in each time shift of stage } j \]
\[ \partial h^i_j : \text{slope of the } i\text{-th part of linearization of the buffer } j \text{ stock cost curve} \]
MILP - parameters - Time dimension

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\[ \sigma_t : \text{working hours for time } t \text{ determined by the corresponding shift } p \]
\[ C_t = C \ast \sigma_t : \text{cost of each operator at time } t \]
\[ f_j : \text{set-up cost of sorting stage } j \]
\[ a_t : \text{quantity of material in kg unloaded from trucks at time } t \]
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MILP - parameters

- Sorting stages dimension

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MILP - parameters

- Stock costs

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\[ T : \text{time horizon partitioned in time shifts with } t \in \{1, \ldots, T\} = T_1 \cup \ldots \cup T_P \]
\[ C : \text{hourly cost of each operator} \]
\[ \sigma_t : \text{working hours for time } t \text{ determined by the corresponding shift } p \]
\[ C_t = C * \sigma_t : \text{cost of each operator at time } t \]
\[ f_j : \text{set-up cost of sorting stage } j \]
\[ a_t : \text{quantity of material in kg unloaded from trucks at time } t \]
\[ \alpha_j : \text{percentage of waste processed in stage } j - 1, \text{received in input by buffer } j \]
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\[ SK_{j,t} = K_j * \sigma_t : \text{operator sorting capacity in sorting stage } j, \text{at time } t \]
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MILP - variables

\( x_{j,t} \in \mathbb{Z}^+ \): operators employed in the sorting stage \( j \) at time \( t \)

\( u_{j,t} \in \mathbb{R}^+ \): processed quantity at stage \( j \) at time \( t \)

\( y_{j,t} \in \{0, 1\} \): equal to 1 if stage \( j \) is activated at time \( t \), 0 otherwise

\( I_{j,t} = I_{j,t}' + I_{j,t}'' \geq 0 \): stock level of material in buffer \( j \) at time \( t \); for each stage \( j \) the corresponding \( I_{j,t}' \) and \( I_{j,t}'' \) represent the inventory level before and after reaching the critical threshold respectively.

\( w_{j,t} \in \{0, 1\} \): equal to 1 if \( I_{j,t}'' > 0 \), 0 otherwise. Indeed, this binary variables are used to model the piece-wise linear functions of the buffer stock costs.
\[ \begin{align*} 
\text{MILP - features} \\
\text{Work shifts scheduling} \\
y_{j,t} \in \{0,1\} : \text{equal to 1 if stage } j \text{ is activated at time } t, \text{ 0 otherwise} \\
\sum_{j \in J} x_{j,t} \leq M \\
E_j y_{j,t} \leq x_{j,t} \leq M y_{j,t} \quad \forall j \in J, t \in T_p, p \in P \\
\sum_{j \in J} x_{j,t} \leq M \quad \forall t \in T \\
u_{j,t} \leq SK_{j,t} x_{j,t} \quad \forall j \in J, t \in T \\
I_{1,t} = I_{1,t-1} + a_t - u_{1,t} \quad \forall t \in T \setminus 0 \\
I_{j,t} = I_{j,t-1} - u_{j,t} + \alpha_j u_{j-1,t} \quad \forall t \in T \setminus 0, j \in J \setminus 1 \\
I_{j,t} = I'_{j,t} + I''_{j,t} \quad \forall j \in J, t \in T \\
LC_j w_{j,t} \leq I'_{j,t} \leq LC_j \quad \forall j \in J, t \in T \\
0 \leq I''_{j,t} \leq (S_j - LC_j) w_{j,t} \quad \forall j \in J, t \in T \\
I_{j,T} \leq \rho_j LC_j \quad \forall j \in J \\
x_{j,t} \in \mathbb{Z}^+ \quad \forall j \in J, t \in T \\
u_{j,t} \in \mathbb{R}^+ \quad \forall j \in J, t \in T \\
y_{j,t} \in \{0, 1\} \quad \forall j \in J, t \in T 
\end{align*} \]
MILP - features

- **Work shifts scheduling**
  \[ y_{j,t} \in \{0,1\} : \text{equal to 1 if stage } j \text{ is activated at time } t, \text{ 0 otherwise} \]

- **Production lot-sizing**
  \[ u_{j,t} \in \mathbb{R}^+ : \text{processed quantity during stage } j \text{ at time } t \]

- **Workforce allocation**
  \[ x_{j,t} \in \mathbb{Z}^+ : \text{operators employed in sorting stage } j \text{ at time } t \]

\[
\begin{align*}
\min Z &= \sum_{j \in J} \sum_{t \in T} C_j x_{j,t} + \sum_{j \in J} \sum_{t \in T} f_j y_{j,t} + \sum_{j \in J} \sum_{t \in T} (\partial h_j^1 l_{j,t} + \partial h_j^2 l_{j,t}^n) \\
\text{s.t.} & \\
E_j y_{j,t} & \leq x_{j,t} \leq M y_{j,t} \quad \forall j \in J, t \in T_p, p \in P \quad (2) \\
\sum_{j \in J} x_{j,t} & \leq M \quad \forall t \in T \quad (3) \\
u_{j,t} & \leq SK_j x_{j,t} \quad \forall j \in J, t \in T \quad (4) \\
i_{j,t} &= i_{j,t-1} + \alpha_{j} - u_{j,t} \quad \forall t \in T \setminus 0 \quad (5) \\
i_{j,t} &= i_{j,t} - u_{j,t} + \alpha_{j} u_{j-1,t} + \alpha_{j} u_{j-1,t} \quad \forall t \in T \setminus 0, j \in J \setminus 1 \quad (6) \\
i_{j,t} &= i_{j,t} + i_{j,t}^n \quad \forall j \in J, t \in T \quad (7) \\
LC_j w_{j,t} & \leq i_{j,t} \leq LC_j \quad \forall j \in J, t \in T \quad (8) \\
0 & \leq i_{j,t}^n \leq (S_j - LC_j) w_{j,t} \quad \forall j \in J, t \in T \quad (9) \\
i_{j,T} & \leq \rho_j LC_j \quad \forall j \in J \quad (10) \\
x_{j,t} & \in \mathbb{Z}^+ \quad \forall j \in J, t \in T \quad (11) \\
u_{j,t} & \in \mathbb{R}^+ \quad \forall j \in J, t \in T \quad (12) \\
y_{j,t} & \in \{0,1\} \quad \forall j \in J, t \in T \quad (13)
\end{align*}
\]

Subject to uncertainty
MILP - ReFormulation

Constraints of stock flows regarding the first buffer consider the parameter $a_t$ being subject to uncertainty.

These constraints are reformulated in order to match the nominal problem form of Berstsimas and Sim robustness theory.

Auxiliary variable $\varepsilon_t$ is introduced.

Constraint (7) guarantees stock PWL costs to be included in the objective function.
Robust Formulation

The protection function is:

\[ z_t \Gamma_t + \sum_{k=0}^{t} p_{t,k} \]

Where \( \Gamma_t \) is the protection parameter (robustness lever).

\( \hat{a}_t \) is the maximum deviation of parameter \( a_t \) being subject to uncertainty

Robustness variables \( z_t \) and \( p_{t,k} \) must respect constraint (31)
Application impact related to $|J_i|$. Moderate impact for small $|J_i| \forall i$. 

The price of robustness

Protection magnitude scenarios: from deterministic to worst case

Price of robustness w.r.t. protection magnitude
Current and future works

- Enlarge experiments and considerations regarding robustness and probabilistic bounds over constraint violations. Considering correlation between parameters that are subject to uncertainty.

- Develop a production model version that includes waste baling press operations.

- Considering production capacity as a function of operators employed.

- Profit patterns recognition through logistic and sorting models integration: framework of models integration for profitability analysis and contract management support.
Models Integration
information flow
diagram for profit patterns dataset creation

SLWA := Scheduling with Lot sizing and Workforce Allocation
PDTW := Pickup and Delivery with Time Windows

Machine Learning estimation of $F$ s.t. $F(\text{Client} + \text{SLWA} + \text{PDTW features}) = \text{Client Cost Performance}$

Client features SLWA instance features PDTW instance features

Cost = $C_{\text{SLWA}} + C_{\text{PDTW}}$