The Target Visitation Problem

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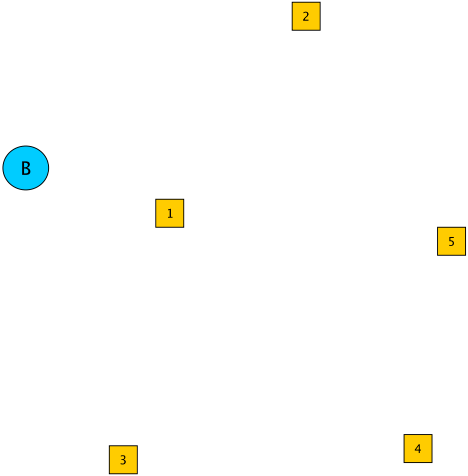
The Problem



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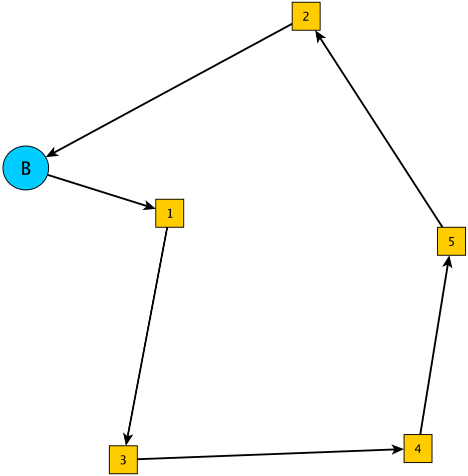

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Applications

- Planning of missions in disaster areas to distribute food and medicine
- Snow clearance
- Town cleaning
- In general: positioning problems with additional preferences

Some Facts:

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- NP-hard
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- NP-hard
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Variables:

TSP Variables:

$$x_{i,j} := \begin{cases} 1 & \text{if } \exists\ k \in \{0, n-2\} \text{ so that } i = \pi(k) \text{ and } j = \pi(k+1) \\ 1 & \text{if } i = \pi(n-1) \text{ and } j = \pi(0) \\ 0 & \text{otherwise} \end{cases}$$

LOP Variables:

$$w_{i,j} := \begin{cases} 1 & \text{if } \exists\ k \in \{1, n-2\} \text{ so that } i = \pi(k) \text{ and } j = \pi(l) \text{ with } k < l \\ 0 & \text{otherwise} \end{cases}$$

Removal of the base node:

- Since the base node is always the first node in our tour we can remove it by adjusting the distance matrix and objective function in the following way.
- Adjust the distance matrix as follows:
  $d'_{ij} = d_{ij} - d_{i0} - d_{0j} \; i, j \in \{1 \dots n\}$
- Adjust the objective function as follows:
  $\max \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} p_{i,j} w_{i,j} - \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} d'_{i,j} x_{i,j} - \sum_{i=1}^{n} d_{i0} - \sum_{i=1}^{n} d_{0i}$
- The TVP is now a combination of the LOP and the Hamiltonian Path problem.

IP model:

$$\max \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} p_{i,j} w_{i,j} - \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} d'_{i,j} x_{i,j} - \sum_{i=1}^{n} d_{i0} - \sum_{i=1}^{n} d_{0i}$$

subject to

$$\sum_{i=1}^{n} \sum_{j=1, i \neq j}^{n} x_{i,j} = n - 1,$$

$$\sum_{i=1}^{n} x_{i,j} \leq 1, \; j \in N$$

$$\sum_{j=1}^{n} x_{i,j} \leq 1, \; i \in N$$

$$\sum_{i \in S} \sum_{j \in S} x_{i,j} \leq |S| - 1, \; \forall S \subset V, \; 2 \leq |S| \leq n$$

$$w_{i,j} + w_{j,k} + w_{k,i} \leq 2, \; i, j, k \in N$$

$$w_{i,j} + w_{j,i} = 1, \; i, j \in N$$

$$x_{i,j} \leq w_{i,j}, \; i, j \in N$$

$$x_{i,j}, w_{i,j} \in \{0, 1\}, \; i, j \in N$$

Polyhedral Results

- The dimension of the TVP_{HP} polytope is: $\frac{3n^2-3n-2}{2}$ for $n \geq 4$
- That means there exist no more equations then the ones we already have in the model
- The examination of the polytope for $n = 4$ yields 1280 facets in 46 classes.
- We were able to generalize some classes.
- For $n = 5$ there are more than 100 Million classes of facets.

Lifting Results

Node i is calles a free node of a facet if all $x_{i,j}, x_{j,i}, w_{i,j}, w_{j,i}$ are zero.

Theorem

Let $ax \leq a_0$ define a facet for the $\mathrm{TVP}_{HP}(\mathrm{n})$ with at least one free node. Then the zero lifting of $ax \leq a_0$ defines a facet for the $\mathrm{TVP}_{HP}(\mathrm{k})$ with $k \geq n$.

With this theorem we can prove that 12 classes of the 46 classes are facets of $TVP_{HP}(n)$.

Name	Facet
C2	$w_{il} + w_{kl} + w_{lj} + x_{ji} + x_{jk} + x_{jl} + x_{li} + x_{lk} \leq 3$
	$w_{il} + w_{kj} + w_{lk} + x_{ji} + x_{jk} + x_{jl} + x_{kl} + x_{li} \leq 3$
C7	$w_{ij} + 2w_{jk} + 2w_{kl} + w_{li} + w_{lj} + x_{il} + x_{ji} + x_{jl} + x_{kj} + x_{lk} \leq 5$
C11	$-x_{ki} \leq 0$
C13	$w_{il} + w_{ji} + w_{jk} + w_{kl} + w_{lj} + x_{ij} + x_{kj} + x_{li} + x_{lj} + x_{lk} \leq 4$
C14	$w_{ij} + 2w_{jk} + w_{ki} + w_{kl} + w_{lj} + x_{ji} + x_{jl} + x_{kj} \leq 4$
C20	$w_{ij} + w_{il} + 2w_{jk} + w_{ki} + w_{lj} + x_{ji} + x_{jl} + x_{ki} + 2x_{kj} + x_{kl} + x_{li} \leq 5$
C25	$w_{il} + w_{jk} + w_{ki} + w_{lj} + x_{jl} + x_{kj} + x_{kl} \leq 3$
C29	$w_{jk} + w_{kl} + w_{lj} + x_{kj} \leq 2$
C30	$2w_{jk} + 2w_{kl} + 2w_{lj} + x_{jl} + x_{kj} + x_{lk} \leq 4$
C39	$w_{ij} + 2w_{jk} + w_{kl} + x_{ji} + x_{ki} + 2x_{kj} + x_{li} + x_{lj} + x_{lk} \leq 4$
C41	$w_{jk} + w_{kl} + x_{kj} + x_{lj} + x_{lk} \leq 2$
C46	$w_{jk} + x_{kj} \leq 1$

Table : Some facets of the $\text{TVP}_{HP}(4)$ polytope for which zero lifting is possible

Observations

- The class of the extended three cycle inequalities $w_{i,j} + w_{j,k} + w_{k,i} + x_{j,i} \leq 2$ can replace the normal three cycle inequalities.
- The extended three cycles imply the subtour elimination constraints.

New constraints

$$\sum_{i=1}^{n} \sum_{j=1, i \neq j}^{n} x_{i,j} = n-1, \; j \in N$$
$$\sum_{i=1}^{n} x_{i,j} \leq 1, \; j \in N$$
$$\sum_{j=1}^{n} x_{i,j} \leq 1, \; i \in N$$
$$w_{i,j} + w_{j,k} + w_{k,i} + x_{j,i} \leq 2, \; i, j, k \in N$$
$$x_{i,j} \leq w_{i,j}, \; i, j \in N$$
$$x_{i,j}, w_{i,j} \in \{0, 1\}, \; i, j \in N$$

Interesting fact: Model could be used for solving TSP with a polynomial number of constraints.

Branch-and-Cut

- In order to solve practical problems we implemented a branch-and-cut algorithm in C++ using CPLEX and ABACUS.
- We were able to solve instance with up to 30 nodes to optimality..
- We try to use different classes of facets as additional cutting planes.
- We use a heuristic for the computation of a lower bounds

Instance	N.	w. c.	T2	T7	T14	T20	T25	T30	T41	o.m
HP13	13	< 1	0:00:01	< 1	< 1	0:00:01	< 1	< 1	< 1	0.29.51
HP19	19	0:00:02	0:00:03	0:00:03	0:00:03	0:00:07	0:00:03	0:00:02	0:00:03	$> 24h.$
HP21	21	0:00:04	0:00:07	0:00:06	0:00:07	0:00:11	0:00:05	0:00:05	0:00:06	$> 24h.$
HP23	23	0:00:05	0:00:06	0:00:05	0:00:06	0:00:11	0:00:07	0:00:05	0:00:05	$> 24h.$
HP25-1	25	0:00:05	0:00:05	0:00:05	0:00:05	0:00:13	0:00:06	0:00:05	0:00:05	$> 24h.$
HP25-2	25	0:36:20	0:39:46	0:38:23	0:43:03	0:53:59	0:42:03	0:37:30	0:38:49	$> 24h.$
HP26-2	26	0:05:57	0:06:42	0:05:50	0:06:27	0:08:12	0:06:20	0:05:31	0:05:57	$> 24h.$
HP26-3	26	1:20:00	1:26:57	1:23:25	1:25:31	1:58:31	1:31:36	1:20:53	1:25:01	$> 24h.$
HP26-4	26	0:00:41	0:00:51	0:00:43	0:00:44	0:01:26	0:00:46	0:00:41	0:00:50	$> 24h.$
HP27	27	0:16:04	0:20:04	0:16:50	0:18:35	0:26:33	0:19:04	0:15:59	0:17:16	$> 24h.$

Table : Overview over the computation results

Extended Formulation

Add new variables:

$$w_{i,j,k} := \begin{cases} 1 & \text{if } \exists\ a, b, c \in \{1, n\} \text{ s. t. } i = \pi(a) \text{ and } j = \pi(b) \text{ and } k = \pi(c) \\ & \text{with } a < b < c. \\ 0 & \text{otherwise} \end{cases}$$

$\rightarrow$ Extension of the linear ordering variables

New constraints

$$\sum_{i=1}^{n} \sum_{j=1, i \neq j}^{n} x_{i,j} = n - 1, \ j \in N$$

$$\sum_{i=1}^{n} x_{i,j} \leq 1, \ j \in N$$

$$\sum_{j=1}^{n} x_{i,j} \leq 1, \ i \in N$$

$$w_{i,j} + w_{j,k} + w_{k,i} + x_{j,i} \leq 2, \ i, j, k \in N$$

$$w_{i,j} + w_{j,i,k} + w_{j,k,i} + w_{k,j,i} = 1, \ 1 \leq i, j, k \leq n, i < j$$

$$x_{i,j} - w_{i,j,k} - w_{k,i,j} \leq 0, \ 1 \leq i, j, k \leq n, i < j$$

$$x_{i,j} \leq w_{i,j}, \ i, j \in N$$

$$x_{i,j}, w_{i,j} \in \{0, 1\}, \ i, j \in N$$

$$w_{ijk} \in \{0, 1\}, \ 1 \leq i, j, k \leq n$$

Advantages and Disandvantages

(+) The polytope has a much simpler description.

(+) The gap closure of the root bound is more than 50 % better compared to our standard model.

(-) Cubic number of variables $\rightarrow$ Pricing is recommended.

Instance	Solution	Bound NF	Gap	Bound EXF	Gap
HP13	410	422.1	2.9%	414.3	1.0%
HP19	894	913.3	2.1%	902.3	0.9%
HP21	1046	1069.1	2.1%	1056.3	0.9%
HP23	1302	1319.9	1.4%	1312.6	0.8%
HP25-1	1586	1604.8	1,1%	1593.5	0.4%
HP25-2	1475	1521.8	3,0%	1508.7	2.2%
HP26-2	1624	1658.1	2.0%	1649.0	1.5%
HP26-3	1679	1725.4	2.6%	1712.1	1.9%
HP26-4	1746	1778.2	1.8%	1767.1	1.2%
HP27	1817	1858.9	2.2%	1846.1	1.5%

Table : Comparison of the root bounds

Problem: If we use column generation, we have to add a cubic number of $w_{i,j,k}$ variables to obtain a feasible LP.

Instance	Solution	Time Branch-and-Price	Time Branch-and-Cut
HP13	410	0:00:19	< 1
HP19	894	0:00.64	0:00:02
HP21	1046	0:00:65	0:00:04
HP23	1302	0:00:02	0:00:05

Table : Computation times of a simple branch-and-price Algorithm

Projection

Idea: We want to project the polytope of the extended formulation to our standard polytope so that we can see which facet classes are implied.

Interesting facts:

- We can eliminate the six variables $w_{i,j,k}$, $w_{i,k,j}$, $w_{j,i,k}$, $w_{j,k,i}$, $w_{k,i,j}$, $w_{k,j,i}$ separately for each i, j, k
- The extended formulation implies two classes of inequalities:
  - $2w_{j,k} + 2w_{k,l} + 2w_{l,j} + x_{j,l} + x_{k,j} + x_{l,k} \leq 4$
  - $w_{j,k} + w_{k,l} + x_{k,j} + x_{l,j} + x_{l,k} \leq 2$

- Both classes are facets for $n \geq 4$
- Moreover they are the only facets on three nodes for $n = 4$ which are not already contained in the model.
- For $n = 5$ this also true.
- open question: Is this true for $n = k$?
- Another open question: Does a model which contains the variables $w_{i_1 i_2 \ldots i_k}$ imply all facets on k nodes ?

Thank you for your attention