A Lagrangian Heuristic for Robust Train Timetabling

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• Motivation
• Nominal Problem
• Robust Problem
• Computational results
The Train Timetabling Problem aims at finding an *optimal schedule* of trains on a railway network, satisfying some track capacity constraints.

In the planning phase, the Infrastructure Manager collects the requests of scheduling trains according to suggested timetables from the Train Operators.

At an operational level, delays can occur, thus the obtained solution might become *infeasible*.

This motivates the study of approaches that call for *robust solutions*, i.e. solutions that allow to avoid delay propagation as much as possible.
Nominal Train Timetabling Problem

**INPUT:**

- **Single Line with a one-way track** (approach easy to extend to railway network)

\[\text{BOLOGNA} \rightarrow \text{MO} \rightarrow \text{RE} \rightarrow \text{PR} \rightarrow \text{PC} \rightarrow \text{MILAN}\]

- **List T of Trains with “ideal timetables”**

<table>
<thead>
<tr>
<th>Train Type</th>
<th>Departure</th>
<th>Arrival</th>
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<tbody>
<tr>
<td>EUROSTAR 1811</td>
<td>BO 7:35</td>
<td>MI 9:10</td>
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<td></td>
<td>PR 8:28</td>
<td>PC 8:55</td>
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</table>

Ideal Timetables are CONFLICTING!!!!
Track Capacity Constraints:

• **no overtaking** between stations (allowed only within stations)

• **min time** between consecutive **departures** from each station

• **min time** between consecutive **arrivals** at each station

**OUTPUT:**

• “Adjusted” non-conflicting timetables with maximum total profit:

  **Train Adjustments:**

  • **shift** departure time from initial station

  • **stretch** stopping time at intermediate stations

  **Train Profit:**

  \[
  \pi_j - \phi_j(shift_j) - \sum_i \phi_{ij}(stretch_{ij})
  \]

  Ideal profit

  Arbitrary monotone functions

  If profit is null or negative cancel the train
Representation on Time-Space Graph

- **source**
- **BO**
- **MO**
- **PC**
- **MI**
- **sink**

- **Initial Arcs**
- **Travel Arcs**
- **Station Arcs**
- **Ending Arcs**

- Departure Nodes
- Arrival Nodes

- **time** (1 minute discretization)

- **Train Timetable Path**
- **Train Profit Path Profit**

- **Ideal profit and Shift cost on initial arcs**
- **Stretch cost on station arcs**
**ILP formulation**

Model without track capacity constraints

\[ x_r \] binary variable associated with arc \( r \in R \)

auxiliary variables for the safeness operational constraints:

\[ y_v, v \in V \]

\[ z_{jv}, j \in T, v \in V^j \]

at most one path for each train

\[
\begin{align*}
\max & \sum_{j \in T} \sum_{r \in R^j} p_r x_r \\
\sum_{r \in \delta^+_j (\sigma)} x_r & \leq 1, \quad j \in T \\
\sum_{r \in \delta^-_j (v)} x_r & = \sum_{r \in \delta^+_j (v)} x_r, \quad j \in T, v \in V^j \setminus \{\sigma, \tau\} \\
z_{jv} & = \sum_{r \in \delta^-_j (v)} x_r, \quad j \in T, v \in V^j \setminus \{\sigma, \tau\} \\
y_v & = \sum_{j \in T : v \in V^j} z_{jv}, \quad v \in V \setminus \{\sigma, \tau\} \\
x_r & \geq 0, \quad \text{binary } \quad r \in R
\end{align*}
\]
Example

\[
\begin{align*}
  w_1 &< w_2, \Delta(w_1, w_2) < d(h) \\
u_1 &< u_2, \Delta(u_1, u_2) < a(h)
\end{align*}
\]
ILP formulation

arrival constraints

\[ \sum_{u \in U(i) : u_1 \preceq u < u_2} y_u \leq 1, \quad i \in S, u_1 \in U(i) \]

\[ u_2 \succ u_1, \Delta(u_1, u_2) = a(i) \]

earliest arrival

first arrival node compatible with \( u_1 \)

minimum headway between consecutive arrivals

departure constraints

\[ \sum_{w \in W(i) : w_1 \preceq w < w_2} y_w \leq 1, \quad i \in S, w_1 \in W(i) \]

\[ w_2 \succ w_1, \Delta(w_1, w_2) = d(i) \]

earliest departure

first departure node compatible with \( w_1 \)

minimum headway between consecutive departures
Example

\[ \min\{\Delta(w_1, w_2), \Delta(w_2, w_1)\} < d(h) \]
\[ \min\{\Delta(v_1, v_2), \Delta(v_2, v_1)\} < a(h) \]

\[ w_1 \prec w_2 \prec v_2 \prec v_1 \]

\[ w_3 \prec w_1, \Delta(w_2, w_3) = d(h) \]
\[ w_4 \prec w_2, \theta(w_1) + t_j + a(i) = \theta(w_4) + t_k \]
ILP formulation

Overtaking constraints

\[
\sum_{w \in W(h) \cap V^j : w_1 \leq w \leq w_3} z_{jw} + \sum_{w \in W(h) \cap V^k : w_2 \leq w \leq w_4} z_{kw} \leq 1,
\]

\(j, k \in T, t_j \geq t_k, w_1 \in W(h) \cap V^j, w_2 \in W(h) \cap V^k\) such that \(z_{jw_1}, z_{kw_2}\) incompatible

\[
\min \{\Delta(w_1, w_2), \Delta(w_2, w_1)\} < d(h)
\]

\[
\min \{\Delta(v_1, v_2), \Delta(v_2, v_1)\} < a(h)
\]

\[
w_3 > w_1, \Delta(w_2, w_3) = d(h)
\]

\[
w_4 > w_2, \theta(w_1) + t_j + a(i) = \theta(w_4) + t_k
\]

earliest departure for train \(j\)

earliest departure for train \(k\)

first departure node compatible with \(w_2\)

first departure node compatible with \(w_1\)

V. Cacchiani, Aussois 2010
Robust Train Timetabling Problem

Same setting as before BUT aims at avoiding delay propagation
Robust Train Timetabling Problem

In the planning phase, insert buffer times that can be used to absorb possible delays occurring at an operational level.

The nominal objective function (efficiency) must be taken into account as well.
Robust Train Timetabling Problem

source

Initial Arcs

Travel Arcs

Station Arcs

Departure Nodes

Arrival Nodes

Departure Nodes

Arrival Nodes

Departure Nodes

Arrival Nodes

time (1 minute discretization)
Robust Train Timetabling Problem

\[
\max \sum_{j \in T} \sum_{r \in R^j} p_r x_r + F \sum_{j \in T} \sum_{r \in R^j} b_r^j x_r
\]

Track capacity constraints are relaxed in a Lagrangian way.

The Lagrangian relaxed problem calls for a set of paths for the trains, each having maximum Lagrangian profit (given by the sum of the original profits for the arcs in the path including the weights for the buffer times, minus the sum of the penalties assigned to the nodes visited by the path).
Robust Train Timetabling Problem

**Heuristic Algorithm**

A Lagrangian-based heuristic algorithm is developed, in a subgradient framework.

Local search procedures are used to improve the solution found.

**Validation Method**

A simulation tool is used to test the robustness of the heuristic solution.

Given a TTP solution, it considers different realistic external delay scenarios and, assuming that all the trains in the solution have to be scheduled and all train precedences are fixed, adapts the solution to make it feasible with the given external delays, evaluating the resulting cumulative delay.
Computational Experiments

Set of real-world instances of the Italian Railways.

Weights of the buffer times that change through the iterations (firstly push efficiency and afterwards robustness).

Weights of the buffer times that change along the path of each train.

\[(1 - e^{-\lambda i})(\text{len}(h) - i)\]

We consider \(\lambda = 3\)

It is observed by Kroon et al. (2007) that buffers that are placed too early are not very useful (since the probability to face any delay at this early position is very small).
## Computational Experiments

<table>
<thead>
<tr>
<th>Instance</th>
<th>Number of trains</th>
<th>Nominal solution (efficiency)</th>
<th>Nominal Cumulative delay</th>
<th>Time (sec)</th>
<th>Robust solution (efficiency)</th>
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Comparison with the approach by Fischetti, Salvagnin and Zanette (2009) (FSZ)

Fast Approaches to Improve the Robustness of a Railway Timetable

They propose an event-based model (by adapting the Periodic Event Scheduling Problem for the periodic case), and investigate different approaches to get robust solutions: stochastic models and a light robustness approach.

Apply the Validation Method to get the cumulative delay.
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Conclusions

We have presented a Lagrangian-based heuristic algorithm for obtaining robust solutions to the Train Timetabling Problem.

The method manages to get many different solutions, in reasonable computing times, thus leaving the opportunity of choosing different levels of trade off between efficiency and robustness.

Compared to previous existing method, it allows us to find robust solutions with comparable/better values of cumulative delay in shorter computing time.

Our approach can also be applied to bi-criteria optimization for finding Pareto optimal solutions.