A New Lower Bound for Curriculum-Based Course Timetabling

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7-11 Jan 2013, Aussois

Dedicated to the memory of Alberto Caprara (1968-2012)

Outline

- Problem description
- Integer Linear Programming (ILP) models with exponentially many variables
- A good ILP model with exponentially many variables
- Computational Results
- Conclusions and Future Research

Determine the best scheduling of university course lectures in a given time horizon (5 or 6 working days).

Curriculum-Based Course Timetabling

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International timetabling competitions ITC2002 and ITC2007 have stated a commonly accepted definition of the problem.

Real-world benchmark instances are available

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Wide amount of research on heuristics

Fairly large gap between upper and lower bounds

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Given:

- a set *C* of courses:
  - number of lectures
  - minimum working days
  - number of students
  - unavailable time periods

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  - courses in each curriculum

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- a set T of teachers
  - courses of the teacher

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Soft Constraints (objective function)

- room capacity: a penalty for each student that cannot have a seat in the room assigned to each course

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- curriculum compactness: a penalty for isolated lectures

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- minimum number of working days: a penalty for each day below the minimum number of working days for each course
- curriculum compactness: a penalty for isolated lectures
- room stability: a penalty for each additional room used for a course

Formulations

- Basic formulation (UD1): room stability is not taken into account
- Extended formulation (UD2)

They have different weights for the penalties of soft constraints violations

Introduced for the ITC2002 and ITC2007 (McCollum et al [2010])

Curriculum-Based Course Timetabling

Literature review

- Many works on heuristic algorithms
- Works on lower bounds:

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    (additional constraints to respect the number of rooms;

    in Surface all the rooms are joined into a single room of multiplicity |R| and the largest capacity;

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    (additional constraints to respect the number of rooms;

    in Surface all the rooms are joined into a single room of multiplicity |R| and the largest capacity;

    in Surface2 rooms are divided into two groups: larger and smaller rooms)
  - decomposition of the model into two stages by Lach and Luebbecke [2012]: first stage: w_{pc} second stage: minimum weight bipartite perfect matching problems or ILP

    it is exact for the Basic formulation

Literature review

• Problem partitioned into k subproblems and relaxation of soft constraints of curriculum compactness and hard constraints of room occupancy and time period conflict: Hao and Benlic [2011]

each subproblem formulated as in Lach and Luebbecke [2012]

Iterated Tabu Search for an effective partition

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Iterated Tabu Search for an effective partition

• Techniques based on propositional satisfiability solvers: Asin Acha and Nieuwenhuis [2012]

• Only column generation based approach by Qualizza and Serafini [2005] on a different version of the problem (objective function with preferences to time periods due to teachers, no soft constraints)

Input:

Courses:
<CourseID> <Teacher> <# Lectures>
<MinWorkingDays> <# Students>

Rooms: <RoomID> <Capacity>

Curricula:
<CurriculumID> <# Courses> <CourseID> ... <CourseID>

Unavailability_Constraints:
<CourseID> <Day> <Day_Period>

```
Name: Toy
Courses: 4
Rooms: 3
Days: 5
Periods_per_day: 4
Curricula: 2
Constraints: 8

COURSES:
SceCosC Ocra 3 3 30
ArcTec Indaco 3 2 42
TecCos Rosa 5 4 40
Geotec Scarlatti 5 4 18

ROOMS:
rA 32
rB 50
rC 40

CURRICULA:
Cur1 3 SceCosC ArcTec TecCos
Cur2 2 TecCos Geotec

UNAVAILABILITY_CONSTRAINTS:
TecCos 2 0
TecCos 2 1
TecCos 3 2
TecCos 3 3
ArcTec 4 0
ArcTec 4 1
ArcTec 4 2
ArcTec 4 3

END.
```

Output:

Curriculum *Cur1* - Courses: *3*

	Day1	Day2	Day3	Day4	Day5
Timeslot1	TecCos *room:* rB		ArcTec *room:* rB	ArcTec *room:* rB	
Timeslot2	ArcTec *room:* rB	TecCos *room:* rB	SceCosC *room:* rB	TecCos *room:* rB	
Timeslot3	SceCosC *room:* rB	TecCos *room:* rB	TecCos *room:* rB		
Timeslot4		SceCosC *room:* rC			

Curriculum *Cur2* - Courses: *2*

	Day1	Day2	Day3	Day4	Day5
Timeslot1	TecCos *room:* rB				
Timeslot2	Geotec *room:* rC	TecCos *room:* rB	Geotec *room:* rA	TecCos *room:* rB	
Timeslot3		TecCos *room:* rB	TecCos *room:* rB	Geotec *room:* rB	
Timeslot4		Geotec *room:* rA		Geotec *room:* rA	

ILP models with exponentially many variables

- Curriculum Schedule ILP
- Time Period Schedule ILP
- 3 Schedule Types ILP: Time Period Schedule, Course Pattern, Curriculum Pattern
- 2 Schedule Types ILP: Time Period Schedule, Course-Curriculum Pattern
- 2 Weekly Schedule Types ILP

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For the basic and
the extended formulations

ILP models with exponentially many variables:

Curriculum Schedule ILP (CS)

x_{sq}

Course 1, Room A

Course 2, Room A

…

Time period 1

Time period 2

Schedule s for curriculum q for the time horizon, specifying which courses of q are given in each room, in each time period

ILP models with exponentially many variables:

Curriculum Schedule ILP (CS)

x_{sq}

Course 1, Room A

Course 2, Room A

…

Time period 1

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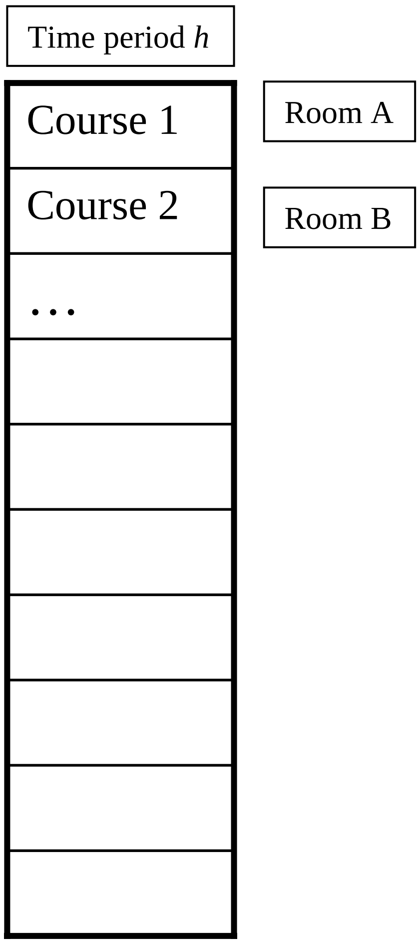
Schedule s for curriculum q for the time horizon, specifying which courses of q are given in each room, in each time period

y_{chr} =1 if course *c* is given in time period *h* in room *r*

ILP models with exponentially many variables:

Time Period Schedule ILP (TS)

x_{hs}



Schedule s for time period h, specifying for each room which course is given

ILP models with exponentially many variables:

Time Period Schedule ILP (TS)

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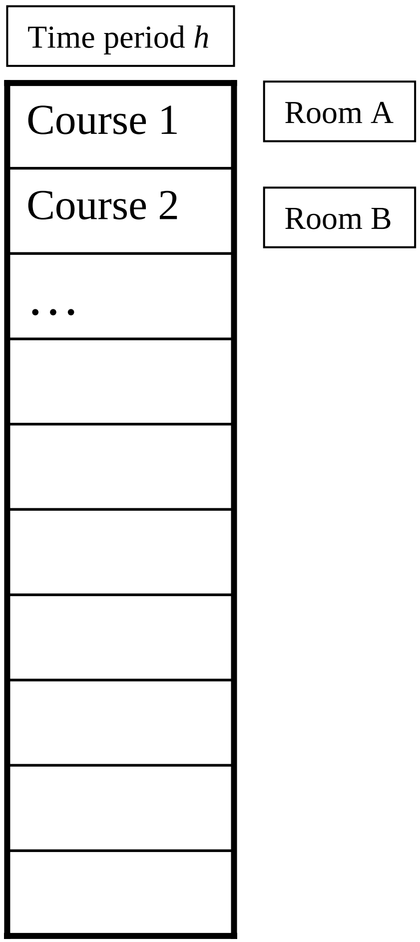


Schedule s for time period h, specifying for each room which course is given

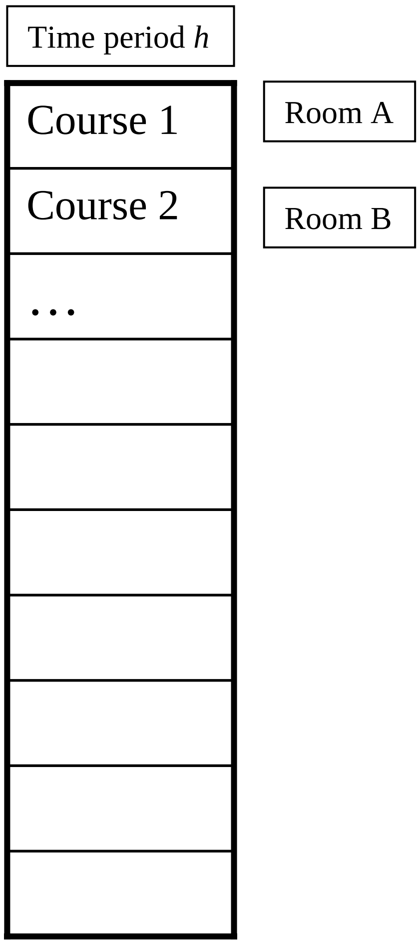
z_c

Counts the number of days below the minimum working days for course c

ILP models with exponentially many variables:

Time Period Schedule ILP (TS)

x_{hs}



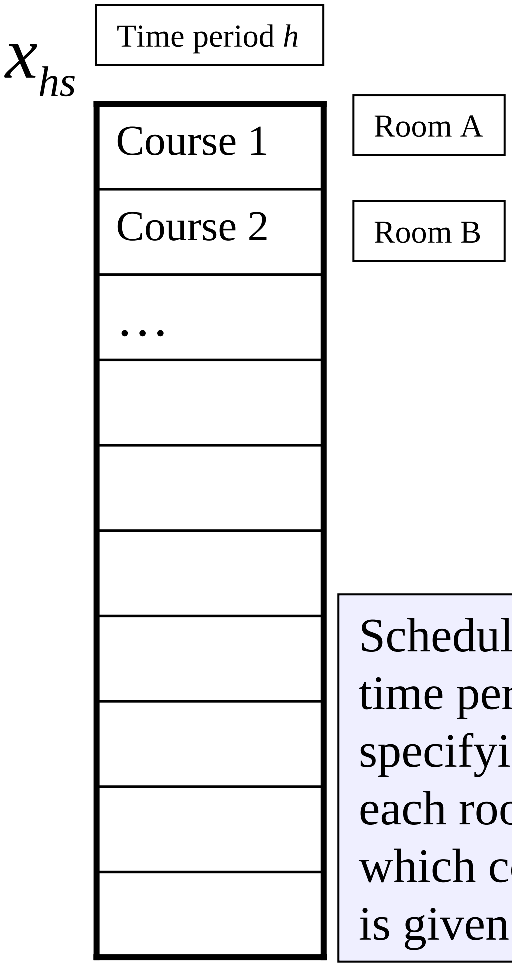
Schedule s for time period h, specifying for each room which course is given

z_c Counts the number of days below the minimum working days for course c

δ_{hq} Assumes value 1 if curriculum q has an isolated lecture in time period h

ILP models with exponentially many variables:

3 Schedule Types ILP: Time Period Schedule, Course Pattern, Curriculum Pattern (3ST)



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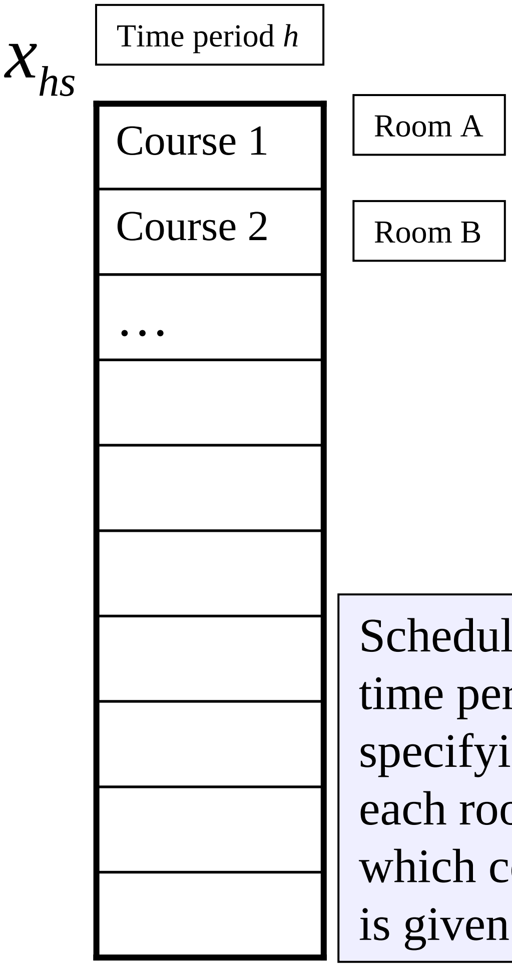

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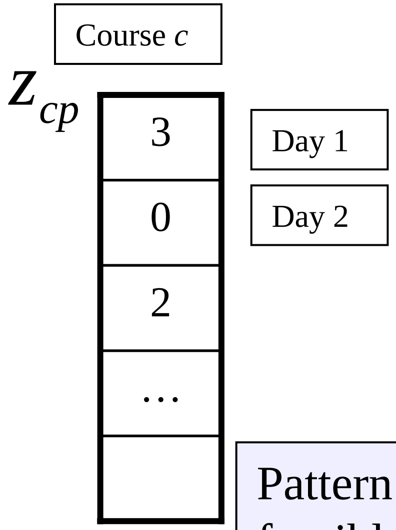

Pattern p of a feasible distribution of the lectures of course c throughout the days: it indicates for each day the number of lectures of course c given in that day

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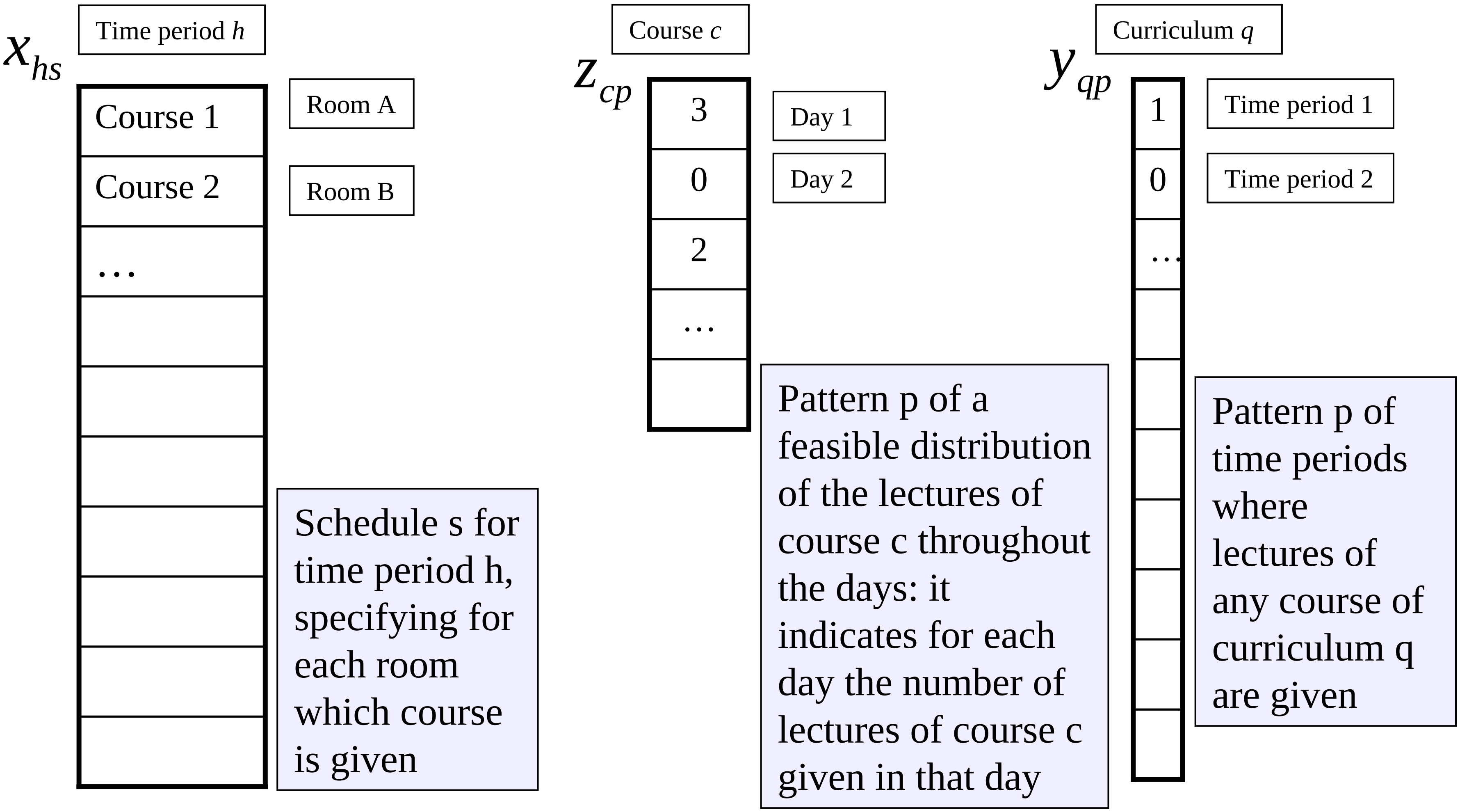
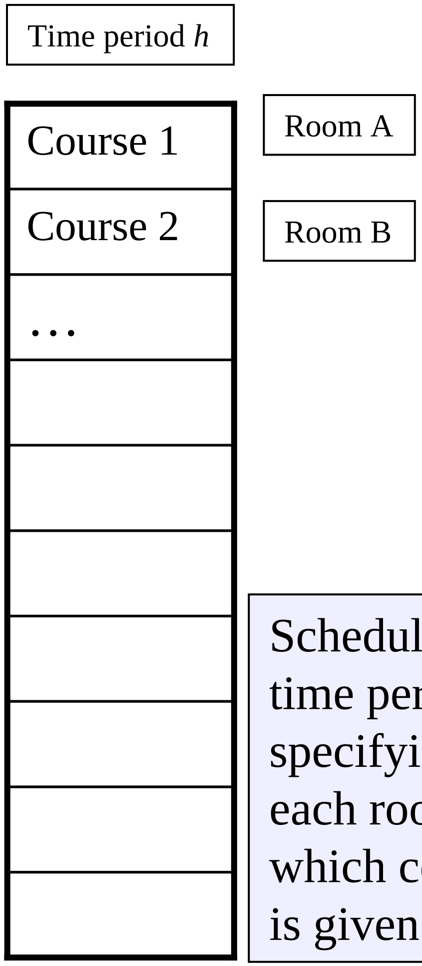


ILP models with exponentially many variables:

2 Schedule Types ILP: Time Period Schedule, Course-Curriculum Pattern (2ST)

x_{hs}



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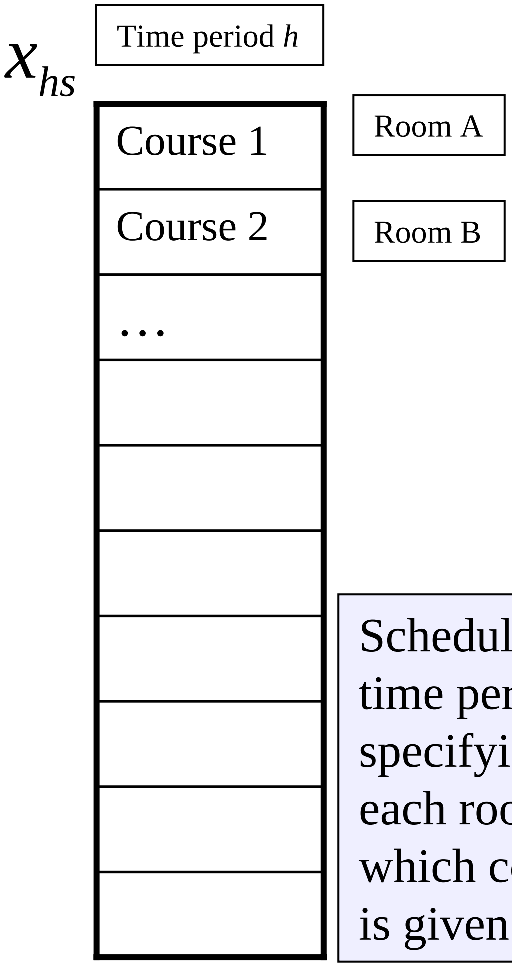

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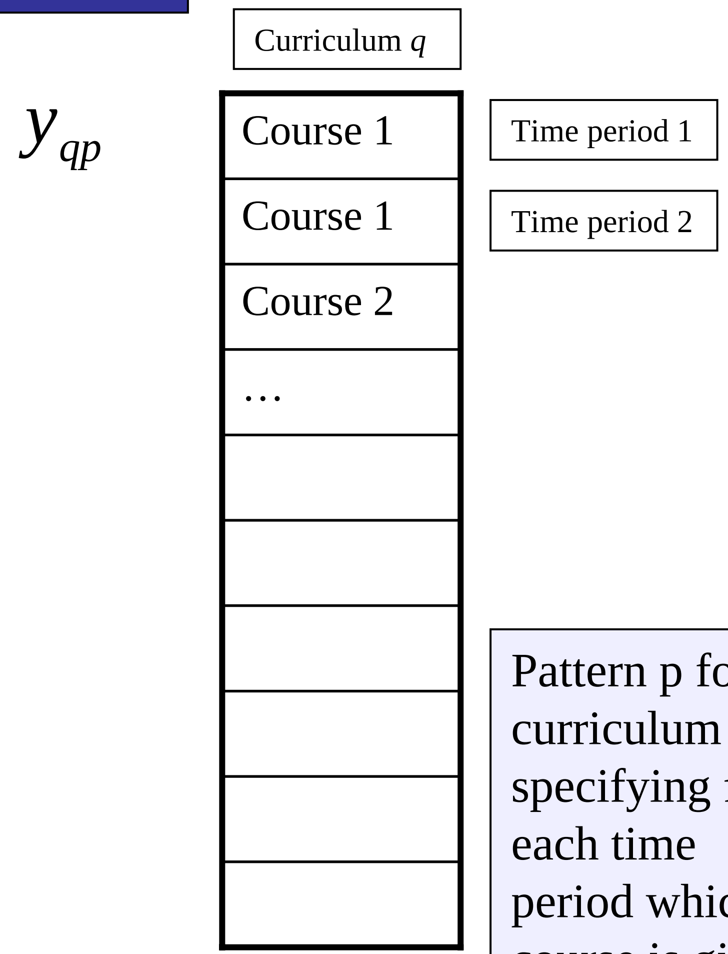

Pattern p for curriculum q specifying for each time period which course is given

ILP models with exponentially many variables:

2 Weekly Schedule Types ILP: the good one! (2WST)

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x_s

Course 1, Room A	Course 2, Room B	…
Course 1, Room B		

Time period 1

Time period 2

Schedule s represents a feasible assignment of lectures to rooms and time periods for the time horizon

ILP models with exponentially many variables:

2 Weekly Schedule Types ILP: the good one! (2WST)

x_s

Course 1, Room A	Course 2, Room B	…
Course 1, Room B		

y_p

Course 1	Course 2	…
Course 1		

Time period 1

Time period 2

Pattern p represents a feasible assignment of lectures to time periods for the time horizon

ILP models with exponentially many variables:

2 Weekly Schedule Types ILP: the good one!

x_s

Takes into account:

Hard constraints:

- at most one lecture per room per time period
- at most one lecture of courses belonging to a certain curriculum per time period
- at most one lecture per teacher per time period
- all lectures must be scheduled for each course

ILP models with exponentially many variables:

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Soft constraints:

- room capacity
- room stability

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Soft constraints:

- minimum working days
- curriculum compactness

ILP models with exponentially many variables:

2 Weekly Schedule Types ILP: the good one!

$$z(2WST) = \min \sum_{s \in S} c_s x_s + \sum_{p \in P} c_p y_p$$

$$\sum_{s \in S} x_s = 1$$

$$\sum_{p \in P} y_p = 1$$

$$\sum_{s \in S} a_s^{ch} x_s - \sum_{p \in P} b_p^{ch} y_p = 0, \quad c \in C, h \in H$$

$$x_s \in \{0,1\}, \quad s \in S$$

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2 Weekly Schedule Types ILP: solution method

We relax the linking constraints.

We solve separately the subproblem for the x variables and the subproblem for the y variables.

ILP models with exponentially many variables:

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$$z(2WST_x) = \min \sum_{s \in S} c_s x_s$$

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$$z(2WST_y) = \min \sum_{p \in P} c_p y_p$$

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We next present:

- an ILP model for the subproblem on the x variables
- an ILP model for the subproblem on the y variables
- an ILP model for the subproblem on the y variables with exponentially many variables

2 Weekly Schedule Types ILP: subproblem for x variables

$$z(2WST_x) = \min \sum_{c\in C}\sum_{h\in H}\sum_{r\in R} d_{cr}\xi_{chr} + W^{RStb}\left(\sum_{c\in C}\sum_{r\in R}\eta_{cr} - |C|\right)$$

=1 if course *c* is scheduled in time period *h* in room *r*

$$\sum_{c\in C(q)}\sum_{r\in R}\xi_{chr} \le 1, \quad h\in H, q\in Q$$

$$\sum_{c\in C}\xi_{chr} \le 1, \quad h\in H, r\in R$$

$$\sum_{c\in C(t)}\sum_{r\in R}\xi_{chr} \le 1, \quad h\in H, t\in T$$

$$\sum_{h\in H}\sum_{r\in R}\xi_{chr} = lect(c), \quad c\in C$$

$$\sum_{h\in H}\xi_{chr} \le lect(c)\eta_{cr}, \quad c\in C, r\in R$$

$$\xi_{chr} \in \{0,1\}, \quad c\in C, h\in H, r\in R$$

$$\eta_{cr} \in \{0,1\}, \quad c\in C, r\in R$$

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=1 if course *c* is scheduled in room *r*

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$$d_{cr} = W^{RCap}\max\{(stud(c) - cap(r), 0\}$$

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$$\sum_{c\in C(q)}\sum_{r\in R}\xi_{chr} \le 1, \quad h\in H, q\in Q$$

$$\sum_{c\in C}\xi_{chr} \le 1, \quad h\in H, r\in R$$

$$\sum_{c\in C(t)}\sum_{r\in R}\xi_{chr} \le 1, \quad h\in H, t\in T$$

$$\sum_{h\in H}\sum_{r\in R}\xi_{chr} = lect(c), \quad c\in C$$

$$\sum_{h\in H}\xi_{chr} \le lect(c)\eta_{cr}, \quad c\in C, r\in R$$

$$\xi_{chr} \in \{0,1\}, \quad c\in C, h\in H, r\in R$$

$$\eta_{cr} \in \{0,1\}, \quad c\in C, r\in R$$

2 Weekly Schedule Types ILP: subproblem for x variables

$$z(2WST_x) = \min \sum_{c \in C} \sum_{h \in H} \sum_{r \in R} d_{cr} \xi_{chr} + W^{RStb} (\sum_{c \in C} \sum_{r \in R} \eta_{cr} - |C|)$$

$$\sum_{c \in C(q)} \sum_{r \in R} \xi_{chr} \leq 1, \quad h \in H, q \in Q$$

$$\sum_{c \in C} \xi_{chr} \leq 1, \quad h \in H, r \in R$$

$$\sum_{c \in C(t)} \sum_{r \in R} \xi_{chr} \leq 1, \quad h \in H, t \in T$$

$$\sum_{h \in H} \sum_{r \in R} \xi_{chr} = lect(c), \quad c \in C$$

$$\sum_{h \in H} \xi_{chr} \leq lect(c) \eta_{cr}, \quad c \in C, r \in R$$

$$\xi_{chr} \in \{0,1\}, \quad c \in C, h \in H, r \in R$$

$$\eta_{cr} \in \{0,1\}, \quad c \in C, r \in R$$

2 Weekly Schedule Types ILP: subproblem for x variables

$$z(2WST_x) = \min \sum_{c\in C}\sum_{h\in H}\sum_{r\in R} d_{cr}\xi_{chr} + W^{RStb}\left(\sum_{c\in C}\sum_{r\in R}\eta_{cr} - |C|\right)$$

$$\sum_{c\in C(q)}\sum_{r\in R}\xi_{chr} \le 1, \quad h\in H, q\in Q$$

$$\sum_{c\in C}\xi_{chr} \le 1, \quad h\in H, r\in R$$

$$\sum_{c\in C(t)}\sum_{r\in R}\xi_{chr} \le 1, \quad h\in H, t\in T$$

$$\sum_{h\in H}\sum_{r\in R}\xi_{chr} = lect(c), \quad c\in C$$

$$\sum_{h\in H}\xi_{chr} \le lect(c)\eta_{cr}, \quad c\in C, r\in R$$

$$\xi_{chr} \in \{0,1\}, \quad c\in C, h\in H, r\in R$$

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2 Weekly Schedule Types ILP: subproblem for x variables

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$$\sum_{c\in C(q)}\sum_{r\in R}\xi_{chr} \le 1, \quad h\in H, q\in Q$$

$$\sum_{c\in C}\xi_{chr} \le 1, \quad h\in H, r\in R$$

$$\sum_{c\in C(t)}\sum_{r\in R}\xi_{chr} \le 1, \quad h\in H, t\in T$$

$$\sum_{h\in H}\sum_{r\in R}\xi_{chr} = lect(c), \quad c\in C$$

$$\sum_{h\in H}\xi_{chr} \le lect(c)\eta_{cr}, \quad c\in C, r\in R$$

$$\xi_{chr} \in \{0,1\}, \quad c\in C, h\in H, r\in R$$

$$\eta_{cr} \in \{0,1\}, \quad c\in C, r\in R$$

2 Weekly Schedule Types ILP: subproblem for y variables

Curricula that do not share any course can be dealt with separately in this model.

Connected components of curricula: groups of curricula that share at least one course.

Solve separately the model for each connected component.

2 Weekly Schedule Types ILP: subproblem for y variables

$$z(2WST_y) = \min \sum_{c \in C(\tilde{q})} W^{Mwd} z_c + \sum_{h \in H} \sum_{q \in \tilde{q}} W^{CComp} \delta_{hq}$$

Counts the number of days below the min working days

$$\sum_{c \in C(q)} \xi_{ch} \leq 1, \quad h \in H, q \in \tilde{q}$$

$$\sum_{c \in C(t)} \xi_{ch} \leq 1, \quad h \in H, t \in T(\tilde{q})$$

$$\sum_{h \in H} \xi_{ch} = lect(c), \quad c \in C(\tilde{q})$$

$$\sum_{h \in H(d)} \xi_{ch} \geq \phi_{cd}, \quad c \in C(\tilde{q}), d \in D$$

$$\sum_{d \in D} \phi_{cd} + z_c \geq mwd(c), \quad c \in C(\tilde{q})$$

$$\sum_{c \in C(q)} (\xi_{ch} - \xi_{ch+1}) \leq \delta_{hq}, \quad h \in H_f, q \in \tilde{q}$$

$$\sum_{c \in C(q)} (-\xi_{ch-1} + \xi_{ch} - \xi_{ch+1}) \leq \delta_{hq}, \quad h \in H \setminus \{H_f \cup H_l\}, q \in \tilde{q}$$

$$\sum_{c \in C(q)} (-\xi_{ch-1} + \xi_{ch}) \leq \delta_{hq}, \quad h \in H_l, q \in \tilde{q}$$

$$\xi_{ch} \in \{0,1\}, \quad c \in C(\tilde{q}), h \in H$$

$$\phi_{cd} \in \{0,1\}, \quad c \in C(\tilde{q}), d \in D$$

$$z_c \in Z_+, \quad c \in C(\tilde{q})$$

$$\delta_{hq} \in \{0,1\}, \quad h \in H, q \in \tilde{q}$$

2 Weekly Schedule Types ILP: subproblem for y variables

$$z(2WST_y) = \min \sum_{c\in C(\tilde{q})} W^{Mwd} z_c + \sum_{h\in H}\sum_{q\in\tilde{q}} W^{CComp}\delta_{hq}$$

=1 if there is an isolated lecture for curriculum *q* in time period *h*

$$\sum_{c\in C(q)} \xi_{ch} \le 1, \quad h\in H, q\in\tilde{q}$$

$$\sum_{c\in C(t)} \xi_{ch} \le 1, \quad h\in H, t\in T(\tilde{q})$$

$$\sum_{h\in H} \xi_{ch} = lect(c), \quad c\in C(\tilde{q})$$

$$\sum_{h\in H(d)} \xi_{ch} \ge \phi_{cd}, \quad c\in C(\tilde{q}), d\in D$$

$$\sum_{d\in D} \phi_{cd} + z_c \ge mwd(c), \quad c\in C(\tilde{q})$$

$$\sum_{c\in C(q)} (\xi_{ch} - \xi_{ch+1}) \le \delta_{hq}, \quad h\in H_f, q\in\tilde{q}$$

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$$\xi_{ch}\in\{0,1\}, \quad c\in C(\tilde{q}), h\in H$$

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$$z_c\in Z_+, \quad c\in C(\tilde{q})$$

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2 Weekly Schedule Types ILP: subproblem for y variables

$$z(2WST_y) = \min \sum_{c\in C(\tilde{q})} W^{Mwd} z_c + \sum_{h\in H}\sum_{q\in\tilde{q}} W^{CComp}\delta_{hq}$$

$$\sum_{c\in C(q)} \xi_{ch} \le 1, \quad h\in H, q\in\tilde{q}$$

=1 if course c is given in time period h

$$\sum_{c\in C(t)} \xi_{ch} \le 1, \quad h\in H, t\in T(\tilde{q})$$

$$\sum_{h\in H} \xi_{ch} = lect(c), \quad c\in C(\tilde{q})$$

$$\sum_{h\in H(d)} \xi_{ch} \ge \phi_{cd}, \quad c\in C(\tilde{q}), d\in D$$

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2 Weekly Schedule Types ILP: subproblem for y variables

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$$\xi_{ch} \in \{0,1\}, \quad c \in C(\tilde{q}), h \in H$$

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2 Weekly Schedule Types ILP: subproblem for y variables

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2 Weekly Schedule Types ILP: subproblem for y variables

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$$z_c \in Z_+, \quad c \in C(\tilde{q})$$

$$\delta_{hq} \in \{0,1\}, \quad h \in H, q \in \tilde{q}$$

=1 if a lecture of course *c* is given on a time period of day *d*

2 Weekly Schedule Types ILP: subproblem for y variables

$$z(2WST_y) = \min \sum_{c\in C(\tilde{q})} W^{Mwd} z_c + \sum_{h\in H}\sum_{q\in\tilde{q}} W^{CComp}\delta_{hq}$$

$$\sum_{c\in C(q)} \xi_{ch} \le 1, \quad h\in H, q\in\tilde{q}$$

$$\sum_{c\in C(t)} \xi_{ch} \le 1, \quad h\in H, t\in T(\tilde{q})$$

$$\sum_{h\in H} \xi_{ch} = lect(c), \quad c\in C(\tilde{q})$$

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$$z_c \in Z_+, \quad c\in C(\tilde{q})$$

$$\delta_{hq} \in \{0,1\}, \quad h\in H, q\in\tilde{q}$$

2 Weekly Schedule Types ILP: subproblem for y variables

The presented ILP model is solved by Cplex but it turned out that it is computationally heavy.

Thus, we use it only for the connected components that contain at most 10 curricula and at most 10 courses globally.

2 Weekly Schedule Types ILP:

subproblem for y variables: column generation based ILP

For the remaining cases we solve the LP-relaxation of an ILP model with exponentially many variables

y_{dp}

Course 1	Course 2	…
Course 1		

Time period 1

Time period 2

Pattern *p* represents a feasible assignment of lectures to time periods for the day *d*

2 Weekly Schedule Types ILP:

subproblem for y variables: column generation based ILP

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y_{dp}

Course 1	Course 2	…
Course 1		

Time period 1

Time period 2

Pattern *p* represents a feasible assignment of lectures to time periods for the day *d*

z_c

Counts the number of days below the minimum working days for course c

2 Weekly Schedule Types ILP:

subproblem for y variables: column generation based ILP

We solve the LP-relaxation of the following model:

$$z(2WST_y) = \min \sum_{d \in D} \sum_{p \in P(d)} c_{dp} y_{dp} + \sum_{c \in C(\tilde{q})} W^{Mwd} z_c$$

$$\sum_{p \in P(d)} y_{dp} = 1, \quad d \in D$$

$$\sum_{d \in D} \sum_{p \in P(d)} a_p^{cd} y_{dp} = lect(c), \quad c \in C(\tilde{q})$$

$$\sum_{d \in D} \sum_{p \in P(d)} b_p^{cd} y_{dp} + z_c \geq mwd(c), \quad c \in C(\tilde{q})$$

$$y_{dp} \in \{0,1\}, \quad d \in D, p \in P(d)$$

$$z_c \in Z_+, \quad c \in C(\tilde{q})$$

2 Weekly Schedule Types ILP:

subproblem for y variables: column generation based ILP

We solve the LP-relaxation of the following model:

$$z(2WST_y) = \min \sum_{d \in D} \sum_{p \in P(d)} c_{dp} y_{dp} + \sum_{c \in C(\tilde{q})} W^{Mwd} z_c$$

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2 Weekly Schedule Types ILP:

subproblem for y variables: column generation based ILP

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$$\sum_{d \in D} \sum_{p \in P(d)} b_p^{cd} y_{dp} + z_c \geq mwd(c), \quad c \in C(\tilde{q})$$

$$y_{dp} \in \{0,1\}, \quad d \in D, p \in P(d)$$

$$z_c \in Z_+, \quad c \in C(\tilde{q})$$

2 Weekly Schedule Types ILP:

subproblem for column generation based ILP

$$\min \sum_{c\in C(\tilde{q})} \sum_{h\in H(d)} -v_c \xi_{ch} + \sum_{h\in H(d)} \sum_{q\in\tilde{q}} W^{CComp} \delta_{hq} - \sum_{c\in C(\tilde{q})} w_c \varphi_c - u_d$$

=1 if course *c* is given in time period *h*

$$\sum_{c\in C(q)} \xi_{ch} \le 1, \quad h \in H(d), q \in \tilde{q}$$

$$\sum_{c\in C(t)} \xi_{ch} \le 1, \quad h \in H(d), t \in T(\tilde{q})$$

$$\sum_{c\in C(q)} (\xi_{ch} - \xi_{ch+1}) \le \delta_{hq}, \quad h \in H_f(d), q \in \tilde{q}$$

$$\sum_{c\in C(q)} (-\xi_{ch-1} + \xi_{ch} - \xi_{ch+1}) \le \delta_{hq}, \quad h \in H(d) \setminus \{H_f(d), H_l(d)\}, q \in \tilde{q}$$

$$\sum_{c\in C(q)} (-\xi_{ch-1} + \xi_{ch}) \le \delta_{hq}, \quad h \in H_l(d), q \in \tilde{q}$$

$$\sum_{h\in H(d)} \xi_{ch} \ge \varphi_c, \quad c \in C(\tilde{q})$$

$$\xi_{ch} \in \{0,1\}, \quad c \in C(\tilde{q}), h \in H(d)$$

$$\delta_{hq} \in \{0,1\}, \quad h \in H(d), q \in \tilde{q}$$

$$\varphi_c \in \{0,1\}, \quad c \in C(\tilde{q})$$

2 Weekly Schedule Types ILP:

subproblem for column generation based ILP

$$\min \sum_{c\in C(\tilde{q})} \sum_{h\in H(d)} -v_c \xi_{ch} + \sum_{h\in H(d)} \sum_{q\in\tilde{q}} W^{CComp} \delta_{hq} - \sum_{c\in C(\tilde{q})} w_c \varphi_c - u_d$$

=1 if there is an isolated lecture for curriculum q in time period h

$$\sum_{c\in C(q)} \xi_{ch} \le 1, \quad h\in H(d), q\in\tilde{q}$$

$$\sum_{c\in C(t)} \xi_{ch} \le 1, \quad h\in H(d), t\in T(\tilde{q})$$

$$\sum_{c\in C(q)} (\xi_{ch} - \xi_{ch+1}) \le \delta_{hq}, \quad h\in H_f(d), q\in\tilde{q}$$

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2 Weekly Schedule Types ILP: subproblem for column generation based ILP

$$\min \sum_{c\in C(\tilde{q})} \sum_{h\in H(d)} -v_c \xi_{ch} + \sum_{h\in H(d)} \sum_{q\in\tilde{q}} W^{CComp} \delta_{hq} - \sum_{c\in C(\tilde{q})} w_c \varphi_c - u_d$$

=1 if course *c* uses one or more time periods of day *d*

$$\sum_{c\in C(q)} \xi_{ch} \le 1, \quad h \in H(d), q \in \tilde{q}$$

$$\sum_{c\in C(t)} \xi_{ch} \le 1, \quad h \in H(d), t \in T(\tilde{q})$$

$$\sum_{c\in C(q)} (\xi_{ch} - \xi_{ch+1}) \le \delta_{hq}, \quad h \in H_f(d), q \in \tilde{q}$$

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2 Weekly Schedule Types ILP: subproblem for column generation based ILP

$$\min \sum_{c \in C(\tilde{q})} \sum_{h \in H(d)} -v_c \xi_{ch} + \sum_{h \in H(d)} \sum_{q \in \tilde{q}} W^{CComp} \delta_{hq} - \sum_{c \in C(\tilde{q})} w_c \varphi_c - u_d$$

$$\sum_{c \in C(q)} \xi_{ch} \leq 1, \quad h \in H(d), q \in \tilde{q}$$

$$\sum_{c \in C(t)} \xi_{ch} \leq 1, \quad h \in H(d), t \in T(\tilde{q})$$

$$\sum_{c \in C(q)} (\xi_{ch} - \xi_{ch+1}) \leq \delta_{hq}, \quad h \in H_f(d), q \in \tilde{q}$$

$$\sum_{c \in C(q)} (-\xi_{ch-1} + \xi_{ch} - \xi_{ch+1}) \leq \delta_{hq}, \quad h \in H(d) \setminus \{H_f(d), H_l(d)\}, q \in \tilde{q}$$

$$\sum_{c \in C(q)} (-\xi_{ch-1} + \xi_{ch}) \leq \delta_{hq}, \quad h \in H_l(d), q \in \tilde{q}$$

$$\sum_{h \in H(d)} \xi_{ch} \geq \varphi_c, \quad c \in C(\tilde{q})$$

$$\xi_{ch} \in \{0,1\}, \quad c \in C(\tilde{q}), h \in H(d)$$

$$\delta_{hq} \in \{0,1\}, \quad h \in H(d), q \in \tilde{q}$$

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2 Weekly Schedule Types ILP: subproblem for column generation based ILP

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Computational Experiments

Set of instances:

- DDS1,…,DDS7 real-life cases from Italian universities
- test1,…,test4 proposed by Di Gaspero and Schaerf [2003]
- comp01,…,comp21 proposed for the ITC2007 (real-life cases from the University of Udine)

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Computational experiments on all instances except from those whose optimal solution is zero.

Tests on Intel Xeon E5310 (Dual Core, 1.6 GHz), 8 GB ram,

Cplex 11.2.1 (one thread only)

Time limit of 60 seconds for subproblem for x variables

Total time limit of 40 CPU time units (22,800 seconds)

Comparison between the proposed ILP formulations

Lower bound

Computing time

	CS		TS		3ST		2ST		2WST	
Inst	*lb*	*T*	*lb*	*T*	*lb*	*T*	*lb*	*T*	*lb*	*T*
test1	212	32	200	9	200	23	212	32	212	4
test2	8	23	0	5	0	28	8	33	8	7
test3	6	20	16	98	13	289	6	42	35	17
test4	0	27	5	152	1	407	1	110	23	29
Avg	57	26	55	66	54	187	57	54	70	14

Computational Experiments

lb_x	Lower bound derived from the solution of the subproblem on the x variables
T_x	Computing time for the subproblem on the x variables
IP	Number of connected components for which we solve the IP model
LP	Number of connected components for which we solve the column generation model
$iter$	Number of times the subproblem for column generation (master) is solved
lb_y	Lower bound derived from the solution of the subproblem (slave)
T_s	Computing time for the solving the subproblem (slave)
T_m	Computing time for solving the column generation (master)
T_y	Computing time for solving the subproblem on the y variables
lb	Obtained lower bound
T	Total computing time
$\%gap$	Average percentage gap

Basic Formulation

	Best-known			Proposed method											
Inst	ub	lb	%gap	lb_x	T_x	IP	LP	iter	lb_y	T_s	T_m	T_y	lb	T	%gap
DDS1	39	39	0.00	0	17	33	1	23,900	33	2,274	5,211	7,486	33	7,503	15.38
DDS4	16	15	6.25	15	60	20	5	3,840	1	107	4	112	16	172	0.00
test1	212	212	0.00	200	1	14	1	330	12	1	1	3	212	4	0.00
test2	8	8	0.00	0	1	9	1	535	8	4	1	6	8	7	0.00
test3	35	35	0.00	0	1	5	2	1,160	35	14	1	16	35	17	0.00
test4	27	27	0.00	0	1	5	2	1,240	23	26	1	28	23	29	14.81

Basic Formulation

	Best-known			Proposed method											
Inst	ub	lb	%gap	lb_x	T_x	IP	LP	iter	lb_y	T_s	T_m	T_y	lb	T	%gap
comp01	4	4	0.00	4	1	3	1	595	0	4	1	6	4	7	0.00
comp02	12	3	75.00	0	1	6	1	3,965	8	3,550	9	3,560	8	3,561	33.33
comp03	38	18	52.63	0	1	6	1	3,985	29	3,085	8	3,094	29	3,095	23.68
comp04	18	18	0.00	0	1	7	3	1,055	18	22	1	24	18	25	0.00
comp05	219	116	47.03	0	1	1	1	1,890	143	19,135	4	22,799	143	22,800	34.70
comp06	14	4	71.43	0	4	7	1	7,085	6	3,566	44	3,611	6	3,615	57.14
comp07	3	3	0.00	0	13	7	1	6,860	3	777	74	852	3	865	0.00
comp08	20	9	55.00	0	1	8	2	1,075	19	30	1	32	19	33	5.00
comp09	54	12	77.78	0	1	5	2	1,610	52	164	1	166	52	167	3.70
comp10	2	2	0.00	0	4	5	1	17,050	1	3,727	663	4,391	1	4,395	50.00
comp12	239	33	86.19	0	2	0	1	3,948	73	19,113	27	22,798	73	22,800	69.46
comp13	32	12	62.50	0	1	6	3	1,730	30	173	1	175	30	176	6.25
comp14	27	22	18.52	0	1	5	1	4,465	26	1,113	16	1,130	26	1,131	3.70
comp15	38	-	100.00	0	1	6	1	3,985	29	3,078	8	3,087	29	3,088	23.68
comp16	11	-	100.00	0	2	7	1	20,520	8	10,249	2,231	12,481	8	12,483	27.27
comp17	30	-	100.00	0	2	8	1	7,105	26	4,707	35	4,743	26	4,745	13.33
comp18	34	-	100.00	0	1	0	1	3,402	30	3,253	4	3,257	30	3,258	11.76
comp19	32	-	100.00	0	1	7	2	1,705	25	128	1	130	25	131	21.88
comp20	2	-	100.00	0	8	6	1	9,075	2	2,896	100	2,997	2	3,005	0.00
comp21	43	-	100.00	0	2	7	1	6,490	39	4,688	28	4,717	39	4,719	9.30

Basic Formulation

The proposed method improves on the best-known LB in 9 out of 13 instances comp01-comp14.

It proves optimality of the best-known solutions for DDS4 and comp20.

Extended Formulation

	Best-known			Proposed method											
Inst	ub	lb	%gap	lb_x	T_x	IP	LP	iter	lb_y	T_s	T_m	T_y	lb	T	%gap
DDS1	48	48[L]	0.00	0	60	33	1	21,585	40	2,173	4,092	6,266	40	6,326	16.67
DDS4	17	17[A]	0.00	15	60	20	5	3,715	2	102	3	106	17	166	0.00
test1	224	24[B]	89.29	200	24	14	1	280	24	1	0	2	224	26	0.00
test2	16	16[B]	0.00	0	60	9	1	515	16	3	1	5	16	65	0.00
test3	67	59[B]	11.94	0	60	5	2	1,310	59	18	1	20	59	80	11.94
test4	73	46[B]	36.99	0	60	5	2	1,300	46	32	1	34	46	94	36.99

Extended Formulation

	Best-known			Proposed method											
Inst	ub	lb	%gap	lb_x	T_x	IP	LP	iter	lb_y	T_s	T_m	T_y	lb	T	%gap
comp01	5	5[B]	0.00	5	19	3	1	395	0	2	1	4	5	23	0.00
comp02	24	16[B]	33.33	0	60	6	1	4,005	16	3,180	8	3,189	16	3,249	33.33
comp03	66	38[L]	42.42	0	60	6	1	3,660	52	3,975	7	3,983	52	4,043	21.21
comp04	35	35[L]	0.00	0	49	7	3	1,110	35	23	2	26	35	75	0.00
comp05	290	211[L]	27.24	0	51	1	1	2,442	166	19,194	5	22,749	166	22,800	42.76
comp06	27	27[B]	0.00	0	60	7	1	7,575	11	3,894	48	3,943	11	4,003	59.26
comp07	6	6[L]	0.00	0	60	7	1	7,920	6	831	89	921	6	981	0.00
comp08	37	37[L]	0.00	0	60	8	2	1,305	37	33	1	35	37	95	0.00
comp09	96	96[L]	0.00	0	60	5	2	1,365	92	111	1	113	92	173	4.17
comp10	4	4[L]	0.00	0	60	5	1	8,175	2	1,332	68	1,401	2	1,461	50.00
comp12	300	99[B]	67.00	0	60	0	1	2,972	100	19,126	14	22,740	100	22,800	66.67
comp13	59	59[B]	0.00	0	60	6	3	1,825	57	156	2	159	57	219	3.39
comp14	51	51[L]	0.00	0	60	5	1	4,675	48	1,010	16	1,027	48	1,087	5.88
comp15	66	41[L]	37.88	0	60	6	1	3,660	52	3,981	7	3,989	52	4,049	21.21
comp16	18	18[B]	0.00	0	60	7	1	7,755	13	3,195	55	3,251	13	3,311	27.78
comp17	56	56[B]	0.00	0	60	8	1	6,875	48	6,296	33	6,330	48	6,390	14.29
comp18	62	61[L]	1.61	0	12	0	1	3,186	52	5,996	3	5,999	52	6,011	16.13
comp19	57	57[L]	0.00	0	60	7	2	1,645	48	114	2	117	48	177	15.79
comp20	4	4[B]	0.00	0	60	6	1	10,450	4	3,207	130	3,338	4	3,398	0.00
comp21	75	74[L]	1.33	0	60	7	1	6,595	68	5,687	29	5,717	68	5,777	9.33

Extended Formulation

The proposed method obtains the best-known or better LBs in 14 out of 26 instances, improving over the best-known LBs in 4 cases.

In 7 out of 16 instances for which the optimal solution is known it is able to get a LB with the same value as the optimal solution.

It proves optimality for the first time for test1.

Comparison with state-of-the-art methods for the Extended Formulation

	Lach and Luebbecke [2012]			Burke et al. [2010]			Hao and Benlic [2011]			Proposed method		
CPU	1T	10T	40T	1T	10T	40T	1T	10T	40T	1T	10T	40T
%Gap	60.45	46.85	35.13	40.38	31.25	28.69	37.79	29.25	27.18	54.34	28.86	22.05
#Best LB			3			6			5			11
#Best LB Only			0			0			0			7

Comparison with Asin Acha and Nieuwenhuis

Asin Acha and Nieuwenhuis [2012]			Proposed method		
T	%Gap	Best LB	T	%Gap	Best LB
56,098	31.10	7	3,726	17.57	11

Conclusion and Future Research

• The proposed method for computing lower bounds is based on splitting the objective function into two parts and on formulating the two parts as ILPs.

• The computational experiments show that the proposed method was able to improve some best known lower bounds and to prove optimality for some instances.

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• The proposed method for computing lower bounds is based on splitting the objective function into two parts and on formulating the two parts as ILPs.

• The computational experiments show that the proposed method was able to improve some best known lower bounds and to prove optimality for some instances.

• Further research can be conducted to determine the optimal solution of more instances, by applying branch and cut and price methods.

• This can be obtained by speeding up the computation through for example stabilization techniques.