Delay-Robust Event-Scheduling
*In Memoriam* of A. Caprara

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Delay-Robust Event-Scheduling
   Robustness
   Framework
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Train Platforming Problem
  In and Out
  TPP deterministic model
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- Delay propagation network
- Buffers linking constraints
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Robust Optimization

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Pros

- no knowledge of the underlying distribution is required
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Cons

Strict robustness is generally overconservative, because:

- solutions must cope with every likely scenarios without any recovery
Informally speaking, a solution to an optimization problem is called recovery robust if it can be adjusted to all likely scenarios by limited recovery action. Thus a recovery-robust solution provides a service guarantee (Liebchen et al. 2009 [3], Stiller 2008 [4]).
Robust Network Buffering

We are interested in the special case in which the recovery problem is a delay ($d_e$) propagation in some directed graph $N = (E, A)$, which is buffered on the arcs against disturbances on the nodes $e \in E$.

**Details** in Liebchen *et al.* 2009 [3], Stiller 2008 [4].
A framework

- A set $\mathcal{E}$ of events to be scheduled.
- Denoting by $F$ the set of feasible schedulings of events $\mathcal{E}$, i.e., the feasible region, a nominal problem of the form $\min \{ c(x) : x \in F \}$.
- A set $\mathcal{S}$ of possible scenarios, where each scenario $s \in \mathcal{S}$ is defined by the external disturbance $\delta_e^s \geq 0$ assigned to each event $e \in \mathcal{E}$.
- A delay-propagation network $N = (\mathcal{E}, A)$, with $(e', e) \in A$ if a delay $d_{e'}$ on event $e'$ may propagate to a delay $d_e$ on event $e$.
- A delay $d_{e'}$ on event $e'$ may propagate to a delay $d_e$ on event $e$ according to the relation $d_e \geq d_{e'} - b((e', e), x)$, where $b((e', e), x)$ is a buffer time function.
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\section*{Delay-Robust Event-Scheduling Framework}

\section*{Mathematical model}

The delay-robust problem can be formulated as:

\begin{equation}
\min c(x) + D, \tag{1}
\end{equation}

subject to

\begin{align*}
x \in F, \tag{2} \\
D & \geq \sum_{e \in \mathcal{E}} d_{e}^{s}, \quad s \in \mathcal{S}, \tag{3} \\
d_{e}^{s} & \geq d_{e}^{s}, \quad e \in \mathcal{E}, \quad s \in \mathcal{S}, \tag{4} \\
d_{e}^{s} & \geq d_{e}^{s} - f(e', e), \quad (e', e) \in A, \quad s \in \mathcal{S}, \tag{5} \\
f(e', e) & = b((e', e), x), \quad (e', e) \in A. \tag{6}
\end{align*}
Scenario set

\[ \{ \delta^s \in \mathbb{R}^{|\mathcal{E}|}_+ : \|\delta^s\|_1 \leq \Delta \} \quad (7) \]

These are uncountably many scenarios, which can however be handled easily thanks to the following observation.

Proposition

For the delay-robust problem (1)-(6), the compact scenario set defined by (7) is equivalent to the finite scenario set \( S \) defined by the vectors

\[ \{ \delta^s \in \{0, \Delta\}^{\mathcal{E}} : \|\delta^s\|_1 = \Delta \}, \]

which contains only \( |\mathcal{E}| \) scenarios.
Quadratic buffer time function

In the most natural case, the buffer time $b((e', e), x)$ only depends on the way in which events $e'$ and $e$ are scheduled in $x$, and is independent of the scheduling of the other events:

$$f(e', e) = \sum_{h' \in \mathcal{H}_e} \sum_{h \in \mathcal{H}_e} \varphi(e', h', e, h) \ x_{e', h'} \ x_{e, h}, \quad (e', e) \in A.$$ 

These quadratic constraints can be linearised in a few ways.
Projecting out delay variables

One may directly optimise over the feasible region obtained by projecting out the $d_e^s$ variables (and removing the constraints (3)-(5)).

Let $\sigma_{\bar{f}}(s, \bar{e})$ be the value of the shortest path from $s$ to $\bar{e} \in \mathcal{E}$ on the delay propagation network $N = (\mathcal{E}, A)$ with arc lengths $\bar{f}_{(e', e)}$ for $(e', e) \in A$. Moreover, let $P_{\bar{f}}(s, \bar{e}) \subseteq A$ denote such a shortest path.

**Lemma**

*Given buffer time values $\bar{f}_{(e', e)}$, an external disturbance equal to $\Delta$ on event $s \in \mathcal{E}$ (and null for the other events) propagates to a delay of value $\max\{0, \Delta - \sigma_{\bar{f}}(s, \bar{e})\}$ on event $\bar{e} \in \mathcal{E}$.***
A direct consequence is:

**Lemma**

Given buffer time values $\bar{f}(e',e)$ and maximum cumulative delay $\bar{D}$, for every scenario $s \in \mathcal{S}$ there exist values of the $d_s^e$ variables satisfying (3)-(5) if and only if $\sum_{e \in \mathcal{E}} \max\{0, \Delta - \sigma_{\bar{f}}(s,e)\} \leq \bar{D}$. Otherwise, for the considered scenario $s$, letting $\mathcal{E}_{\bar{f}}(s) \subseteq \mathcal{E}$ be the set of events such that $\Delta - \sigma_{\bar{f}}(s,e) > 0$, a valid inequality that is violated by $\bar{f}(e',e)$ and $\bar{D}$ is

$$\sum_{e \in \mathcal{E}_{\bar{f}}(s)} \sum_{a \in P_{\bar{f}}(s,e)} f_a \geq \Delta |\mathcal{E}_{\bar{f}}(s)| - D.$$
Problem definition

The objective of train platforming is assigning trains to platforms in a railway station.
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- for a specific railway station
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The objective of train platforming is assigning trains to platforms in a railway station. Platforming is carried out:

- for a specific railway station
- after the timetable has been defined
In and Out
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Input

- Train schedule: arrival, departure times, directions and allowed shifts
- Railway station topology: platforms, paths and directions
In and Out

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- Train schedule: arrival, departure times, directions and allowed shifts
- Railway station topology: platforms, paths and directions

Output

- Assign each train a platform and two paths for arrival and departure s.t. no operational constraint is violated
Train schedule

The train schedule of a railway station contains info on arrival and departure times, directions and allowed shifts of each train passing through it.
Railway station topology

The topology of a railway station includes platforms, paths and directions.
Resources and operational constraints

Platform conflicts are forbidden, path conflicts are allowed to some extent.

A pattern $P$ for a train $t$ is a 5-tuple defining: platform, arrival/departure paths and shifts. Operational constraints can be expressed using an incompatibility graph among patterns.
TPP deterministic model

\[
\min \sum_{t \in T} \sum_{P \in \mathcal{P}_t} c_{t,P} \ x_{t,P}
\]

s.t.

\[
\sum_{P \in \mathcal{P}_t} x_{t,P} = 1, \quad t \in T
\]

\[
\sum_{(t,P) \in K} x_{t,P} \leq 1, \quad K \in \mathcal{K}
\]

\[
x_{t,P} \in \{0,1\}, \quad t \in T, P \in \mathcal{P}_t
\]

**Details** in Caprara *et al.* 2011 [1].
Delay propagation network

The platforming gives rise to a network in which the delay caused by disturbances propagates.
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We consider two type of disturbances: (i) the train arrives late at the station, and (ii) the platform operations take longer than required.
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Hence, the following two events are associated with each train $t \in T$ and define $\mathcal{E}$:

- **arrival $a_t$:** the train occupies a given arrival path at a given instant
- **departure $p_t$:** the train frees its platform at a given instant
Example

Assume that in the nominal solution trains $t'$ and $t$ stop at the same platform, event $p_{t'}$ is scheduled at 10:00, and event $a_t$ at 10:04 with 3 minutes to travel along the arrival path, so that $t$ occupies the platform at 10:07. Moreover, assume that the headway time that must elapse between $t'$ freeing the platform and $t$ occupying it is 2 minutes and that the 3-minute travel time for $t$ along its arrival path is fixed. If the departure path of $t'$ and the arrival path of $t$ are compatible, the buffer time $f(p_{t'}, a_t)$ is equal to 5 minutes, as a delay of up to 5 minutes on $p_{t'}$ does not affect $a_t$, whereas a larger one does.
Delay-robust mathematical model

\[
\min \sum_{t \in T} \sum_{P \in \mathcal{P}_t} c_{t,P} \ x_{t,P} + D
\]

subject to

\[
\sum_{P \in \mathcal{P}_t} x_{t,P} = 1, \quad t \in T,
\]

\[
\sum_{(t,P) \in K} x_{t,P} \leq 1, \quad K \in \mathcal{K},
\]

\[
x_{t,P} \in \{0, 1\}, \quad t \in T, \ P \in \mathcal{P}_t,
\]

\[
f(e', e) = \sum_{P_1 \in \mathcal{P}_{t_1}} \sum_{P_2 \in \mathcal{P}_{t_2}} c_{P_1,P_2,a} \ x_{t_1,P_1} x_{t_2,P_2}, \quad (e', e) \in A,
\]

\[
D \geq \sum_{e \in \mathcal{E}} d^s_e, \quad s \in \mathcal{I},
\]

\[
d^s_e \geq \delta^s_e, \quad e \in \mathcal{E}, \ s \in \mathcal{I},
\]

\[
d^s_e \geq d^s_{e'} - f(e', e), \quad (e', e) \in A, \ s \in \mathcal{I}.
\]
A straightforward link between the buffer value of a given arc $a \in A(N)$ associated with train pair $(t_1, t_2) \in T^2$ and the choice of patterns for the given pair of trains is the following:

$$\sum_{P_1 \in \mathcal{P}_{t_1}} \sum_{P_2 \in \mathcal{P}_{t_2}} c_{P_1,P_2,a} x_{t_1,P_1} x_{t_2,P_2}$$

where $c_{a,P_1,P_2}$ is a constant associated to arc $a$ and to the corresponding choice of patterns $(P_1, P_2)$ for trains $(t_1, t_2)$. 
Bufflers linking constraints

\[ f_a \leq \sum_{P_1 \in \mathcal{P}_{t_1}} \alpha^a_{P_1} x_{t_1,P_1} + \sum_{P_2 \in \mathcal{P}_{t_2}} \beta^a_{P_2} x_{t_2,P_2} - \gamma^a, \quad a \in A(N), \ (\alpha, \beta, \gamma) \in \mathcal{F}_a \quad (8) \]

Following Caprara et al. 2011 [1], the separation of Constraints (8) is done by a sort of polyhedral brute force, given that, for each pair of trains \( t_1, t_2 \), and for each arc \( a \in A(N) \) the number of vertices in \( Q_{t_1,t_2,a} \) is small. Specifically, \( Q_{t_1,t_2,a} \) has \( |\mathcal{P}_{t_1}| |\mathcal{P}_{t_2}| \) vertices and lies in \( \mathbb{R} |\mathcal{P}_{t_1}| + |\mathcal{P}_{t_2}| + 1 \), so we can separate over it by solving an LP with \( |\mathcal{P}_{t_1}| |\mathcal{P}_{t_2}| \) variables and \( |\mathcal{P}_{t_1}| + |\mathcal{P}_{t_2}| + 1 \) constraints.
Computational results: Palermo C.Le.

<table>
<thead>
<tr>
<th>instance</th>
<th>[1] Solution</th>
<th>Delay-Robust LP</th>
<th>Delay-Robust Solution</th>
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<tbody>
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<td></td>
<td>$D$</td>
<td>time</td>
<td>$D$</td>
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<td>1112</td>
<td>5</td>
<td>451.88</td>
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<tr>
<td>PA II</td>
<td>694</td>
<td>5</td>
<td>252.00</td>
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<tr>
<td>PA III</td>
<td>756</td>
<td>5</td>
<td>285.00</td>
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<tr>
<td>PA IV</td>
<td>525</td>
<td>5</td>
<td>207.00</td>
</tr>
</tbody>
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**Table:** Results for the real-world instances of Rete Ferroviaria Italiana.
Computational results: Genova P.Principe

<table>
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<tbody>
<tr>
<td></td>
<td>$D$</td>
<td>time</td>
<td>$D$</td>
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<tr>
<td>GE I</td>
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<td>GE II</td>
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<td>GE III</td>
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<tr>
<td>GE IV</td>
<td>758</td>
<td>5</td>
<td>231.00</td>
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Computational results: Bari C.Le.

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<th>Delay-Robust Solution</th>
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<tbody>
<tr>
<td>BA I</td>
<td>770 5</td>
<td>341.00 68</td>
<td>592 42% 23% 1095</td>
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<td>508.08 118</td>
<td>959 47% 0% 5</td>
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<tr>
<td>BA III</td>
<td>711 5</td>
<td>293.91 115</td>
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<tr>
<td>BA IV</td>
<td>733 5</td>
<td>308.40 103</td>
<td>686 55% 6% 2431</td>
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<td>BA V</td>
<td>786 5</td>
<td>245.26 165</td>
<td>734 67% 7% 3204</td>
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<tr>
<td>BA VI</td>
<td>819 5</td>
<td>312.00 51</td>
<td>819 62% 0% 5</td>
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<tr>
<td>BA VII</td>
<td>1025 5</td>
<td>315.18 77</td>
<td>743 58% 27% 4126</td>
</tr>
<tr>
<td>BA VIII</td>
<td>714 5</td>
<td>240.00 45</td>
<td>580 59% 19% 535</td>
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Table: Results for the real-world instances of Rete Ferroviaria Italiana.
### Computational results: Milano C.Le.

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<tr>
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<th>[1] Solution $D$</th>
<th>Delay-Robust LP $D$</th>
<th>Delay-Robust Solution $D$</th>
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<th>%reduction</th>
<th>time</th>
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<tr>
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<td>25%</td>
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<td>816</td>
<td>49%</td>
<td>9%</td>
<td>618</td>
</tr>
</tbody>
</table>

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Questions

Thank you!
References


