Reverse Chvátal-Gomory rank

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joint work with

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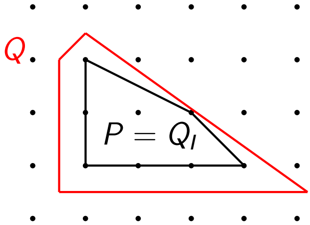
R. Grappe, Paris XIII

Aussois, 10th Jan, 2013

Integer hull

Theorem (Meyer 1974) The integer hull of a rational polyhedron is a polyhedron.



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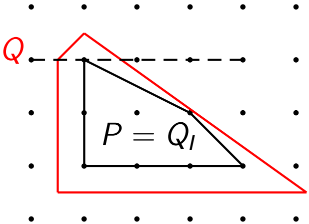

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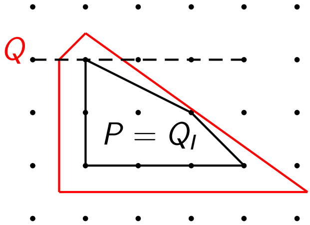

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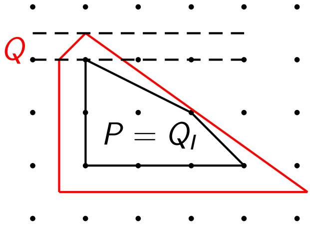

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Chvátal-Gomory closure of Q: the set $Q^{(1)}$ obtained by applying CG rounding to all valid inequalities for Q.

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Theorem (Schrijver 1980) If Q is a rational polyhedron, then $Q^{(1)}$ is a rational polyhedron.

Iteratively define $Q^{(k)}$ as the Chvátal-Gomory closure of $Q^{(k-1)}$.

Theorem (Schrijver 1980) For every rational polyhedron Q, there is an integer k such that $Q^{(k)} = Q_I$.

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The smallest k such that $Q^{(k)} = Q_I$ is the Chvátal-Gomory rank of Q, denoted $r(Q)$.

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- If $Q \subseteq [0,1]^n$:
  - Bockmayr, Eisenbrand, Hartmann, Schulz 1999: $O(n^3 \log n)$; Eisenbrand, Schulz 1999/2003: $O(n^2 \log n)$;
  - Eisenbrand, Schulz 1999/2003: $\Omega(n)$; Pokutta, Stauffer 2011 $\Omega(n)$, constructive; Rothvoß, Sanità 2012: $\Omega(n^2)$;
- $Q \subseteq [0,\ell]^n$
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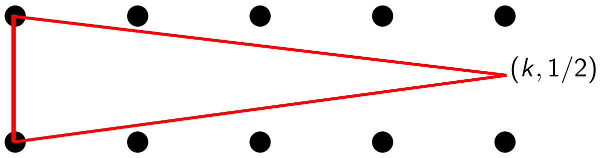
  - global bounds for the general case?

Global bounds?



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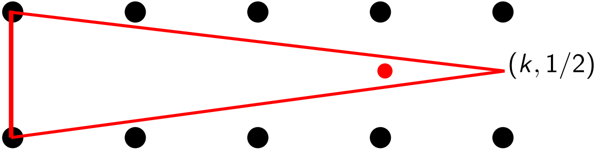

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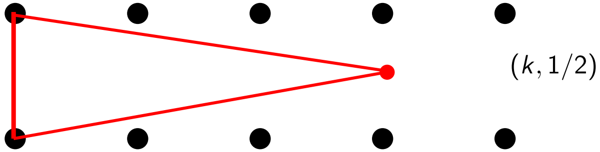

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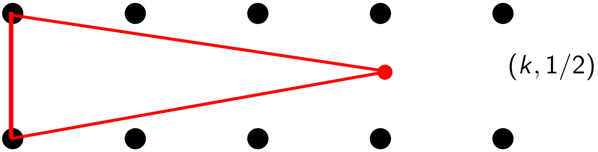

$r(Q_k) \geq k$ $\quad \rightarrow$ no global bounds exists, even in dimension two.

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Questions

- When is $r^*(P) < +\infty$?
  - Recall: $r^*(P) < +\infty$ for $P = \emptyset$.
- When $r^*(P) < +\infty$, what does $r^*(P)$ depend on?

Main result

Theorem

Let $P \subseteq \mathbb{R}^n$ be a non-empty integral polytope.
Then $r^*(P) = +\infty$ if and only if there exists $v \in \mathbb{Z}^n$ such that $P + \langle v \rangle$ is lattice-free.

- $\langle v \rangle$ is the 1-dimensional linear space generated by v;
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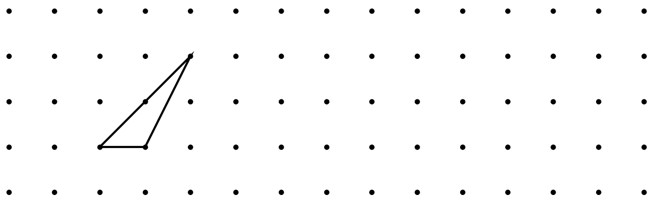

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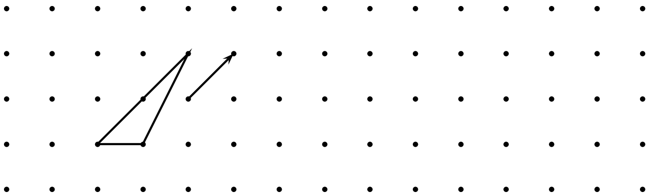

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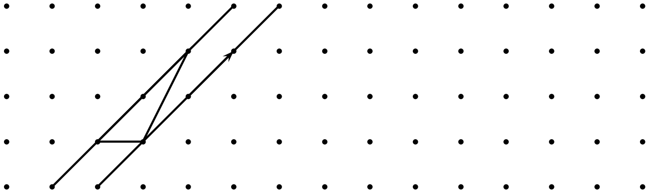

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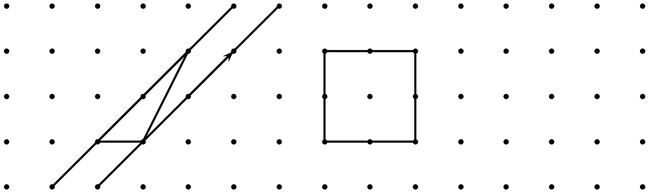

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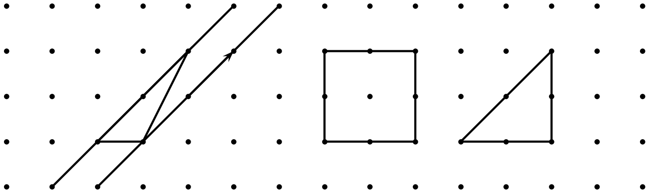

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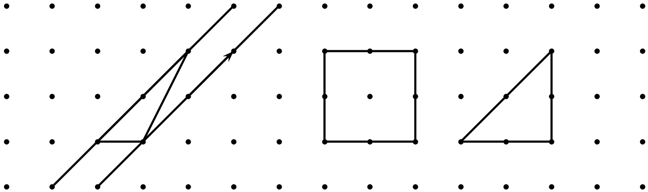
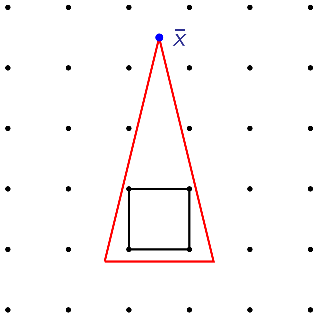
Let $P \subseteq \mathbb{R}^n$ be a non-empty integral polyhedron.
Then $r^*(P) = +\infty$ if and only if there exists $v \in \mathbb{Z}^n \setminus \text{rec}(P)$ such that $P + \langle v \rangle$ is lattice-free.

- $\langle v \rangle$ is the 1-dimensional linear space generated by v;
- a set is lattice-free if it has no integer point in its relative interior;
- $\text{rec}(P)$ is the recession cone or characteristic cone of P, i.e., the set of unbounded directions of P.



Lower bound lemma

Let Q be a polyhedron and $\bar{x}$ be a point in Q.
Let v be an integer vector and $\bar{t} = \min\{t \geq 0 : \bar{x} + tv \in Q_I\}$.
Then $r(Q) \geq \lceil \bar{t} \rceil$.



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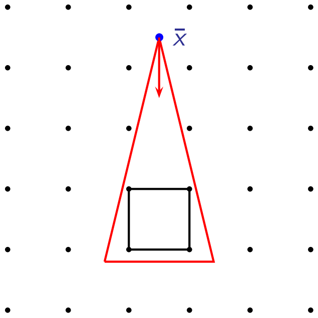

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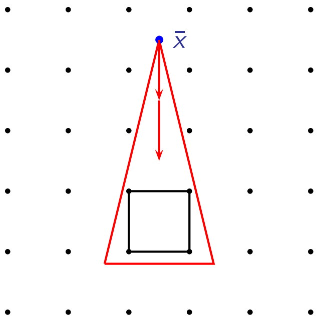

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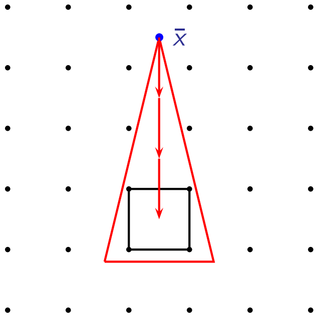

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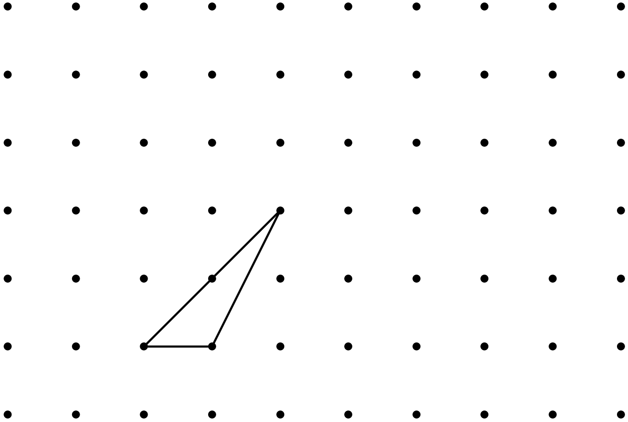

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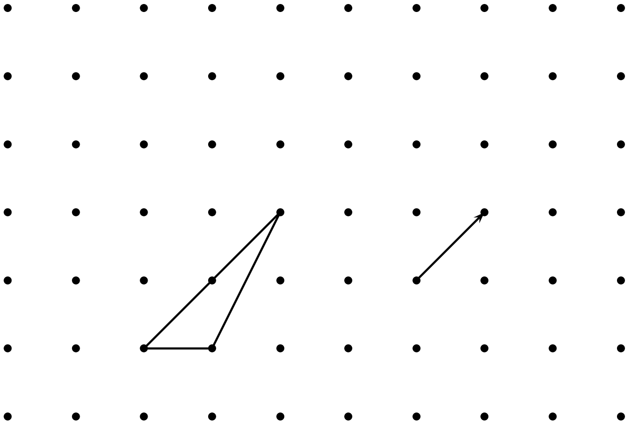

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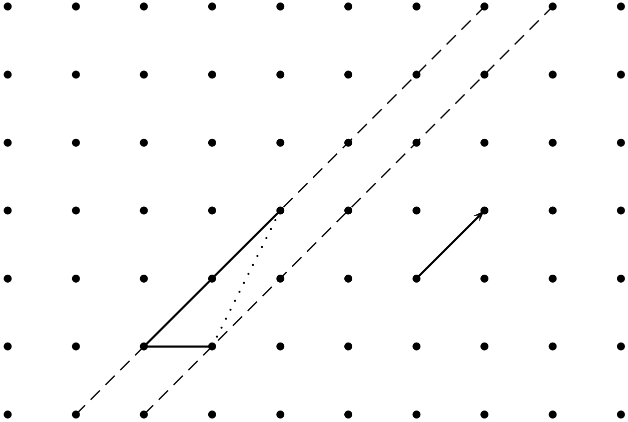

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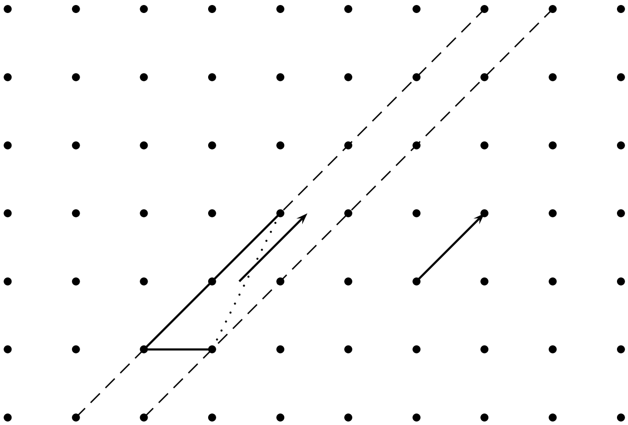

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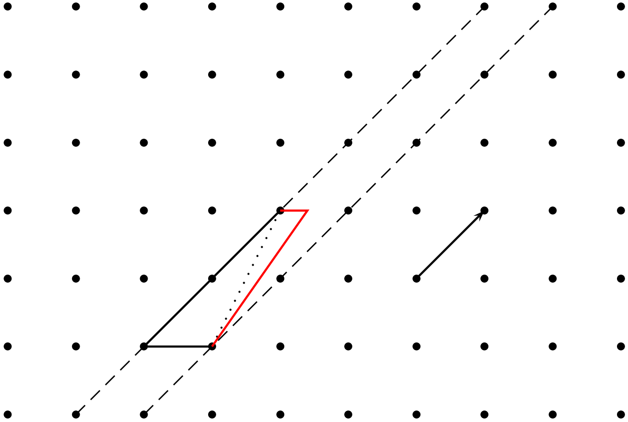

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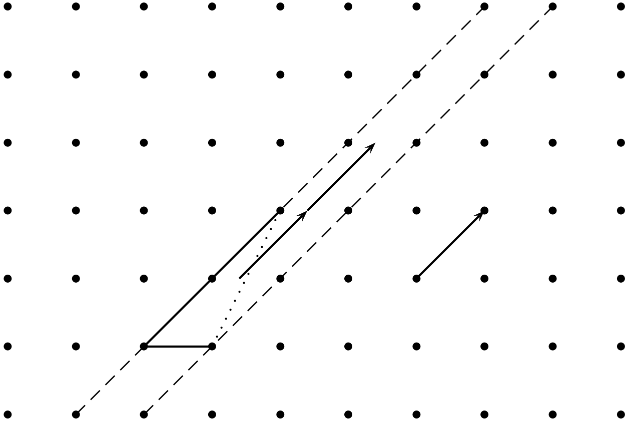

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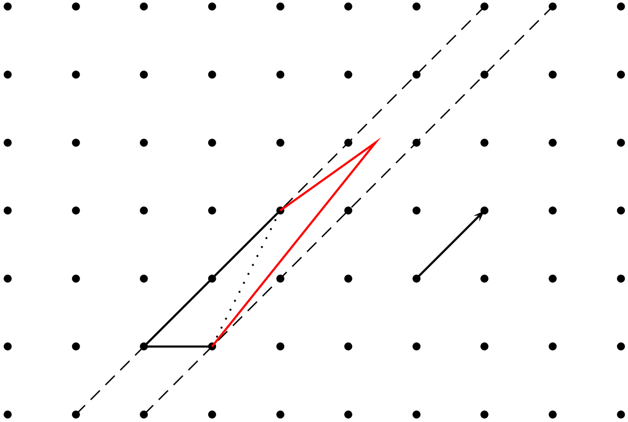

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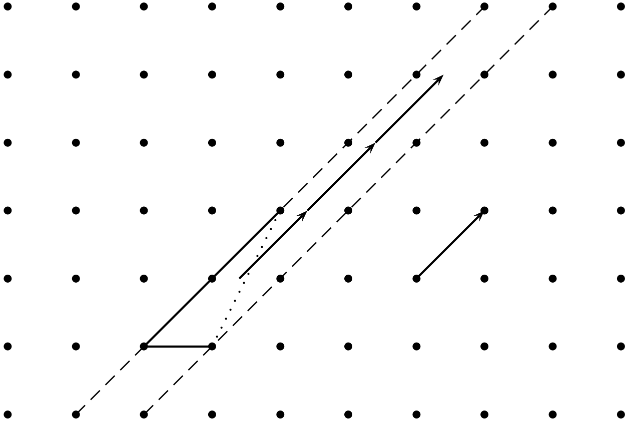

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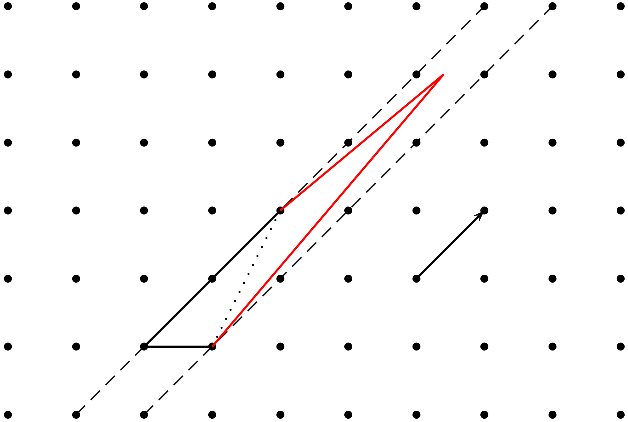

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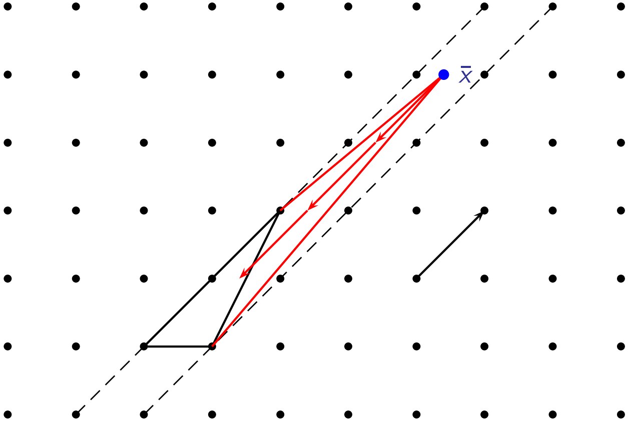

Upper-bound lemma

Let $P = \{x : Ax \leq b\} \subseteq \mathbb{R}^n$ be an integral polyhedron. If there exists $k \in \mathbb{N}$ such that each relaxation of P is contained in $\{x : Ax \leq b + k \cdot \mathbf{1}\}$, then $r^*(P) < +\infty$.

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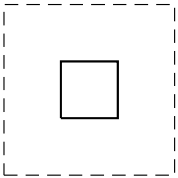

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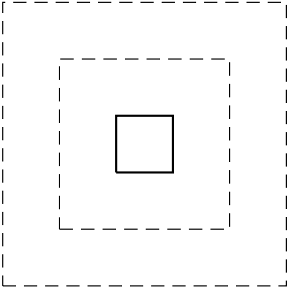

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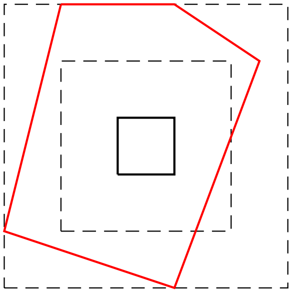

Necessity for full-dimensional polytopes

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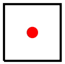

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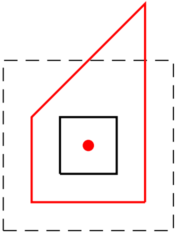

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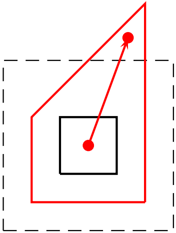

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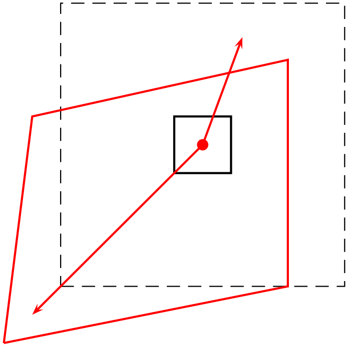

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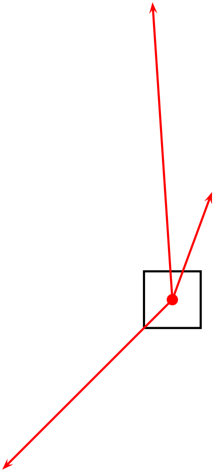

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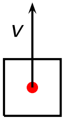

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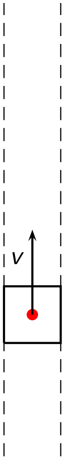

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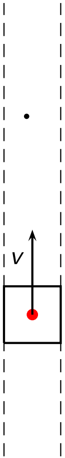

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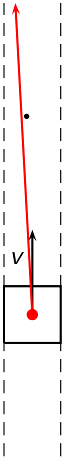

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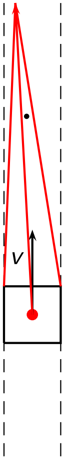

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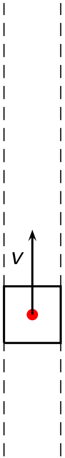

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3. we can assume that $v \in \mathbb{Z}^n$.

   (using the characterization of maximal lattice-free convex sets, Basu, Conforti, Cornuéjols, Zambelli, 2010)

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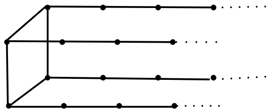

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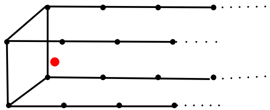

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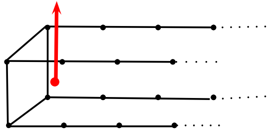

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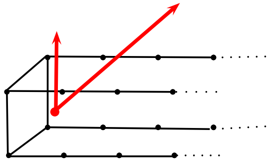

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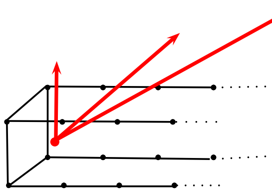

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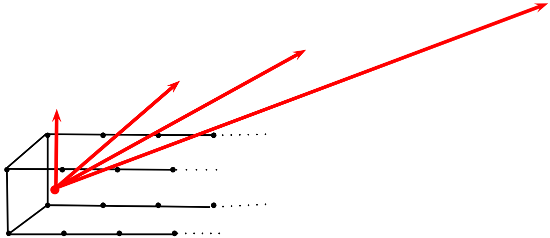

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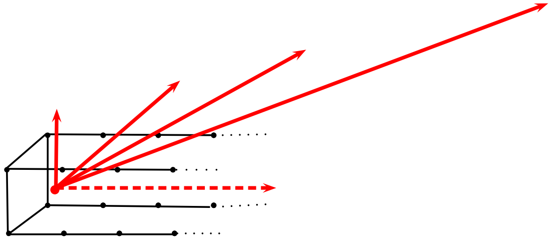

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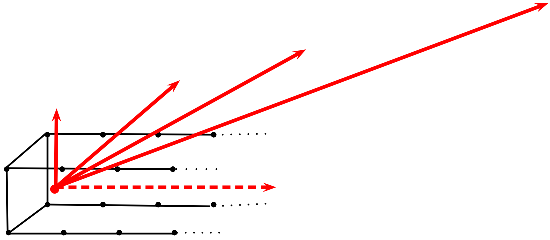

- Extension to non-full-dimensional polyhedra: how many CG rounds does a relaxation need to lie in the affine hull of its integer hull? (Averkov,CDDF 2012).

Further results and future work

Bounds on r^*

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Given P, for which $v \in \mathbb{Z}^n$ is $P + \langle v \rangle$ lattice-free?

What about the Reverse Split rank?

Thank you for your attention.

Thank you for your attention. Paper available here:

http://arxiv.org/abs/1211.0388