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A MATHEMATICAL MODEL FOR CONTAINERS FORWARDING FROM MEGA-SHIPS TO DRY PORTS WITH LIMITED NETWORK CAPACITY AND TIME PERIOD DEPENDENT TRAVEL COST FUNCTION

R. 20-03, July 2020

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This work has been partially supported by PRIN program of the Ministry of Instruction, University, and Research (MIUR) through the project SPORT Smart PORt Terminals [Grant No. 2015XAPRKF]

ISSN: 1128–3378
Abstract

This work investigates the impact of mega-ships on the port surrounding network and proposes a mixed integer programming model to optimize the forwarding of containers toward dry port destinations. The aim is to give a decision support for operational planning with limited capacity of both the yard and the network, with respect to the quantity of containers transferred per time unit. The model considers time dependent costs and traveling times to reduce congestion. In particular, congestion is limited by delaying departing times and exploiting the storage capacities of the nodes. As a further novel issue, the problem is modelled as a multi-period network problem where arc costs and traveling times change during specific time slots. More precisely, new linear functions are derived as tangents to the nonlinear convex components of a classical travelling time function proposed in the literature. The model is used to study the container dispatching process in a terminal which is going to be the main Italian container terminal equipped to manage mega-ship traffic.

*Key words:* Mega-ships, Inland Forwarding, Multi-commodity Flow, Linearization
1. Introduction

Container vessels account for about a quarter of the world's total fleet and are therefore essential for the international transport of goods. The massive development of multilateral trade originated the development in shipping business, thus causing the resulting phenomenon of naval gigantism, leading to the construction of naval cargo up to 500,000 tons. Accordingly, the size of the current generation of containerships could potentially further increase capacity to 22,000 TEUs. The current trends in liner shipping confirm such economies of scale, which means larger full-containerships and noticeable call sizes in ports.

Critical issues of naval gigantism concern a possible increase in costs paid by ports and their hinterland more than proportional to the growth of the naval size. In fact, mega ships have led to a series of consequences in terms of infrastructural adjustments of ports and information technology systems for the organization of the handling operations, the peaks of port congestion and the environmental impact [14]. Therefore, it is necessary to analyze the trade-offs between the benefits offered by mega-containerships and their cost over the entire transport chain. Such costs include pollution and traffic congestion of the inland transportation networks [32, 14].

As already said, container terminal operations are going to be highly affected by the mega-ships trend. Mega-ships allow a clear reduction in transport costs but require investments in handling and short term storing capacity. Thousands of unloaded containers must be dispatched from the yard to the required destinations in short time. In particular, the role of the port in coordinating the hinterland’s logistics activities has become a necessary aspect to maintain competitiveness and defend the customer portfolio. Therefore, terminal operators must be able to handle containers as quickly as possible, not only with reference to berth performances, but also to the inland forwarding [26, 15].

In order to cope with this need and being able to face the technological demands of containerization, around the world, the port industry has invested a lot. In particular, modern container terminals have been built and new and more efficient organizational forms have been adopted to speed up port operations [14]. These developments took place in the understanding of governments and local authorities that ports now constitute the most important node in the overall door-to-door supply chain. Thus, bottlenecks and inefficiencies in maritime terminals can easily invalidate all benefits derived from economies of scale due to naval gigantism. In particular, connections with the inland transportation often become the weak point for a port, with the risk of nullifying the efforts of the terminal operator in the optimization of quay-side and yard operations.

Moreover, while mega-ships allow a reduction in transport costs, their deployment requires investments in handling and short term storing capacity. Thus, as already mentioned, one of the main problems seaports face today is the lack of space at maritime terminals and the growing congestion on their access routes with the inland connections, especially considering the road modality [10]. In view of the foreseen increasing trend of freight transport in the next years throughout Europe up to 80% [5], in the recent literature many authors proposed the use of dry ports and hub and spoke networks in order to improve the performances and the competitiveness of the inland shipping. Suggested papers are, among others, [13, 30, 2, 19]. In this scenario, the importance of ports as international logistic nodes will increase too; therefore, higher freight flows among ports and hinterlands have to be even more efficiently managed. Note that connections between a port and its hinterland are not just important in terms of efficiency but also for the overall costs due to different transport solutions, accounting not only for direct private costs but also for external ones [10, 15]. In this context, it is strongly required to promote trans-
port policies able to shift part of the volumes destined to road transport to other modalities. However, in many countries, in particular in Italy, road transport is still the most convenient shipping modality [26, 24]. Therefore, there is a real need to consider the social costs derived from such a high increasing volume of goods traveling every day on the main highway connections, merging them with the private and commercial vehicles and at the same time making the road transportation network more sustainable. In this direction, in the last decade a number of research works focused on the evaluation of the negative impact of high volumes of containerized flows shipped by using the road modality [7, 17, 29, 4].

In this paper, we consider a road transportation network connecting ports to inland destinations in which some constraints related to both the network and the vehicles play a relevant role (see, e.g. [3, 11]). We propose a mathematical programming model to optimize the containers forwarding towards possible dry ports and final destinations. The objective of the mathematical model is the minimization of the road congestion which is measured by the sum of the flow of container trucks multiplied by the arc travelling time. The aim is to give a decision support for operational planning in the presence of limited capacity in both the yard of the terminal for container storage and the network due to the quantity of containers transferred per time unit. Other research works have been recently proposed in this direction. In particular, [31] propose a Tabu search heuristic for a trucking company with a homogeneous fleet to receive inbound containers or ship outbound ones. [12] present a mixed integer linear programming model for dispatching 20 and 40 feet containers, with the aim of minimizing either the travelling distance or the operation time of the different types of trucks involved in the transportation process. As a novel issue, the proposed model considers time dependent costs and traveling times varying according to the amount of flow on the network. The purpose is to consider in the decisions the reduced capacity of the road transport network that could result in congestion, especially in the links originating from the ports. In particular, we include in the model a possible delay in the departing time of the vehicles from the origin ports as soon as a congestion level on the surrounding arcs is close to given threshold values. Shipment delays have been previously considered in [16] with the aim to explore the effect of limited resources and modal transit time variability on hub networks in the presence of service time requirements. Further, [33] proposed a two-phase game model to study the ocean carriers decision about the free detention time and the time when the container arrived at the inland terminal is dispatched to the sea container terminal. The present model allows us to analyse the containers dispatching process in a case study derived from APM-VL (Vado Ligure) which is going to be the main Italian container terminal equipped to manage mega-ship traffic; APM-VL container terminal is part of the Port Authority of the Western Ligurian Sea, which also includes the ports of Genoa, Savona and Pra. The terminal is active from the second half of December 2019. The expected throughput of the terminal, expressed in terms of the number of containers handled per year, is about 800,000 TEUs. The main performance indices of the terminal under consideration have been recently analyzed in [25] by using a discrete event simulation study considering different ship arrival profiles. In the present work, the output performance indices data derived from [25] are considered for tuning the origin-destination containerized flow demand in different daily time periods. Further, we consider the limitations given by the network and the vehicles availability as in [3] and [11].

The remainder of this work is as follows. In section 2 the problem is presented and its linear formulation is described. In section 3 the application case is described while in section 4 results are discussed. Section 5 concludes the paper and gives outlines for future works.
2. Models for container dispatching

The literature on network flows and congestion analysis and optimization considers different modeling approaches based on equilibrium problem, simulation, and optimization. As for the approach proposed in our work, one key milestone is represented by the work of [23] that modeled the congestion caused by vehicles over a network flow, and congestion costs are minimized in a nonlinear formulation. In the model, the objective function is the minimization of the sum of non-negative, non-decreasing, continuous, convex functions depending on the flow for each arc in each given time period. Congestion is represented by functions acting on flow balancing constraints (through travel time). These functions are considered as non-decreasing, continuous, concave and represent the physical phenomenon of congestion. In order to solve the model, the authors linearize it by use of piece-wise linear approximation. The model is single source and single sink. As reported by the survey of [28], the fundamental work of [23] has been expanded in several directions in terms of methodologies and applications. The principal methodologies used to study the dynamic equilibrium assignment problem are mathematical programming, variational inequality, optimal control, and simulation-based, while the most studied applications are the traffic assignment, real-time deployment, and planning. [8] and [9] demonstrated the non convexity of the dynamic traffic assignment problem and clarified several problems of the mathematical programming approach when dealing with FIFO. FIFO means that, in the mathematical programming models, no overtaking is allowed among vehicles traveling an arc. Holding-back is considered in real situations and it means to favor certain traffic movements over others in order to minimize system-wide travel delays. An interesting research direction has been the introduction of stochastic features or uncertainties. In particular [6] extend the [23] problem and consider the stochastic case by assuming that the origin - destination matrix is not completely known for the entire planning horizon. A control theory based approach is discussed by [27] who considers as variables the split rates in subflows, allowing the computation of route guidance information. [9] introduce another important achievement for this research line. In their single sink model the authors investigate the impact of capacity and the relation between link flow, trip time and cost. Other extensions of the problem are related to different applications, as for example the one of [1] where the problem of electric vehicle routing and charging is discussed.

In this section we introduce a linear programming based approach to plan the dispatching of containers from the port to the destinations with the aim of minimizing congestion. We take the approach of mathematical programming and consider a more appropriate network flow model in which capacity constraints are considered for nodes and arcs while multi-sinks are modeled. The main issue when planning containers forwarding is to consider if the traveling time is or not dependent on the flow that is going to plan. Probably the most famous model of travel time is the Bureau of Public Roads (BPR) [22] function, which considers the travel time as dependent by the flow $q$ with the following polynomial relation:

$$
\tau = T_{ff} \left[ 1 + \alpha \left( \frac{q}{q_{pc}} \right)^{\beta} \right]
$$

Function [1] defines the traveling time $\tau$ of one arc with maximum capacity of $q_{pc}$ when a flow $q$ is present; $\alpha$ and $\beta$ are parameters, $T_{ff}$ is the traveling time when free flow is considered. The function is almost flat before the threshold $\beta$ is reached, then it starts to increase significantly. This function is the base of several convex optimization problems with the aim to solve the traffic equilibrium problem. Often, flow dependent or time dependent travel times should be considered.
in settings with more general planning problems. In these settings, the nonlinear parameter can be addressed in several ways in order to reduce the complexity, and the particular shape of the BPR function allows a straightforward piece-wise approximation.

2.1. The linear programming model

Having in mind the traveling time equation given in (1), in this section we propose a new model aimed at minimizing the total traveling time spent by all trucks along arcs of a given graph $G$ within the planning time horizon. The following assumptions apply.

**Assumption 2.1.** The flows generated by the trucks have no considerable effect on the traveling time in the time slot considered.

**Observation 2.1.** Assumption 2.1 means that we can consider a multi-period linear model. Practically, the planning horizon is discretized into $T$ time slots; the traveling time on the network varies during this time horizon, but it is independent of the output flow of the decision model.

**Assumption 2.2.** The number of containers required to be moved can be easily translated in a number of trucks used; if a fraction of the travels can be done with 2 truck capacity, a proportional reduction of demand is considered.

The network is represented as an oriented graph $G = (N, A)$ where $N$ is the set of nodes and $A$ is the set of directed arcs. For each node $j \in N$ and each time slot $t \in T$, a required demand $d^t_j$ of containers is given.

**Assumption 2.3.** We suppose all the deliveries are available at the source at time 0.

The demand should be delivered from the source $s \in N$, which corresponds to the unloading point of the port. Another parameter considered in the model is the traveling time $\tau_{ij}^t$, $\forall t \in T$, $\forall (i, j) \in A$, given for each arc of the network and for each time slot of the planning horizon.

**Definition 2.1.** The value $\tau_{ij}^t$ of the traveling time is defined in terms of numbers (integer $\geq 1$) of time slots, since it is considered to be divisible by the single time slot.

**Example 2.1.** For example if we divide the time horizon in time slots of 15 minutes, the traveling time of 60 minutes will be equal to $\tau_{ij}^t = 4$.

The objective of the problem is to minimize the total time spent on the network by the containers to reduce the contribution to congestion. Congestion can be reduced through the availability of buffer zones where containers can be “parked” on their route to destination. Each node $j \in N$ is associated with a buffer with limited capacity $\text{cap}_j$. For nodes corresponding to dry ports this capacity is greater than zero while for all other network nodes the capacity will be set to zero. Capacity is also considered for the arcs. So, for each arc $\forall (i, j) \in A$ and for each time slot $t \in T$ a maximum number of trucks allowed to travel that arc is given and denoted with $\text{cap}_{ij}^t$. The given parameter is time dependent.

Defined the network and the parameters of the problem, in the proposed model, we consider the following decision variables:

- $x_{ij}^t$, is the number of containers (trucks) entering arc $(i, j)$ during time slot $t$;
$y^t_j$ is the number of containers (trucks) in the buffer of node $j$ in time $t$.

Figure [1] reports a schema of the network flow and the related parameters and variables. The nodes are depicted as points, arrows denote flows, while the overturned triangle depicts a buffer node - a node of the network where containers can be temporarily stored, as for example a dry-port - and the dashed arrow denotes the demand $d^t_j$ to be satisfied. The figure schematizes the flow balancing equation: $x^t_{ij}$ plus $y^t_j$, minus the number of containers stored in the node $j$ at time $t - 1$, that is $y^{t-1}_j$, must be equal to the number of containers used to satisfy $d^t_j$, and the sum of the flow exiting from $j$ and entering to nodes $j'$ at time $t$, where $j'$ are all the direct successors of $j$.

The proposed model, in the afterwards named P0, can be then written as follows:

**Model P0**

\[
\begin{align*}
\min \quad & Z = \sum_{t \in T} \sum_{(i,j) \in A} \tau^t_{ij} x^t_{ij} \\
\text{s.t.} \quad & \sum_{i \in \delta^-(j); t=(t-\tau^t_{ij}) \geq 0} x^t_{ij} + y^{t-1}_j = y^t_j + \sum_{k \in \delta^+(j)} x^t_{jk} + d^t_j \\
& 0 \leq y^t_j \leq \text{cap}_i \\
& 0 \leq x^t_{ij} \leq \text{cap}_A^t_{ij} \\
& z^t_{ij} \geq 0, \quad y^t_j, x^t_{ij} \in \mathbb{Z} \geq 0
\end{align*}
\]

In model P0, equation (2) defines the objective function, meaning that the congestion is measured by the total traveling time spent by trucks on the network. Equation (3) is the flow conservation constraints set, which consider the variable traveling time; the symbols $\delta^-(j)$ and $\delta^+(j)$ are the notations used to indicate the arcs entering and exiting a node $j$, respectively. Equation (4) defines the capacity constraint for buffers of each node, while (5) imposes to the flow to respect the arcs capacity. Equations (6) specify the domain of the decision variables.
2.2. Modeling the traveling time

The traveling time of an arc can be considered as a scalar parameter only as first approximation. In this section we propose an improved model where the assumption 2.1 is removed and the following assumption is introduced.

**Assumption 2.4.** Traveling time is affected at least by 3 distinct features: i) daytime \( t \), ii) type of arc \((i,j)\), and iii) flow \( x_{t}^{ij} \).

While the first two features are represented in model P0 by parameter \( \tau_{t}^{ij} \), the third feature requires the introduction of a nonlinearity. In particular the BPR equation (1) is then considered and the new model, P1, becomes as follows:

Model P1:

\[
\begin{align*}
\min \ Z &= \sum_{t \in T} \sum_{(i,j) \in A} \tau_{t}^{ij} x_{t}^{ij} \\
\text{s.t.} \quad & \sum_{i \in \delta^{-}(j)} x_{t}^{ij} + y_{t}^{ij} = y_{t}^{ij} + \sum_{k \in \delta^{+}(j)} x_{t}^{jk} + d_{t}^{j} \quad j \in N, t \in T \\
& 0 \leq y_{t}^{ij} \leq \text{cap}_{i} \\
& 0 \leq x_{t}^{ij} \leq \text{capA}_{ij} \\
& \tau_{t}^{ij} = T_{ff} \left[ 1 + \alpha \left( \frac{x_{t}^{ij}}{q_{pc}} \right)^{\beta} \right] \\
& y_{t}^{ij}, x_{t}^{ij} \in \mathbb{Z}_{\geq 0} 
\end{align*}
\]

In this updated model, equation (8) represents the nonlinearity affecting the objective function (7), and it is defined through the constraints set (8). Model P1 can be linearized by a piece-wise linear approximation. The linearization can be done by upper, mean, or lower approximations. In the following, we describe a lower approximation by 1-degree linearization using tangents to each nonlinear convex term.

**Assumption 2.5.** In order to linearize the objective function given in (7), we suppose \( 0 \leq x_{t}^{ij} \leq \text{capA}_{ij} \) to be continuous.

**Observation 2.2.** The lower approximation can be done by tangents which can be easily computed because each objective function term \( Z(x_{t}^{ij}) = \tau_{t}^{ij} x_{t}^{ij} \) is a \( \beta + 1 \) grade polynomial which is continuous and differentiable under the assumption 2.5.

**Definition 2.2.** Let \( \tilde{A} \in A \) be the subset of arcs for which the traveling time \( x_{t}^{ij} \) is assumed to be nonlinear. We define \( \gamma_{ij}^{tm}, m = 1 \ldots M \) the set of points where we linearize the objective function term, with \( \gamma_{ij}^{t1} = 0 \), and \( \gamma_{ij}^{tM} = \text{capA}_{ij} \).

**Definition 2.3.** \( \forall \tilde{A} \in A \) we define the tuples \((a_{ij}^{mt}, b_{ij}^{mt}) \in M_{ij}^{t} \) such that the equations \( a_{ij}^{mt}\gamma_{ij}^{tm} + b_{ij}^{mt} = Z(\gamma_{ij}^{tm}) \) are verified.

Thus, for each tuple \((i,j,t)\) where a nonlinear term is defined, we select \( Z(\gamma_{ij}^{tm}) \) intersect points for the function to linearize. Then, by using the derivative of the objective function term
Proposition 2.4. \( \forall (i, j) \in \bar{A}, \forall m \in \{1, \ldots, M\} \) the slope \( a_{ij}^{tm} \) and the intercept \( b_{ij}^{tm} \) of the tangents are identified by:

\[
a_{ij}^{tm} = T_{ff} \left[ 1 + \alpha (\frac{\gamma_{ij}^{tm}}{q_p c}) \right] \beta
\]

and

\[
b_{ij}^{tm} = -\alpha T_{ff} \beta \gamma_{ij}^{tm} \left( \frac{\gamma_{ij}^{tm}}{q_p c} \right)^\beta
\]

Proof. The tangent in a point \( \gamma_{ij}^{tm} \) is computed by the equation

\[
y - Z(\gamma_{ij}^{tm}) = \frac{\partial Z}{\partial x_{ij}^t}(\gamma_{ij}^{tm}) (x_{ij}^t - \gamma_{ij}^{tm})
\]

from which we obtain

\[
a_{ij}^{tm} = \frac{\partial Z}{\partial x_{ij}^t}(\gamma_{ij}^{tm})
\]

and

\[
b_{ij}^{tm} = Z(\gamma_{ij}^{tm}) - \frac{\partial Z}{\partial x_{ij}^t}(\gamma_{ij}^{tm}) \gamma_{ij}^{tm}
\]

Considering that

\[
Z(x_{ij}^t) = T_{ff} \gamma_{ij}^{tm} \left[ 1 + \alpha \left( \frac{x_{ij}^t}{q_p c} \right) \beta \right]
\]

and that

\[
\frac{\partial Z}{\partial x_{ij}^t}(\gamma_{ij}^{tm}) = T_{ff} \gamma_{ij}^{tm} \left[ 1 + \alpha (\beta + 1) \left( \frac{\gamma_{ij}^{tm}}{q_p c} \beta \right) \right]
\]

by simple algebraic steps we obtain the equations (9) and (10).

Figure 2 shows an example of tangent approximation. Dashed lines in the figure represent the set of tangents computed and used to approximate the objective function term. As depicted, the lines are very close at the beginning and at end of the curve. Using a proper number of tangents, and intersect points, we can easily control the error and have a very tight approximation as demonstrated in the test case. This result in a very powerful mean to introduce nonlinearity as representation of congestion while controlling the complexity of the model and its usability as decision support tool.

Therefore, we can define a set of linear functions \( a_{ij}^{tm} x_{ij}^t + b_{ij}^{tm} \) as defined in (2.3) and demonstrated in (2.4), computed with the assumption (2.5). After the linearization we remove the assumption (2.5), so by setting back the integer property of \( x_{ij}^t \) we introduce the MILP model P2 as follows.
10.

Figure 2: Example of tangent approximation for convex polynomial term $\tau_{tij}^tx_{tij}$

Model P2

$$\min Z = \sum_{t \in T} \sum_{(i,j) \in \tilde{A}} \tau_{tij}^tx_{tij}^{t} + \sum_{t \in T} \sum_{(i,j) \in A} z_{tij}^{t}$$  \hspace{1cm} (11)

s.t.

$$\sum_{i \in \delta^{-}(j); t(t-\tau_{tij}) \geq 0} x_{tij}^{t} + y_{tj}^{t-1} = y_{tj}^{t} + \sum_{k \in \delta^{+}(j)} x_{tkj}^{t} + d_{tj}^{t} \quad j \in N, t \in T$$ \hspace{1cm} (3’)

$$0 \leq y_{tj}^{t} \leq \text{cap}_{ij} \quad j \in N, t \in T$$ \hspace{1cm} (4’)

$$0 \leq x_{tij}^{t} \leq \text{capA}_{ij}^{t} \quad (i,j) \in A, t \in T$$ \hspace{1cm} (5’)

$$z_{tij}^{t} \geq a_{ij}^{mt}x_{tij}^{t} + b_{ij}^{mt} \quad (a_{ij}^{mt}, b_{ij}^{mt}) \in M_{ij}^{t}, (i,j) \in \tilde{A}, t \in T$$ \hspace{1cm} (12)

$$z_{tij}^{t} \geq 0, \quad y_{tj}^{t}, x_{tij}^{t} \in \mathbb{Z}^{>0} \quad (i,j) \in A, t \in T$$ \hspace{1cm} (13)

Variables $z_{tij}^{t}$ and constraints set [12] in model P2 are used to define the linearization as tangent approximation as just explained. Given that the $\tau_{tij}^tx_{tij}$ function is non-decreasing convex, they are sufficient to define the approximation. We use this model in the study of the application case defined in the following section.

3. The case study

In this section we describe how the application case is addressed and how the baseline instance has been built. The aim is twofold. The first objective is to build an instance as close as possible
to the real scenario which is going to be investigated once the terminal is fully operational. The second objective is to let the instance be replicable and scalable in order to test the model on larger instances and to modify parameters once the terminal is in production and further data can be easily collected.

The application case considers the forwarding of containers from a terminal located in the port of Vado Ligure that is part of the Port Authority of the Western Ligurian Sea, which also includes the ports of Genoa, Savona and Pra (see [http://www.apmterminals.com](http://www.apmterminals.com/) for more details). Figure 3 shows the map of the area where the APM-VL terminal is located. The port of Vado Ligure is connected to the northernmost part of Italy via the highway network, consisting of the A6 Savona-Turin highway, the Brennero highway via the A33 Cuneo-Asti, and the A10 coastal highway. The new built terminal is designed to host mega-ships. The expected throughput of the terminal, expressed in terms of number of containers handled per year, is about 800,000 TEUs. Unloaded containers from each mega-ship must be forwarded to inland destinations in a short time. The demand and the locations are derived from estimations done in the previous works of [3], and [25]. The forwarding destinations are reached by using trans-shipment nodes (with or without buffer capacity) where are located dry ports or important shipment crossroads. The source points, buffers, and gate points of the container terminal enrich the selection of nodes. The gate points play an important role in the model because they are used to represent scenarios with flow dependent traveling times and congestion effects for the baseline instance. These data of the application case are reported in Table 1. The table reports, for each node of the network, the identification number, the name, the information if the node is a source, or a target node, the capacity, and the required demand. A negative value for the demand column denotes a source node. The total demand indicated in Table 1 is estimated to be of 2500 TEU per day, taking into account data from previous works, which estimate a total of 900,000 TEU over 360 days. These real case data are very important to validate the model as a decision support tool, and can be used to generate additional large scale instances if needed. For the planning
of the containers’ forwarding, we consider a 48 hour horizon with a discretization based on a time frame of 10 minutes \((TimeFrame)\). The 10 minutes discretization guarantees the model to capture enough variations in traveling time registered by the traffic system. The traveling times have been computed by using the data of two different sources. The first one is the Open Source Routing Machine - OSRM \([21]\), which has been used to compute the origin-destination distance matrix. The second one is Google Maps\(^\text{TM}\) used to compute information on max, min, and average traveling times extracted for each hour of the day of arcs. The arc capacity for the baseline instance has been set to \(q_{pc} = TimeFrame \times 60/10 = 60\) trucks every 10 minutes.

In the baseline instance, the arc incident with the port gate is considered to follow the BPR function. As stated in definition \((2.3)\), for that arc we have then linearization points such that \(\gamma_{ij}^1 = 0\), and \(\gamma_{ij}^M = 60\). We deemed appropriate to set \(M = 5\) and let the intersect points to be equidistant. In this case we have a linear function every \(60/4 = 15\) units of flow.

### 3.1. Estimation of parameters

The estimation of the parameters has been made by considering several aspects of the case study related to the Vado Container terminal. Particular attention has been devoted to the estimation of the travel time \(\tau_{ij}\) which, as already mentioned, can be assumed depending on day time, type of arc, and flow. We used Google Maps\(^\text{TM}\) web service to obtain the travel time for each hour of the day only for a selection of arcs because we wanted to extract a travelling time pattern which can be used to generate large instances without relying on the web service. Thus, we developed a procedure to extend to all other arcs the computation of travelling time, as explained in the following. The selected arcs are the ones incident with the terminal arc and others with different lengths, assuming that longer arcs have a higher average speed if compared to shorter ones. As can be noticed in the example of Figure 4, the arcs connecting the terminal to Rivalta Scrivia and Genova (Genoa) are somehow shorter and have a higher fraction of their path on normal roads, leading to a lower speed, while arcs connecting it to farthest destinations, such as Ginevra (Geneva), and Ravenna, are characterized by a greater speed. After selecting arcs and divided them in these two groups, we queried the google web service in order to obtain the traveling time at each hour of the day. Then, we computed the relation time/distance for each hour of the day and type of arc in order to generate the travel time of the remaining arcs. For larger networks we can extract this information for a selection of arcs and apply it to other arcs, according to their classification, in order to estimate travel times. Moreover, it is worth noting that, in respect of the travel times returned by the web services, a scale factor, CarToTruck (CTT), is applied to consider the car to truck ratio, which is the reduction in speed of a truck if compared to the standard speed of a car. The estimation takes into account the maximum speed allowed, and our setting for the baseline instance is CTT = 0.8. Generally speaking, the value of CTT can be set considering the type of road and travelling rules applied to it. In particular, the maximum car speed in Italy along highway is 130 km/h, while for medium trucks (from 3.5 to 12 tons capacity) is 100 km/h and for heavy trucks (more than 12 tons capacity) is 80 km/h. Note that the trucks addressed in this study have a speed limit of 80 km/h, so the minimum value of CTT would be 0.6. This value is a lower bound and 0.8 would be a better estimation because not all the paths are on highway. Moreover, the average speed for cars are lower than 130 km/h while trucks tend to stay to their maximum allowed speed. The data obtained by the google map and OpenStreetMap web services allowed also to set the parameters for the BPR function \([1]\). The parameters are \(T_{ff}\), \(q_{pc}\), \(\alpha\), and \(\beta\). In order to compute these parameters, we need the estimation of the maximum and minimum speed \(s_{\text{max}}\), \(s_{\text{min}}\) and the maximum traveling
time $T_{\text{max}}$ for a given arc in which we want to model the BPR function. The estimation of the traveling time for free flow and for $T_{\text{max}}$ are, respectively, $T_{\text{ff}} = 60(\text{dist/1000})/s_{\text{max}}$, and

\[ T_{\text{max}} = T_{\text{ff}}(s_{\text{max}}/s_{\text{min}}) \] (14)

The estimation of the parameters $q_{\text{pc}}$, $\alpha$, and $\beta$ has been done by looking to recent literature results such as those found by [20] and by matching them with the available data for our case study. The maximum arc capacity $q_{\text{pc}}$ has been set to 360 trucks per hour (6 trucks per minute); therefore the TimeFrame is 60 trucks every 10 minutes.

The literature consider $\beta \approx 4$. For the purpose of our application we tested different values of $\alpha$ and we were able to put it in relation with $T_{\text{max}}$ and $T_{\text{ff}}$, defining as the most appropriate value of $\alpha$ to be as in equation (15):

\[ \alpha = \frac{T_{\text{max}}}{T_{\text{ff}}} - 1 \] (15)

Equation (15) in our setting is congruent with the estimations found by [20]. For the purpose of our model, in the case of $T_{\text{max}} = T_{\text{ff}}$, from equation (15) we obtain $\alpha = 0$, and then, from equation (1) $\tau = T_{\text{ff}}$, meaning that we have a linear model for traveling time. As opposite, if we suppose a maximum speed of 70 km/h and a minimum speed of 15 km/h (as we assumed for nonlinear terms, also thanks to the available data), considering equations (14) and (15), we will get $\alpha = 70/15 - 1 = 3.67$. While these equations and these procedures to generate data are proved to be very useful and realistic, it is clear that future followups of this work will benefit of more proper data analytics based estimations with the VADO ligure system data available.

Table 1: Node list, buffer capacity, and demand of the baseline instance

<table>
<thead>
<tr>
<th>id</th>
<th>name</th>
<th>source</th>
<th>target</th>
<th>capacity</th>
<th>demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vado Ligure</td>
<td>Y</td>
<td>N</td>
<td>0</td>
<td>-5000</td>
</tr>
<tr>
<td>2</td>
<td>Buffer Terminal 1</td>
<td>N</td>
<td>N</td>
<td>5000</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>gate enter</td>
<td>N</td>
<td>N</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>gate exit</td>
<td>N</td>
<td>N</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>Alessandria</td>
<td>N</td>
<td>N</td>
<td>1000</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>Novi Ligure</td>
<td>N</td>
<td>N</td>
<td>1000</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>Rivalta scrivia</td>
<td>N</td>
<td>N</td>
<td>2500</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>Piacenza</td>
<td>N</td>
<td>N</td>
<td>2500</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>Novara</td>
<td>N</td>
<td>N</td>
<td>2500</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
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<td>N</td>
<td>N</td>
<td>2500</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>Rubiera</td>
<td>N</td>
<td>N</td>
<td>2500</td>
<td>0</td>
</tr>
<tr>
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<td>Bologna</td>
<td>N</td>
<td>N</td>
<td>2500</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>Torino</td>
<td>N</td>
<td>Y</td>
<td>2500</td>
<td>1285</td>
</tr>
<tr>
<td>14</td>
<td>Milano</td>
<td>N</td>
<td>Y</td>
<td>2500</td>
<td>760</td>
</tr>
<tr>
<td>15</td>
<td>Genova</td>
<td>N</td>
<td>Y</td>
<td>2500</td>
<td>465</td>
</tr>
<tr>
<td>16</td>
<td>Varese</td>
<td>N</td>
<td>Y</td>
<td>2500</td>
<td>455</td>
</tr>
<tr>
<td>17</td>
<td>Ravenna</td>
<td>N</td>
<td>Y</td>
<td>2500</td>
<td>310</td>
</tr>
<tr>
<td>18</td>
<td>Roma (Orte)</td>
<td>N</td>
<td>Y</td>
<td>2500</td>
<td>205</td>
</tr>
<tr>
<td>19</td>
<td>Cuneo</td>
<td>N</td>
<td>Y</td>
<td>2500</td>
<td>180</td>
</tr>
<tr>
<td>20</td>
<td>Ginevra</td>
<td>N</td>
<td>Y</td>
<td>2500</td>
<td>1340</td>
</tr>
</tbody>
</table>
4. Experimental results

Model P2 has been implemented in Gurobi™ 8.0, python interfaces 3.6.9 on Ubuntu 16.04 64bit. The computing machine used in our computational experimentation is an Intel Core™ model i7-5600U CPU with 2.60 GHZ, RAM 8 GB. The maximum running time was set to 200 seconds. The baseline instance produced a MIP model of 7200 rows, 91316 columns and 167354 non-zeros elements which was solved in less than 10s.

The first analysis has been devoted to the linearization. As already mentioned in Section 3, we used 5 linearization points for each nonlinear objective term; the linearization always allowed obtaining a light model easily to solve with modern solvers and with a detailed approximation level. In order to check how the tangent approximation affected the solution, we compared it with the upper approximation, obtained with piece-wise linear functions intersecting the nonlinear terms at the same points where the lower approximation has been done with tangents. The gap between the aforementioned approximations is very tight, ranging from 0.52% to 0.56% if we consider, respectively, 1 and 22 nonlinear terms in the objective function. This means that we modeled progressively more arcs as nonlinear. We have to point out that considering more nonlinear terms in the studied instance is not useful as congestion is only well represented in the neighbourhood of the terminal gate. With regard to computational time, we noticed a remarkable increase as depicted in the plot in Figure 5; nevertheless, this increase is very far to make the computational time to explode. The second part of the test campaign has been devoted to validate the model as a decision support tool for the port authority. The model has been tested to analyze the effect of different parameters on congestion and overall model perfor-
mance. By using the base settings as described in the previous section, also named the baseline instance, we firstly investigated the impact of the buffer capacity on both the congestion (i.e. the objective function) and the running time. To do so we changed the capacity of the buffers, which are listed in the fifth column of Table 1, generating different instances. The new buffer capacity is progressively set from 50% of the original capacity to 250% of the original capacity. To identify the instances, we introduce the parameter “cap factor” which is the ratio between the new buffer capacity and the original buffer capacity. The results demonstrate that, in the 48h planning horizon, the capacity of the buffers in the baseline instance is enough to manage the congestion and only a reduction of 50% of the capacity affects the performance. The results are depicted in Figure 6.

Another important issue to consider in our study is the potential shut-down of the gates during night shifts, when inland forwarding is planned. At this aim we considered three test cases: no shut-down, 4 hour shut-down, and 8 hour shut-down. As expected, the congestion rise over in these three scenarios, as plotted in Figure 7. The results when the hours of shut-down are changed can be also analysed in detail and for each single arc. These results are described in Figures 8 and 9, where the plots report the number of trucks travelling each single arc respectively if a shut-down is planned from 23:00 to 7:00 (Figure 8) and with no shut-down (Figure 9). In the Figures the arcs are identified with their progressive id number. Readers can note that with no shutdown, the number of arcs with high flow is remarkably reduced if compared to the scenario with shutdown from 23:00 to 07:00.

Another analysis considers the variation of the arc capacity from its standard settings of 60 vehicles per 10 minutes considered in the baseline instance. In this test campaign, we progressively augmented the arc capacity from 20 to 120 vehicles per 10 minutes. The results are depicted in

Figure 5: Model runtime in the baseline instance when changing the number of nonlinear terms considered, and linearized with tangent approximation
Figure 6: Object value for the optimal solution of P2 model when capacity of buffers changes. In abscissa “cap factor” is the ratio between the capacity of the buffers of the considered instance and the capacity of the buffers in the baseline instance. “cap factor” equal to 1 stands for the baseline instance.

Figure 10 which plots the optimal solution (measuring the congestion) when the arc capacity varies. As demonstrated by the results, a reduction of the standard capacity may highly affect the overall congestion while, as expected, no effects are reported if we augment too much the arc capacity. However, it is important to notice that in these experiments the nonlinearity is considered only at the arc representing the gate.

5. Conclusion

In this paper we discussed a mixed integer programming model for planning the inland forwarding of containers unloaded by mega-ships. The aim is to give a support to the planners in order to minimize congestion effects caused by mega-ship trend and the need to dislocate containers in short time. The inland forwarding with congestion can be classified as a “first mile” logistics problem and may cause errors in planning if not accounted correctly; the modeling of travel time is critical in order to represent and control congestion effects. Congestion can be considered only if nonlinear terms are represented in the model. Nonlinear terms may cause complexities in the model which can be managed by using linearization and mixed integer programming. Our proposed tangent approximation allowed to maintain the model easy to be solved with a very tight gap if compared with upper bounds. As a result, our approach allowed to produce a light model which can be effectively used for macro planning of forwarding operations. The application case is the Container Terminal of VADO Ligure, which is going to be the main Italian hub for mega-ships. We used different studies and data sources in order to design the baseline instance of a 48h planning problem. The parameters of the model have been systematically defined by integrating studies on containers to be handled in the VADO Ligure container terminal, by studying the literature on congestion functions, and by studying the map and the data available on well known map services (such as Google Maps™ and OpenStreetMap). We were
Figure 7: congestion with different gate shut-down scenarios

Figure 8: Impact of shut down hours on objective: flow distribution at gate with shutdown 23:00 - 07:00
Figure 9: Impact of shut down hours on objective: flow distribution at gate with no shut down

Figure 10: Congestion related to arc capacity in vehicles per 10 minutes
able to find relations on the parameters which allow to generalize their estimations thus facing
the travelling time function in a simpler way than in other previous methods proposed in the
recent literature. Starting from the implemented instance we tested the behavior of the model
under different scenarios. In particular, we found the importance of controlling shut-down hours
in order to lower congestion. Other test results confirmed congestion peaks that can be caused
by reductions in buffer and arc capacities. As a followup of this work we are going into three
main directions. First, we think it is important to integrate the consideration of environmental
impact. Further, from this general model a more detailed multicommodity one can be derived
when detailed data will be available. Finally, a simulation - optimization approach, not yet
presented in the literature in this field, is in progress to better synchronize the arrivals of the
containers at the terminal with their inland forwarding.

6. Acknowledgment

This work has been partially supported by PRIN program of the Ministry of Instruction, Uni-
versity, and Research (MIUR) through the project SPORT  Smart PO Rt Terminals [Grant No.
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