# A heuristic which works even though it should not

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- The unit-commitment problem
- Resolution via Lagrangian relaxation
- Primal recovery from relaxed solution
- Illustrations
- Theoretical background:
  the primal proximal algorithm

# Unit-commitment problem

- $I$ [$\simeq 200$] power plants* 

| | |
|---|---|
| nuclear | 80% |
| coal-fuel | 3% |
| hydro | 17% |

- $T$ [$= 96$] time periods (2 days $\times$ 48 half-hours)

- $p(i) = \left(p^1(i), \ldots, p^T(i)\right)$ planning of plant $i$

- $p(i) \in P(i)$ technological constraints

- $c_i(p(i))$ production cost
  includes purchases-sales

- $d^t$ demand (known, to be met exactly)

$$\min \sum_{i=0}^{I} c_i(p(i)), \quad p(i) \in P(i), \quad \sum_{i=0}^{I} p^t(i) = d^t$$

- $+$ other linking constraints
  spinning reserves, . . .

# * Notes

- = total available power

Typical daily production: 50 000 MW
85% nuclear, 5% coal-fuel, 10% hydro

- 1 hydro = 1 valley = 1 tree
  includes pumping
  cost ill-defined

- ∃ gas; very expensive, emergency only
  out of optimization model

- Technological constraints must be satisfied

- Linking constraints: softer

# Large-scale + heterogeneous

## Fossil Discrete dynamic system

- Finite set of production levels
- Max # ons-offs per day
- Max # jumps per day
- Max # simultaneous ons
- Min duration of ons-offs
- Min duration between jumps
- Setup times depend on history
- Complicated setup protocols
- Costs vary from 1 to 20

suited to standard dynamic programming

## Hydro-valleys Nonlinear optimal control

impossible but tolerable convex approximation
suited to interior points

$\Longrightarrow$ **decomposition necessary**

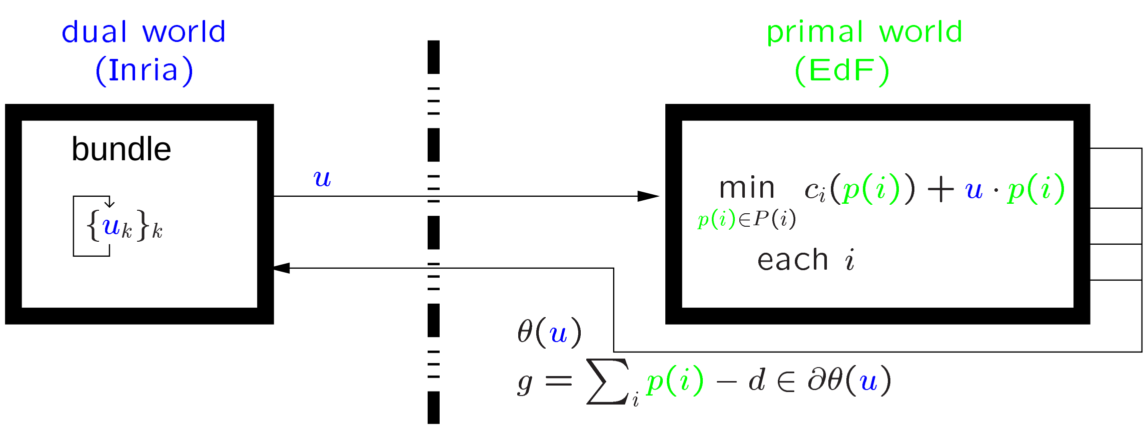
# Lagrangian Relaxation

- Form Lagrangian

$$L(p,u) := \sum_i c_i(p(i)) + u \cdot \left(\sum_i p(i) - d\right)$$

- Compute $\theta(u) :=$ min$_p\, L(p,u)$
  = decomposed problem

$$\sum \left( \min_{p(i)\in P(i)} C_i(p(i)) + u^\top p(i) \right) - u^\top d$$




- Dual algorithm max $\theta(u)$ yields optimal $\hat{u}$

# Dual optimality certificate

Get also from Inria

- selected $p_k$'s approx. optima of $L(\cdot, \hat{u})$
- convex multipliers $\{\alpha_k\}_k$
- such that $\sum_k \alpha_k \sum_i p_k(i) \simeq d$

cf. column generation!

$\Longrightarrow$ pseudo-planning $\hat{p}(i) := \sum_k \alpha_k p_k(i)$

approx. feasible . . . but not a planning! Z

# Theorem

$\hat{p}$ optimal solution of relaxed primal problem

# Primal heuristic

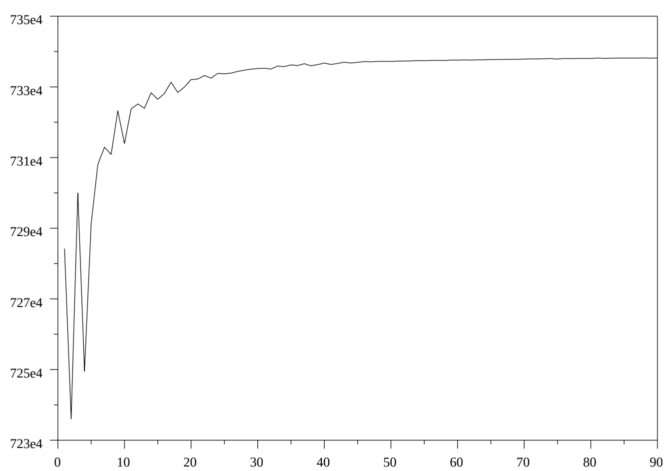
- Perturb $c_i(p(i))$ to $c_i(p(i)) + \frac{r}{2}|p(i) - \hat{p}(i)|^2$
- Solve perturbed dual
- Record "good" plannings | approx. feasible, cheap

# Illustration: dual algorithm

- Data for a Friday+Saturday
- Problem with 296 relaxed constraints
- Run on Sun Blade 900MHz

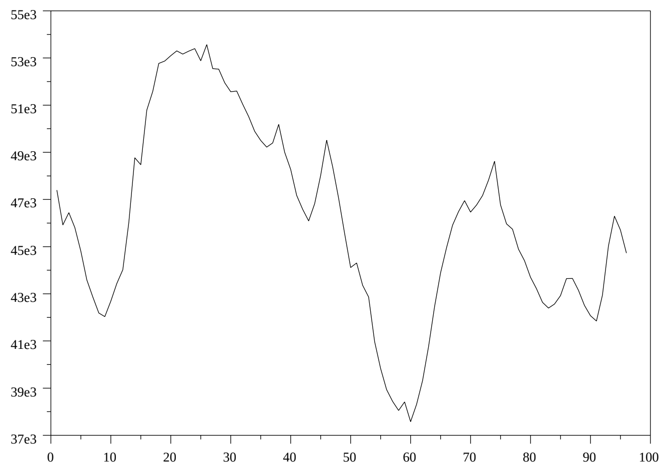
| | Initial | 90% | final |
|---|---|---|---|
| $\theta$ (MF) | 72.842 | 73.344 | 73.385 |
| CPU | 0 | 99 s | 660 s |
| # min $L$ | 0 | 30 | 200 |

$$\frac{\text{CPU(Inria)}}{\text{CPU(L)}} = \varepsilon$$

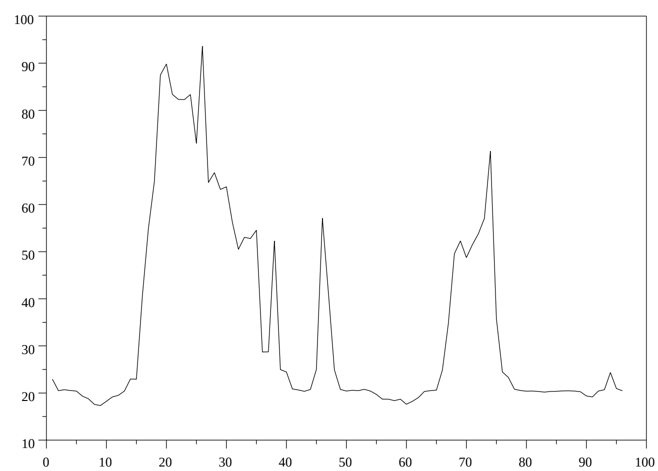



Evolution of $\theta$

**Warning fake data**




Demand
$\in [37, 54]$ GW




$\widehat{u}$
$\in [0.2, 0.9]$ F/kW

**Warning fake data**

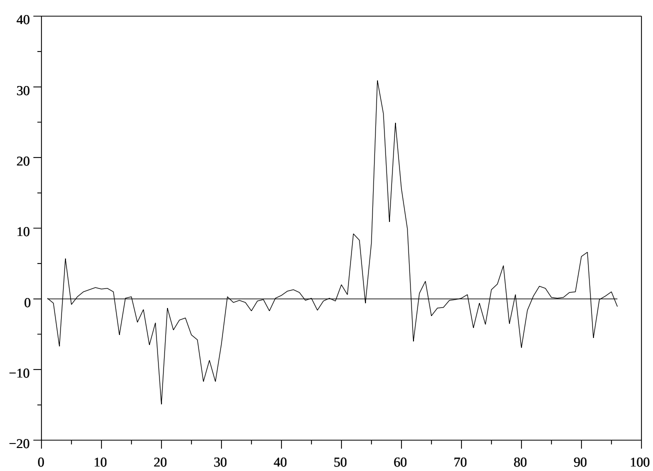
# Illustration: primal recovery

- Phase II converged after
  38 $L$-minimizations
  = 125 s CPU in addition to Phase I (660 s)

- Best planning: Duality gap 14 kF ($< 0.02\%$)
  Constraint violation

| | + | − |
|---|---|---|
| Average (MW) | 4.0 | 3.3 |
| Max (MW) | 31 | 15 |
| Max (%) | 0.02 | 0.03 |

⇝ 50 GW average




$\sum_i p(i) - d$

**Warning fake data**

# Theoretical Background

Given problem: $\boxed{\min f(p)}$

Basic idea: introduce

$$f_r(p,q) := f(p) + \frac{r}{2}|p-q|^2$$

$f_r(\cdot,q)$ "more convex" than $f(\cdot)$

$$\varphi(q) := \min_p f_r(p,q)$$

# Facts

- $\min f \iff \min \varphi$
- Argmin$\varphi$ : $p(q) = q \iff q$ local min of $f_r$

  minimize $\varphi$: fixed point of $q \mapsto p(q)$
- $f_r$ convex jointly $\Longrightarrow \varphi$ convex

  for the record!

# The proximal algorithm:

Assume $F$ computable **Z**

$$q_{k+1} = p(q_k)$$

start from $q_0 =$ relaxed solution

# Good news

- $f(q_{k+1}) \leqslant f(q_k) - \frac{r}{2}|q_{k+1} - q_k|^2$
- Terminates at some local min of $\varphi$
- which is feasible in original problem
- here is the heuristic: provides
  feasible point
  better than relaxed solution
- and even optimal if $r$ small enough

...yes but

# Actual computation

Approximate $\varphi$ via duality

introduces parasitic extreme point

# Bad news

Good properties of $\varphi$ are killed:

- $f$ no longer decreasing

- $q_k$ may stop at any extreme point

  for example the relaxed solution!

# Last hope

Give a meaning to "$r$ small enough"