Simultaneous Embedding with Fixed Edges

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Graph Drawing

Input
One graph $G$

Output
Layout of $G$
Simultaneous Graph Drawing

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A simultaneous embedding with fixed edges (SEFE) of graphs $G_1, \ldots, G_k$ consists of drawings $\Gamma_1, \ldots, \Gamma_k$ with

- $\Gamma_i$ is a planar drawing of $G_i$,
- every node in $G_i \cap G_j$ is drawn equally in $\Gamma_i$ and $\Gamma_j$ and
- every edge in $G_i \cap G_j$ is drawn equally in $\Gamma_i$ and $\Gamma_j$. 

## Known results

### Positive results

<table>
<thead>
<tr>
<th>Guaranteed SEFE for</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(tree, path)</td>
<td>[Erten and Kobourov 2004]</td>
</tr>
<tr>
<td>(outerplanar graph, cycle)</td>
<td>[Di Giacomo and Liotta 2005]</td>
</tr>
<tr>
<td>(planar graph, tree)</td>
<td>[Frati 2006]</td>
</tr>
</tbody>
</table>

### Negative result

<table>
<thead>
<tr>
<th>Example pair without SEFE for</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>two outerplanar graphs</td>
<td>[Frati 2006]</td>
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</table>
NP-completeness

Theorem

To decide SEFE for three graphs is NP-complete.

Open problem

The complexity for two graphs.
Let $P_{SEFE}$ be the set of all planar graphs, that share a SEFE with any planar graph.

\[ G_1 \in P_{SEFE}, \ G_2 \text{ planar} \implies (G_1, G_2) \text{ has SEFE.} \]
Theorem

$P_{SEFE}$ is the set of all planar graphs that do not contain the node-disjoint union of a cycle and an edge.
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Construction SEFE:

- $G_1$ planar, $G_2 \not\supseteq 1$

1. Planar drawing $D_1$ of $G_1$
2. Start $D_2$ of $G_2$ with $G_1 \cap G_2$
3. Insert remaining edges of $G_2$
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Theorem

\( P_{SEFE} \) consists of all

- forests,
- circular caterpillars,
- bi-stars, and
- subgraphs of \( K_4 \).
do not contain
To show:

Proof: Case distinction
Thanks.

Thank you very much for your attention.