Timetable Combinatorial Optimization

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Motivation

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Temporal Extensions

Meaningful way to incorporate the dimension of time in Combinatorial Optimization problems.

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- Weights $w_a \in \mathbb{Z}_+$ for every $a \in A$.

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Duration property w.r.t a is satisfied for all $a \in A$.

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Example: Matchings

- A is the edge set of a graph $G = (V, A)$
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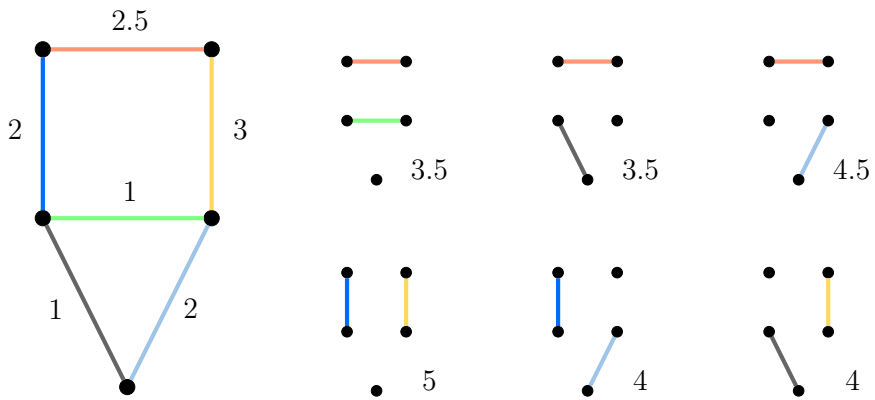

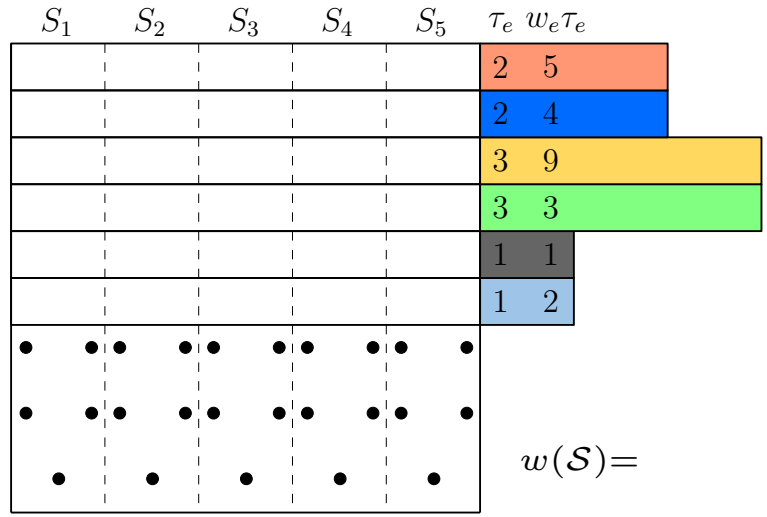

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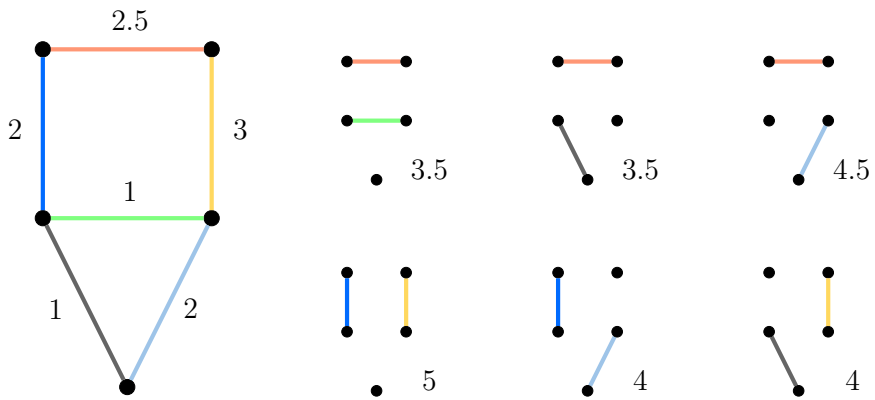

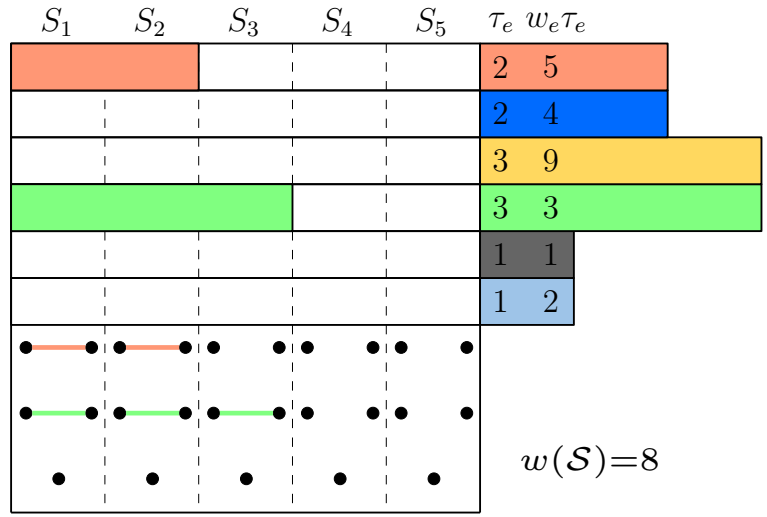

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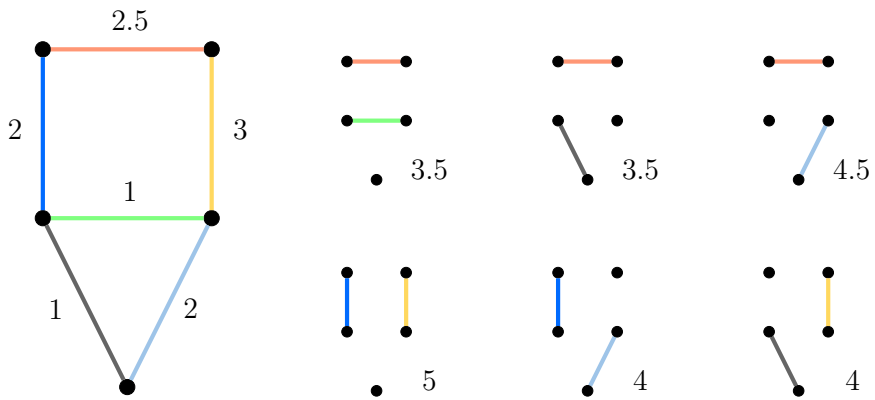

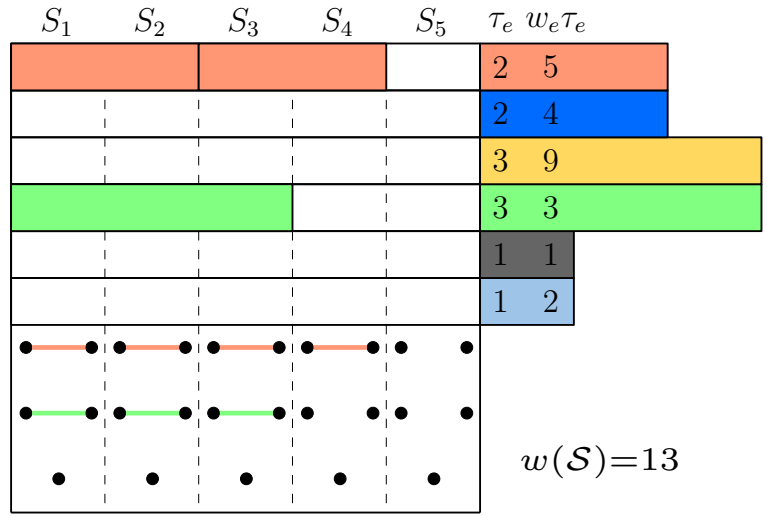

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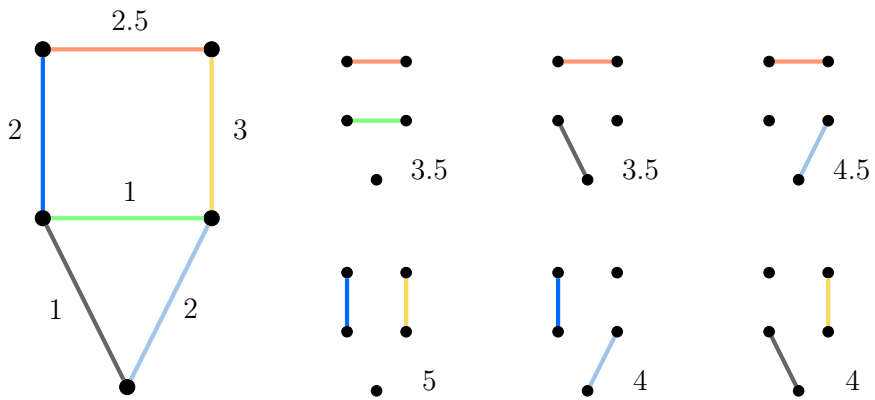

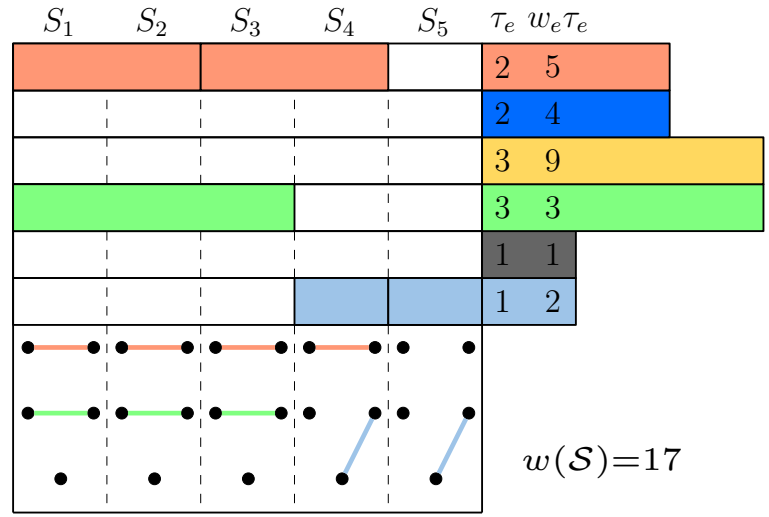

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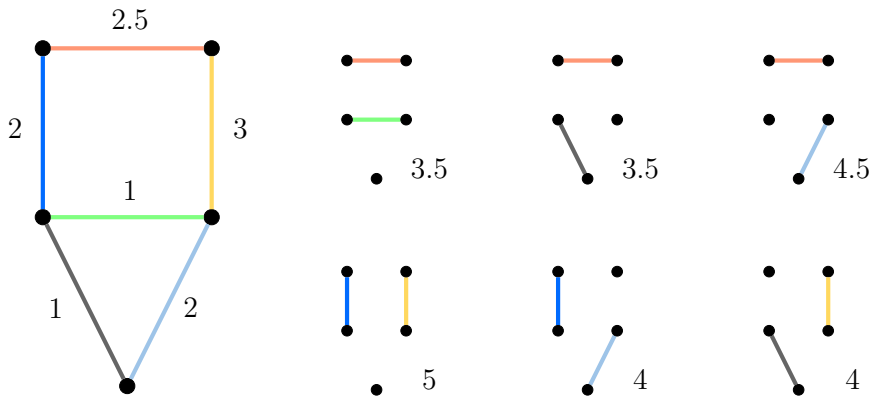

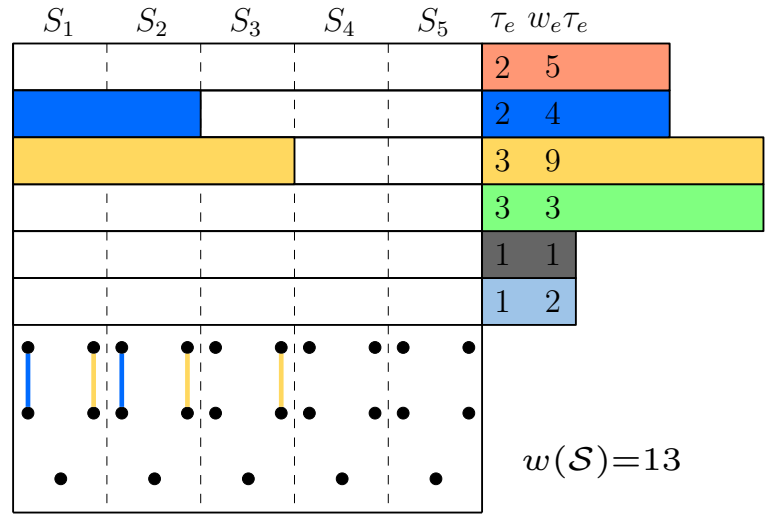

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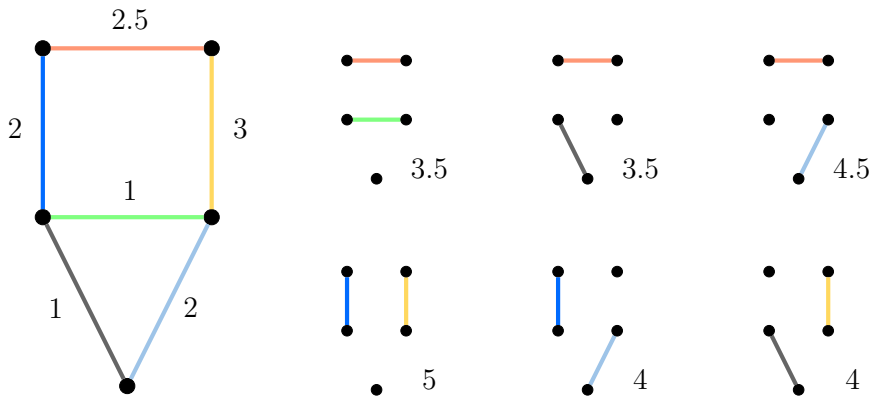

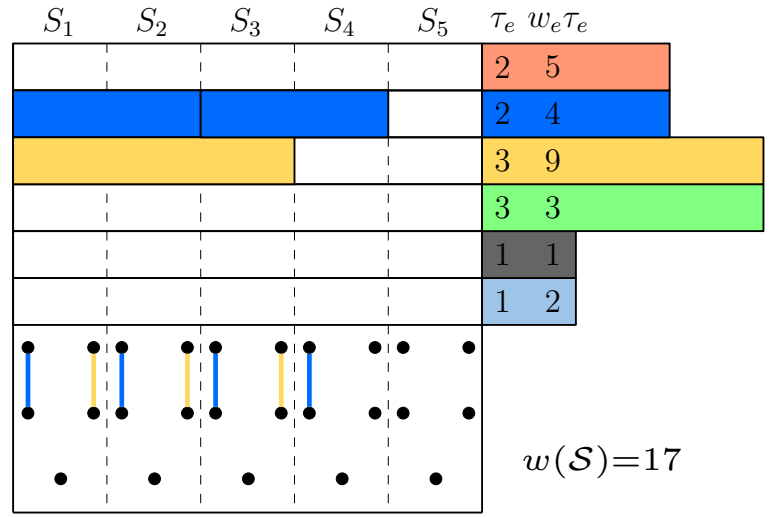

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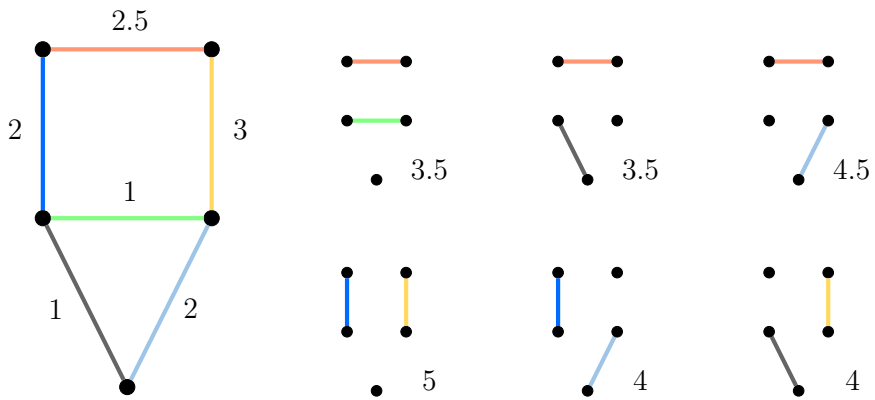

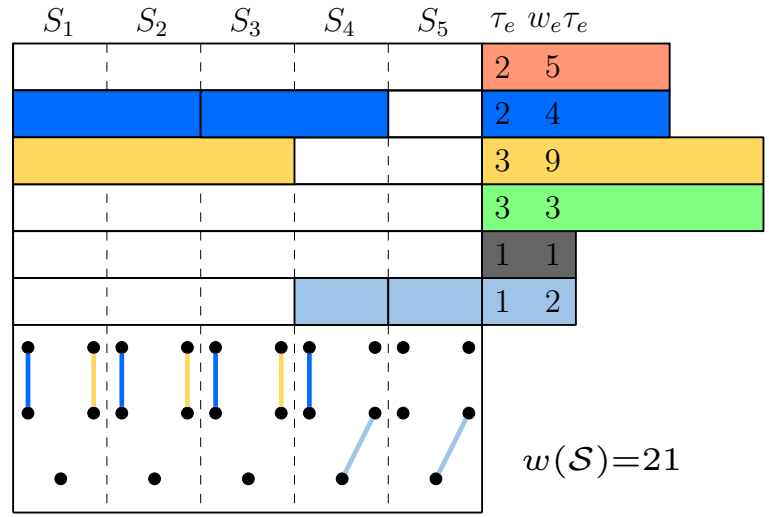

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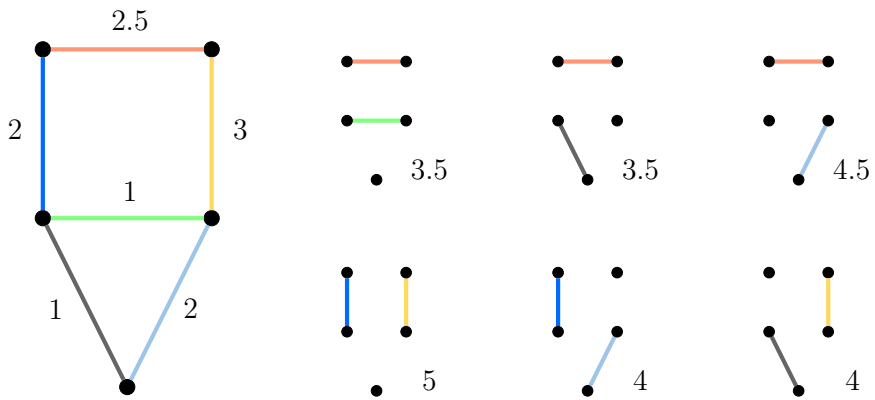

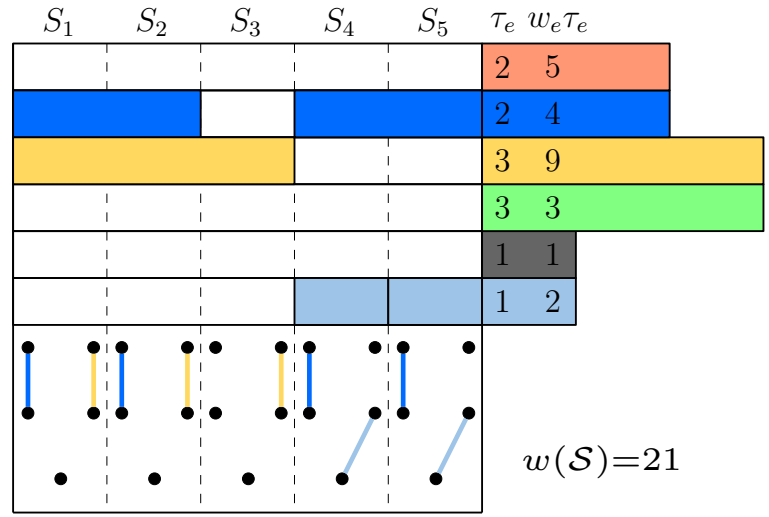

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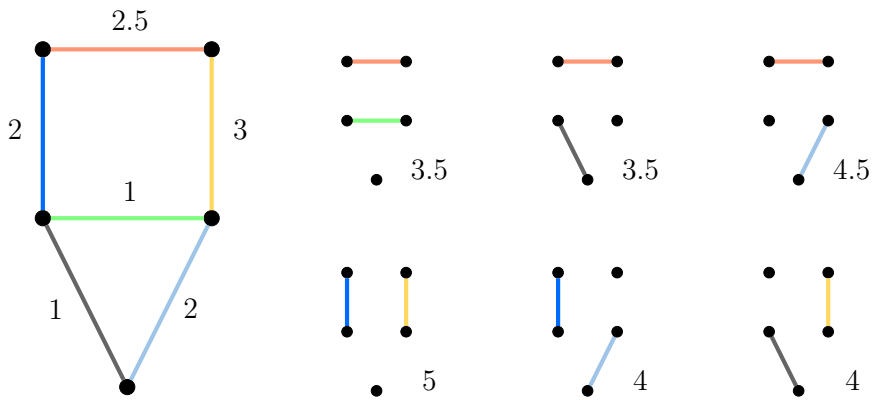

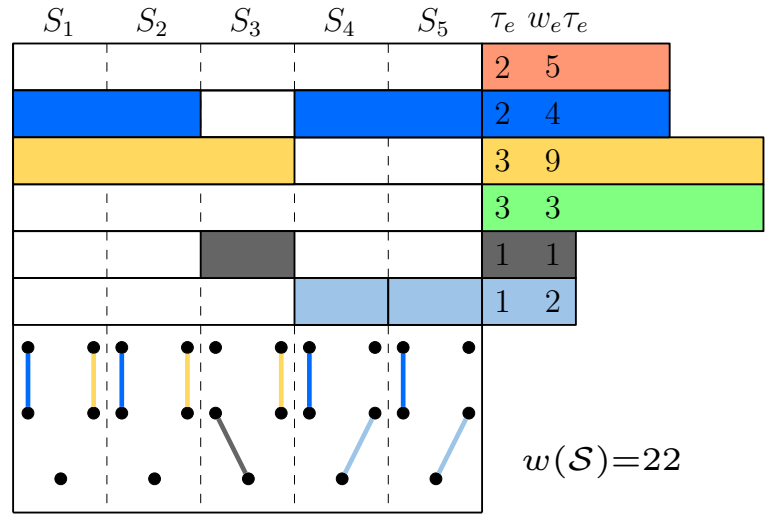

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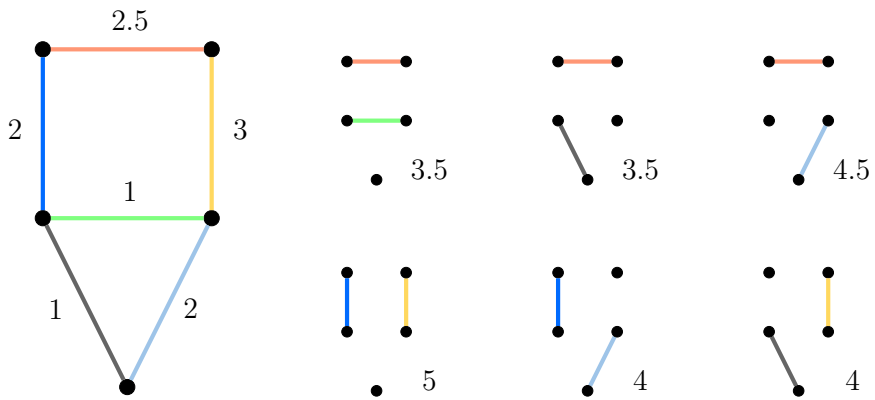

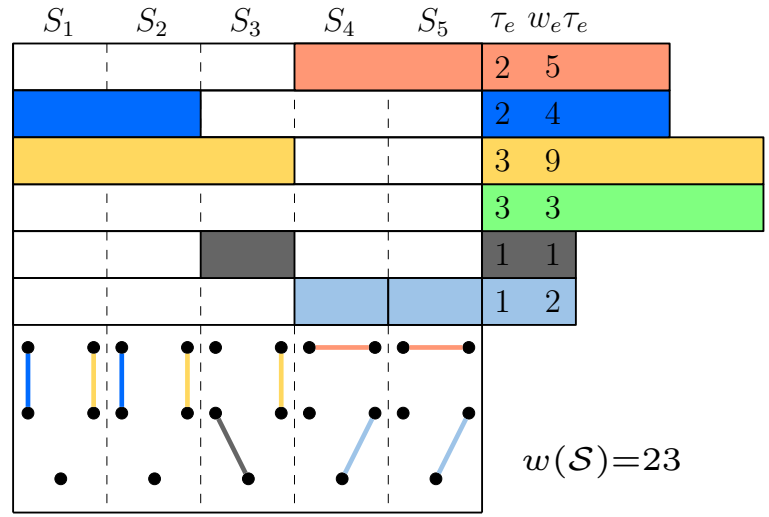

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- Let $\mathcal{X} = \{S \subseteq A : |S| \leqslant 1\}$, the rank-1 uniform matroid
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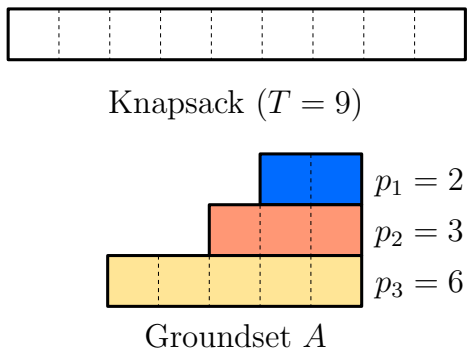

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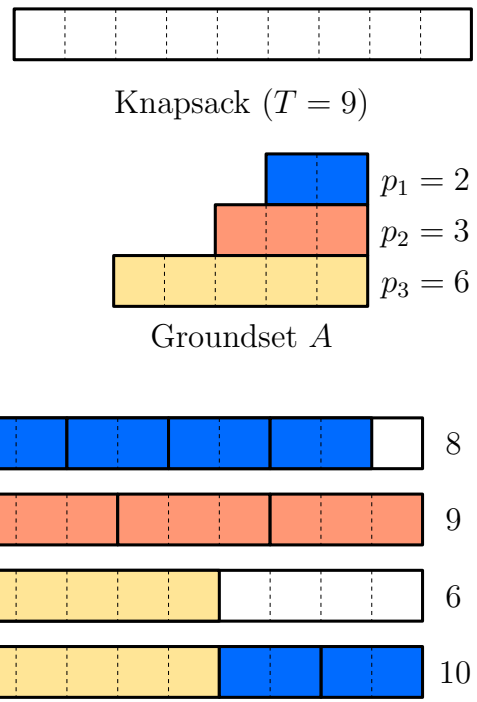

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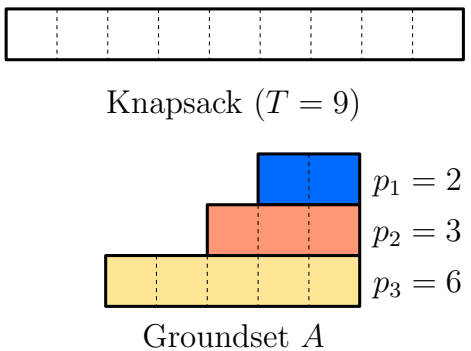

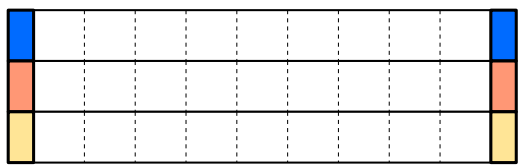

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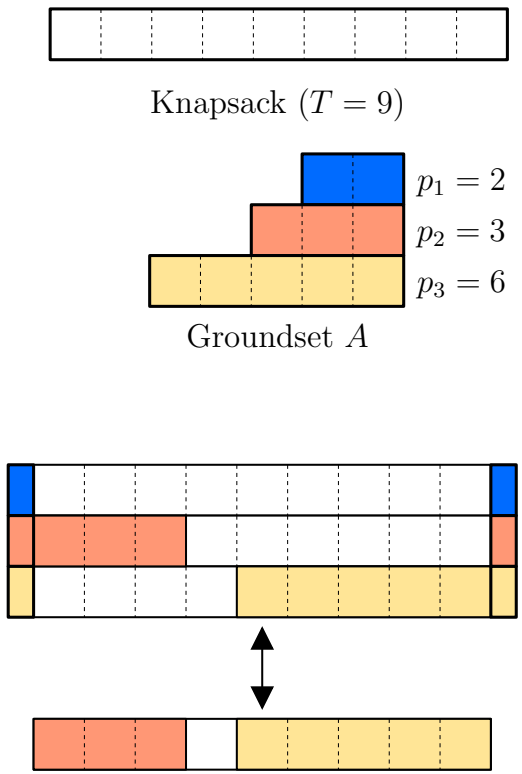

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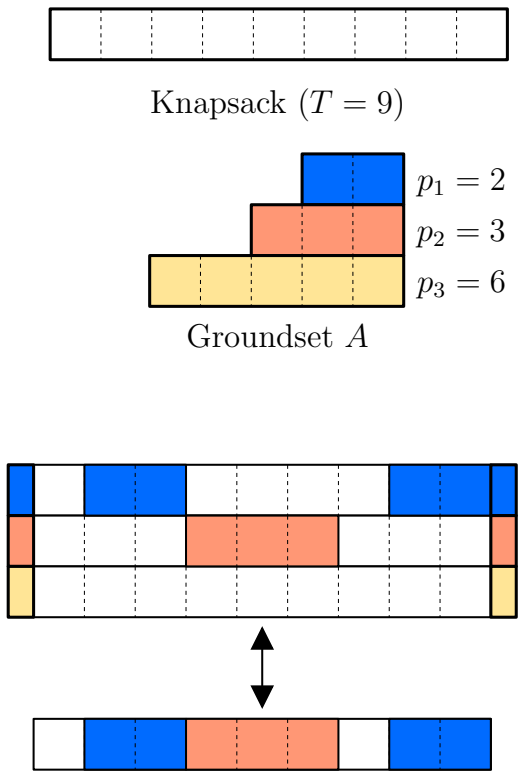

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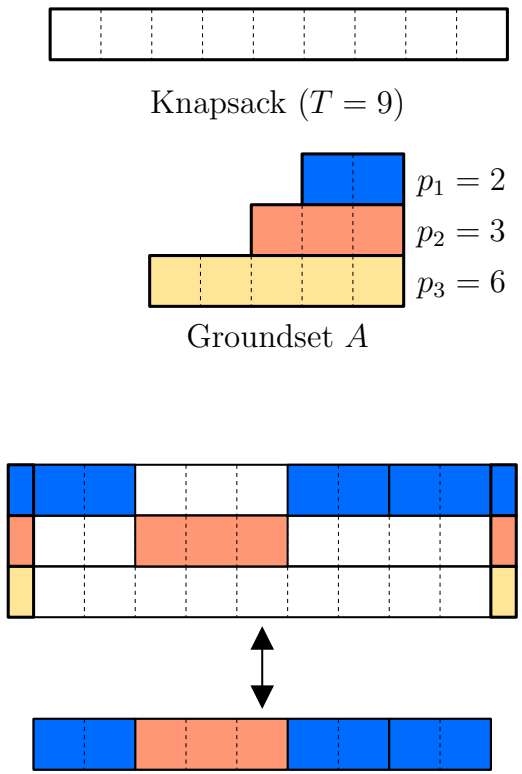

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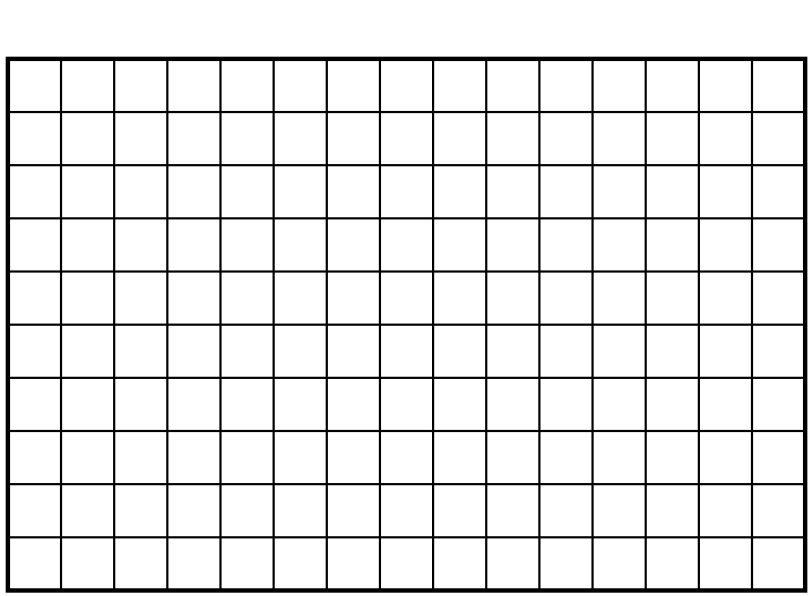

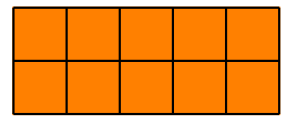

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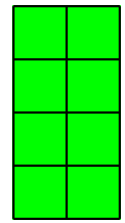
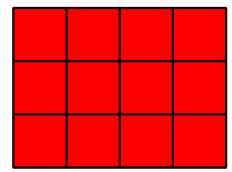


$w_2 = 5$



$w_3 = 2$

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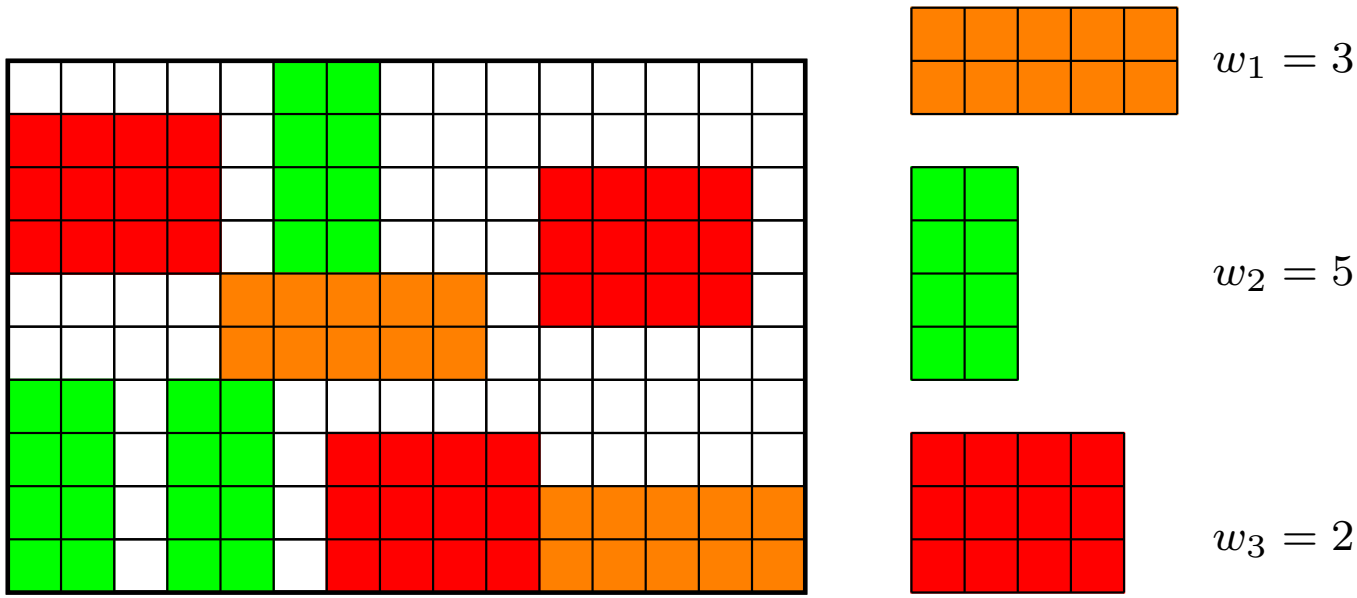

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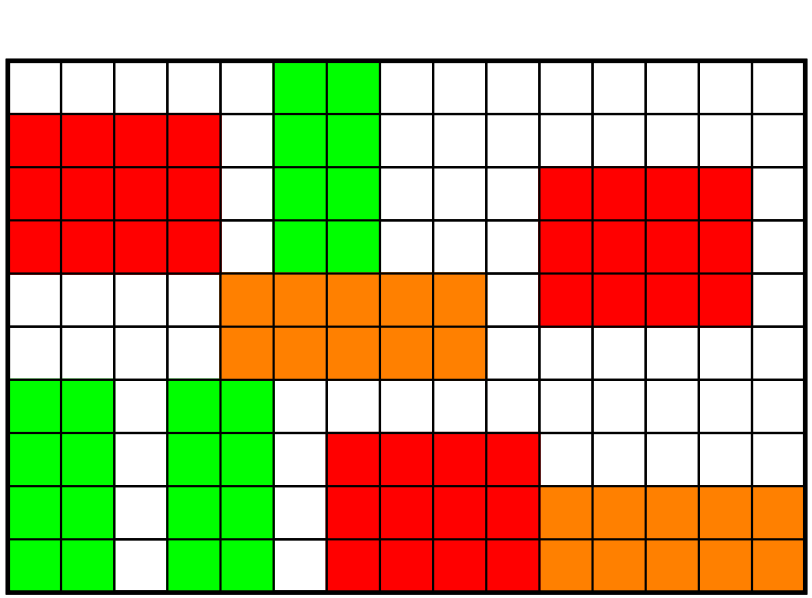

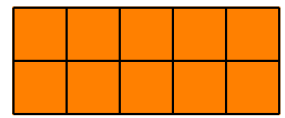

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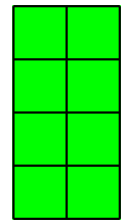

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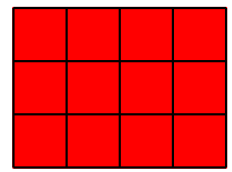

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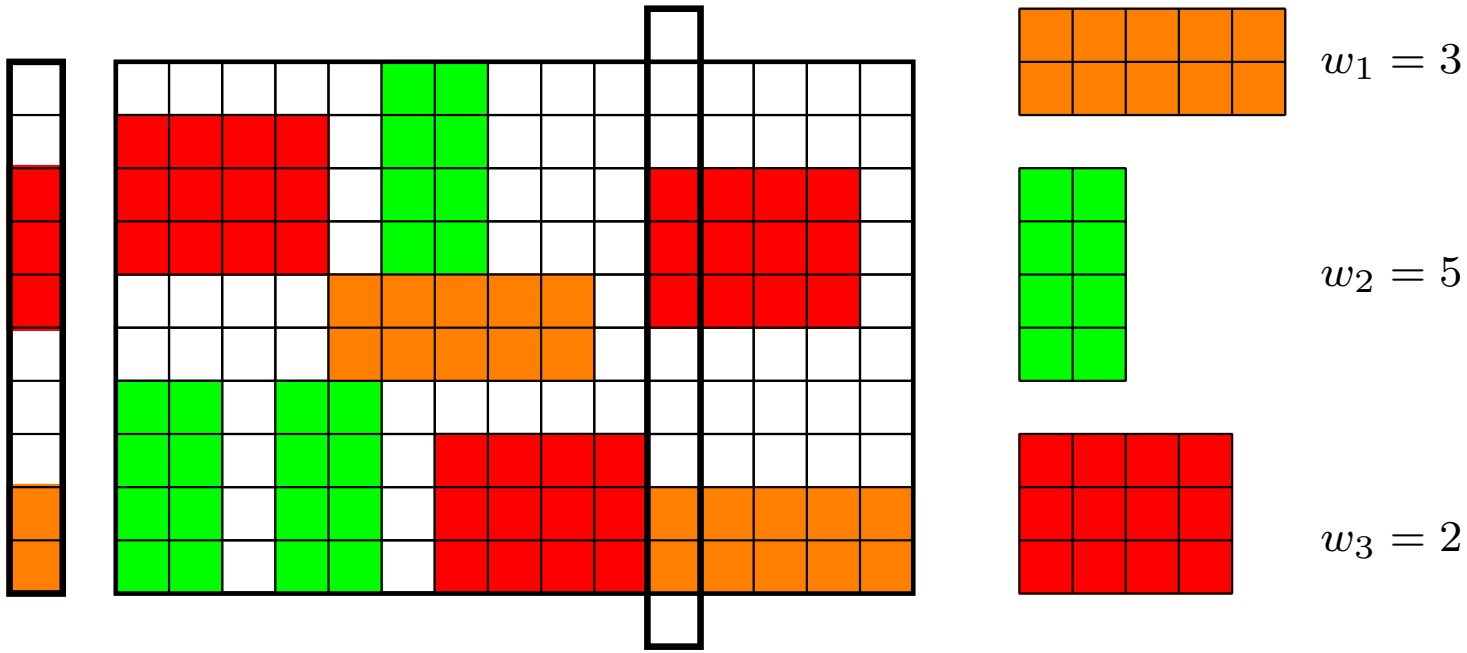

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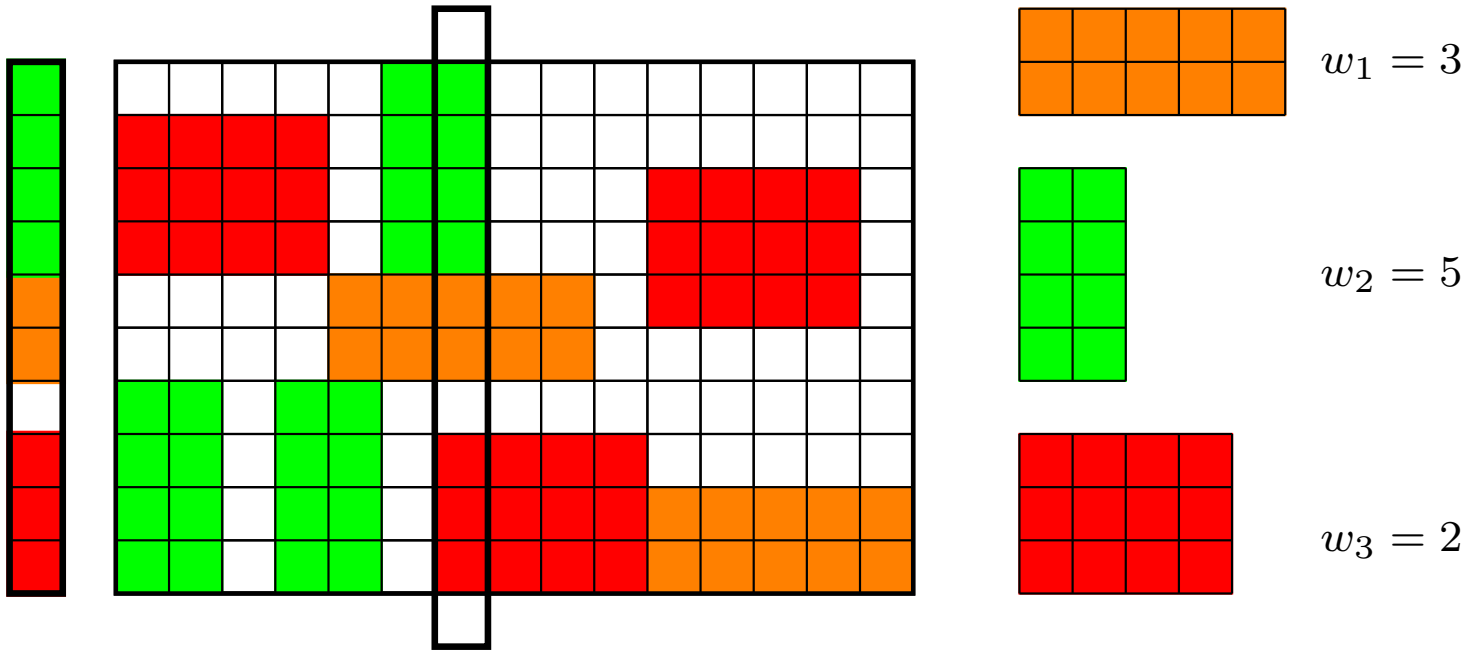

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Approximation Algorithm:

Theorem 1

If $(A, \mathcal{X})$ are independence systems and there is β-approximation algorithm for the maximization problem, then there exists a $\alpha\beta$-approximation algorithm for the timetable counterpart, with $\alpha \approx 1.691$.

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- Timetable Matroid Optimization is NP-hard.
- Timetable Matchings is APX-hard.
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Approximation Algorithm:

Theorem 1

If $(A, \mathcal{X})$ are independence systems and there is β-approximation algorithm for the maximization problem, then there exists a $\alpha\beta$-approximation algorithm for the timetable counterpart, with $\alpha \approx 1.691$.

Ingredients

- Simple algorithm based on black-box execution of β-approximation.

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Paper:

A.D., Bosio S. and Weismantel R. **Timetable combinatorial optimization: a complexity and approximability study**. *Submitted*.

TCO with Upper Bounds

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Binary Knapsack: repeat TCO reduction for IK and set $c_a = 1$ for all elements.

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Graph Balancing:

Given a graph $G = (V, E)$ and edge weights w_e, obtain a direction $D = (V, E')$ of G so as to minimize

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$$x_u^e + x_v^e = 1 \qquad \forall e = (u, v) \in E$$

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Ebenlendr, Krčál and Sgall (2007):

- NP-hard to approximate within $1.5 - \epsilon$.
- Integrality gap of IP1 is 2.
- A 1.75-approximation algorithm.

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$$x_u^e + x_v^e \leqslant 1 \qquad \forall e = \{u,v\} \in E \tag{2}$$

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$$x_u^e, x_v^e \in \{0,1\} \qquad \forall e = \{u,v\} \in E. \tag{4}$$

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$\bar{x}$ can be efficiently rounded to a solution $\hat{x}$ with the properties

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Phase2: Perform the rounding and obtain $\hat{x}$.

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To obtain a factor 3 one needs to work a bit more...

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Thank You!