Reformulation of a market clearing problem avoiding complementarity constraints, and its consequences

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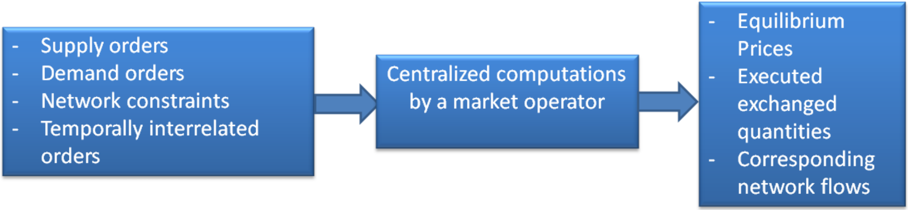
Introduction

Day-ahead Markets:

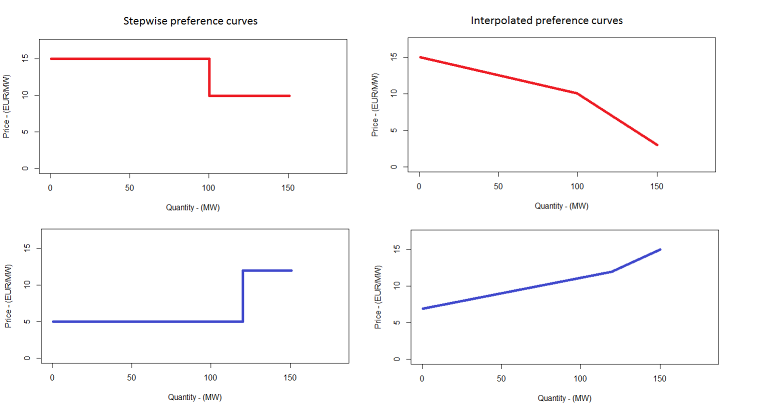
- 24 periods (23 or 25 once a year)
- several areas/locations for bids + network constraints
- Both demand and offer bids (elastic demand)



We are interested in *uniform/linear prices*.

Main kinds of orders:

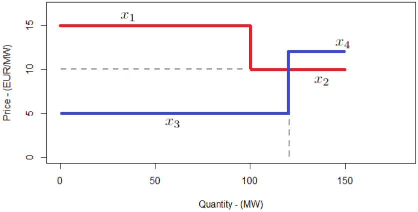
Hourly orders - (Monotonic) DEMAND / SUPPLY preference curves



Block orders, i.e. 'binary orders'

Span multiple periods and "fill-or-kill condition": order must be entirely accepted or rejected.

A simple matching for hourly orders:



$$\max_{x_i \geq 0} \quad (100 \times 15)x_1 + (50 \times 10)x_2 - (120 \times 5)x_3 - (30 \times 12)x_4$$

$$\begin{aligned} s.t. \quad & x_i \leq 1 && \forall i \in \{1,2,3,4\} \quad [s_i] && (1) \\ & 100x_1 + 50x_2 = 120x_3 + 30x_4 && [p] && (2) \end{aligned}$$

optimum value $= 1100$ for $x_1 = 1, x_2 = \dfrac{2}{5}, x_3 = 1, x_4 = 0$

Quantity exchanged: $100 + \dfrac{2}{5}50 = 120$ MW.

$$\min_{s_i \geq 0, p} \quad s_1 + s_2 + s_3 + s_4$$

$$\begin{aligned} s.t. \quad & s_1 + 100p \geq 1500 \quad [x_1] & (3) \\ & s_2 + 50p \geq 500 \quad [x_2] & (4) \\ & s_3 - 120p \geq -600 \quad [x_3] & (5) \\ & s_4 - 30p \geq -360 \quad [x_4] & (6) \end{aligned}$$

optimum value = 1100 for $p = 10, s_1 = 500, s_2 = 0, s_3 = 600, s_4 = 0$

Marginal Price: 10 EUR/MW

s_i **is the surplus of order** i

Equilibrium: 120 MW exchanged at 10 EUR/MW.

Quantity exchanged: primal program

Equilibrium Price: dual program

Welfare maximization $\Leftrightarrow$ Walrasian equilibrium

(see dual and *complementarity constraints*)

Primal (welfare maximizing) program:

$$\max_{x_i, y_j} \quad \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j$$

subject to:

$$x_i \leq 1 \qquad \forall i \in I \quad [s_i] \tag{7}$$

$$y_j \leq 1 \qquad \forall j \in J \quad "[s_j]" \tag{8}$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0 \qquad [p_m] \tag{9}$$

$$x_i, y_j \geq 0, \quad y_j \in \mathbb{Z}. \tag{10}$$

$Q < 0$ for sell orders and $Q > 0$ for buy orders !

Dual (continuous relax.):

$$\min_{s_i, s_j} \sum_i s_i + \sum_j s_j$$

subject to:

$$s_i + Q^i p_m \geq Q^i P^i \quad \forall i \in I \quad [x_i] \tag{11}$$

$$s_j + Q^j p_m \geq Q^j P^j \quad \forall j \in J \quad [y_j] \tag{12}$$

$$s_i, s_j \geq 0 \tag{13}$$

Complementarity Constraints:

$$s_i(1 - x_i) = 0 \qquad \forall i \in I \tag{14}$$

$$s_j(1 - y_j) = 0 \qquad \forall j \in J \tag{15}$$

$$x_i(s_i + Q^i p_m - Q^i P^i) = 0 \qquad \forall i \in I \tag{16}$$

$$y_j(s_j + Q^j p_m - Q^j P^j) = 0 \qquad \forall j \in J \tag{17}$$

If $x_i = 1 \longrightarrow$ complem. constr. of type (16)

$x_i = 1 \Rightarrow s_i = Q^i(P^i - p_m) \geq 0$

- $p_m \geq P^i$ for sell orders (when $Q < 0$)
- $P^i \geq p_m$ for buy orders (when $Q > 0$)

Order is In-The-Money (ITM) or At-The-Money (ATM)

If $0 < x_i < 1 \longrightarrow$ complem. constr. of type (14) and (16)

$(0 < x_i < 1) \Rightarrow s_i = Q^i(P^i - p_m) = 0 \Rightarrow P^i = p_m$ (order is ATM)

If $x_i = 0 \longrightarrow$ complem. constr. of type (14) and dual (11)

$(x_i = 0) \Rightarrow (s_i = 0) \Rightarrow Q^i(P^i - p_m) \leq 0$ (order is ATM or OTM)

Or also: ITM orders are executed !

Classical MPCC formulation Of European market rules

$$\max_{x_i, y_j, p_m, s_i, s_j} \quad \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j$$

subject to:

$$\begin{array}{lll}
x_i \leq 1 & \forall i \in I & [s_i] \\
y_j \leq 1 & \forall j \in J & "[s_j]" \\
\sum_i Q^i x_i + \sum_j Q^j y_j = 0 & & [p_m] \\
x_i, y_j \geq 0, \ \boxed{y_j \in \mathbb{Z}} & &
\end{array}$$

Primal constraints (feasible dispatch)

$$\begin{array}{lll}
s_i + Q^i p_m \geq Q^i P^i & \forall i \in I & [x_i] \\
\boxed{s_j + Q^j p_m \geq Q^j P^j} & \forall j \in J & [y_j] \\
s_i, s_j \geq 0 & &
\end{array}$$

Dual program (prices)

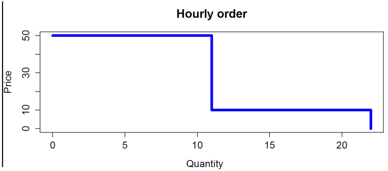
$$\begin{array}{ll}
s_i(1 - x_i) = 0 & \forall i \in I \\
\cancel{s_j(1 - y_j) = 0} & \forall j \in J \\
x_i(s_i + Q^i p_m - Q^i P^i) = 0 & \forall i \in I \\
y_j(s_j + Q^j p_m - Q^j P^j) = 0 & \forall j \in J
\end{array}$$

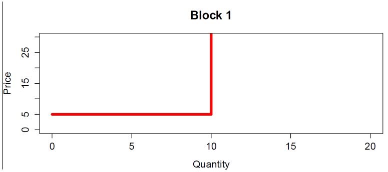
Related compl. constraints

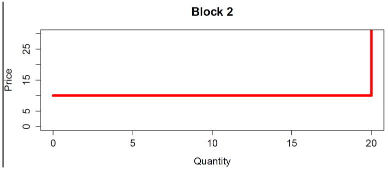
Walrasian equilibrium

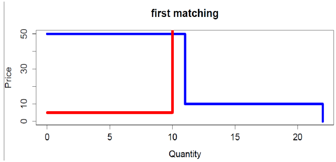
Paradoxically rejected block orders allowed

Equilibrium with block orders ?...

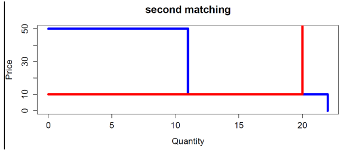








price = 50,
welfare = 450, op. cost (block 2) = 800



price = 10
welfare = 11 x 40 = 440, op. cost (block 1) = 50

Classical MPCC formulation Of European market rules

$$\max_{x_i, y_j, p_m, s_i, s_j} \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j$$

subject to:

$$x_i \leq 1 \qquad \forall i \in I \quad [s_i]$$

$$y_j \leq 1 \qquad \forall j \in J \quad "[s_j]"$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0 \qquad [p_m]$$

$$x_i, y_j \geq 0, \quad \boxed{y_j \in \mathbb{Z}}$$

Primal constraints (feasible dispatch)

$$s_i + Q^i p_m \geq Q^i P^i \qquad \forall i \in I \quad [x_i]$$

$$\boxed{s_j + Q^j p_m \geq Q^j P^j} \qquad \forall j \in J \quad [y_j]$$

$$s_i, s_j \geq 0$$

Dual program (prices)

Walrasian equilibrium

$$s_i(1 - x_i) = 0 \qquad \forall i \in I$$

$$\cancel{s_j(1 - y_j) = 0} \qquad \forall j \in J$$

$$x_i(s_i + Q^i p_m - Q^i P^i) = 0 \qquad \forall i \in I$$

$$y_j(s_j + Q^j p_m - Q^j P^j) = 0 \qquad \forall j \in J$$

Related compl. constraints

Paradoxically rejected block orders allowed

Primal-dual framework:

Feasible Set *LMM* defined by:

$$x_i \leq 1 \qquad \forall i \in I$$

$$y_j \leq 1 \qquad \forall j \in J$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0$$

$$x_i, y_j \geq 0, \quad y_j \in \mathbb{Z}$$

$$s_i + Q^i p_m \geq Q^i P^i \qquad \forall i \in I$$

$$s_j + d_{0_j} - \cancel{d_{1_j}} + Q^j p_m \geq Q^j P^j \qquad \forall j \in J$$

$$d_{0_j} \leq M_j(1 - y_j) \qquad \forall j \in J$$

d_{0_j} *upper bound* on the opportunity cost of order j

$$\cancel{d_{1_j} \leq M_j y_j} \qquad \forall j \in J$$

d_{1_j} *upper bound* on the actual loss of executed order j

$$s_i, s_j, d_{0_j}, d_{1_j} \geq 0, \quad param.: \; M_j >> 0$$

$$\sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j \geq^{\cdots=} \sum_i s_i + \sum_j s_j - \cancel{\sum_{j_1 \in J_1} d_{1_{j_1}}}$$

Strong duality <-> 'relaxed complementarity constraints'

Maximizing Welfare, new formulation:

$$\max_{x_i, y_j, p_m, s_i, s_j} \quad \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j$$

subject to:

$$x_i \le 1 \qquad \forall i \in I \quad [s_i] \tag{18}$$

$$y_j \le 1 \qquad \forall j \in J \quad \text{"}[s_j]\text{"} \tag{19}$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0 \qquad [p_m] \tag{20}$$

$$x_i, y_j \ge 0, \quad y_j \in \mathbb{Z} \tag{21}$$

$$s_i + Q^i p_m \ge Q^i P^i \qquad \forall i \in I \quad [x_i] \tag{22}$$

$$s_j + Q^j p_m \ge Q^j P^j - M_j(1 - y_j) \qquad \forall j \in J \quad \text{"}[y_j]\text{"} \tag{23}$$

$$\sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j \ge \sum_i s_i + \sum_j s_j \tag{24}$$

$$s_i, s_j \ge 0, \quad param.\ M_j >> 0 \tag{25}$$

Maximizing Welfare, new formulation, quadratic case:

$$\max_{x_i, y_j, p_m, s_i, s_j} \quad \sum_i Q^i P^i x_i + \sum_i Q^i (P_1^i - P_0^i) \frac{x_i^2}{2} + \sum_j Q^j P^j y_j$$

subject to:

$$x_i \leq 1 \tag{26}$$

$$y_j \leq 1 \tag{27}$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0 \tag{28}$$

$$x_i, y_j \geq 0, \quad y_j \in \mathbb{Z} \tag{29}$$

$$s_i + Q^i p_m \geq Q^i P^i + Q^i (P_1^i - P_0^i) x_i \tag{30}$$

$$s_j + Q^j p_m \geq Q^j P^j - M_j (1 - y_j) \tag{31}$$

$$\sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j + \sum_i Q^i (P_1^i - P_0^i) x_i^2 \geq \sum_i s_i + \sum_j s_j \tag{32}$$

$$s_i, s_j \geq 0, \quad param. \ M_j >> 0 \tag{33}$$

Primal-dual framework:

Feasible Set *LMM* defined by:

$$x_i \leq 1 \qquad \forall i \in I$$

$$y_j \leq 1 \qquad \forall j \in J$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0$$

$$x_i, y_j \geq 0, \quad y_j \in \mathbb{Z}$$

$$s_i + Q^i p_m \geq Q^i P^i \qquad \forall i \in I$$

$$s_j + d_{0_j} - \cancel{d_{1_j}} + Q^j p_m \geq Q^j P^j \qquad \forall j \in J$$

$$d_{0_j} \leq M_j(1 - y_j) \qquad \forall j \in J$$

d_{0_j} *upper bound* on the opportunity cost of order j

$$\cancel{d_{1_j} \leq M_j y_j} \qquad \forall j \in J$$

d_{1_j} *upper bound* on the actual loss of executed order j

$$s_i, s_j, d_{0_j}, d_{1_j} \geq 0, \quad param.: \; M_j >> 0$$

$$\sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j \geq^{\ldots=} \sum_i s_i + \sum_j s_j - \cancel{\sum_{j_1 \in J_1} d_{1_{j_1}}}$$

Strong duality <-> 'relaxed complementarity constraints'

block order selection $J = J_0 \dot{\cup} J_1$
Primal:

$$\max_{x_i, y_j} \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j$$

subject to:

$$
\begin{aligned}
& x_i \leq 1 && \forall i \in I \quad [s_i] && (34)\\
& y_j \leq 1 && \forall j \in J \quad [s_j] && (35)\\
& y_{j_0} \leq 0 && \forall j_0 \in J_0 \subseteq J \quad [d_{0_{j_0}}] && (36)\\
& -y_{j_1} \leq -1 && \forall j_1 \in J_1 \subseteq J \quad [d_{1_{j_1}}] && (37)\\
& \sum_i Q^i x_i + \sum_j Q^j y_j = 0, && [p_m] && (38)\\
& && x_i,\ y_j \geq 0 && (39)
\end{aligned}
$$

$$\textbf{Dual:}\ \min \sum_i s_i + \sum_j s_j - \sum_{j_1 \in J_1} d_{1_{j_1}}$$

subject to:

$$s_i + Q^i p_m \geq Q^i P^i \qquad \forall i \in I \quad [x_i] \tag{40}$$

$$s_{j_0} + d_{0_{j_0}} + Q^{j_0} p_m \geq Q^{j_0} P^{j_0} \qquad \forall j_0 \in J_0 \quad [y_{j_0}] \tag{41}$$

$$s_{j_1} - d_{1_{j_1}} + Q^{j_1} p_m \geq Q^{j_1} P^{j_1} \qquad \forall j_1 \in J_1 \quad [y_{j_1}] \tag{42}$$

$$s_i, s_j, d_{j_0}, d_{j_1}, u_m \geq 0 \tag{43}$$

Complementarity constraints

$$s_i(1 - x_i) = 0 \qquad \forall i \in I \tag{44}$$

$$s_{j_0}(1 - y_{j_0}) = 0 \qquad \forall j \in J \tag{45}$$

$$s_{j_1}(1 - y_{j_1}) = 0 \qquad \forall j \in J \tag{46}$$

$$x_i(s_i + Q^i p_m - Q^i P^i) = 0 \qquad \forall i \in I \tag{47}$$

$$y_{j_0}(s_{j_0} + d_{0_{j_0}} + Q^{j_0} p_m - Q^{j_0} P^{j_0}) = 0 \qquad \forall j \in J_0 \tag{48}$$

$$y_{j_1}(s_{j_1} - d_{1_{j_1}} + Q^{j_1} p_m - Q^{j_1} P^{j_1}) = 0 \qquad \forall j \in J_1 \tag{49}$$

$$y_{j_0} d_{0_{j_0}} = 0, \quad (1 - x_{j_1}) d_{1_{j_1}} = 0 \qquad \forall j_0 \in J_0, \forall j_1 \in J_1 \tag{50}$$

With primal, dual and complementarity constraints:

- d_{0_j} is an *upper bound* on the opportunity cost of order j
- d_{1_j} is an *upper bound* on the actual loss of (executed) order j

Block order selection not known 'ex ante'.
Goal: according to some criterion/obj. fun., determine:

- a block selection $J = J_0 \dot{\cup} J_1$
- optimal primal and dual points for this block selection (pair of problems above)
- Linear constraints, no new binary variables

...Strong duality + Big M's:

- complementarity constraints $\leftrightarrow$ equality of objective functions
- dual constraints adapted to the block order selection $\leftrightarrow$ Big M's...

Primal-dual framework:

Feasible Set *LMM* defined by:

$$x_i \leq 1 \qquad \forall i \in I \quad (51)$$

$$y_j \leq 1 \qquad \forall j \in J \quad (52)$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0 \qquad (53)$$

$$x_i, y_j \geq 0, \quad y_j \in \mathbb{Z} \qquad (54)$$

$$s_i + Q^i p_m \geq Q^i P^i \qquad \forall i \in I \quad (55)$$

$$s_j + d_{0_j} - d_{1_j} + Q^j p_m \geq Q^j P^j \qquad \forall j \in J \quad (56)$$

$$d_{0_j} \leq M_j (1 - y_j) \qquad \forall j \in J \quad (57)$$

$$d_{1_j} \leq M_j y_j \qquad \forall j \in J \quad (58)$$

$$s_i, s_j, d_{0_j}, d_{1_j} \geq 0, \quad param.: \ M_j >> 0 \qquad (59)$$

$$\sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j \geq^{\cdots=} \sum_i s_i + \sum_j s_j - \sum_{j_1 \in J_1} d_{1_{j_1}} \qquad (60)$$

Minimization problems under European rules

European market rules ? No paradoxically accepted block orders:
add to LMM (the 'primal-dual framework') constraints $d_{1_j} = 0 \; \forall j \in J$

$d_{0_j} \geq 0$ *upper bound* on the opportunity cost of block order j...

Minimizing opportunity costs ? $\min \sum d_{0_j}$

Minimizing # PRBs ? $\min \sum z_{0_j}$ s.t. $z_{0_j} \geq d_{0_j}$ & $z_{0_j} \in \mathbb{Z}, \quad \forall j \in J$

- No complementarity constraints, no more binary variables than block orders
- Has been tested to maximize welfare using real data from Apx-Endex: works great, solves real large-scale (MILP) instances
- Also works when adding a network model (several areas, several periods)

- Could consider balance between multiple criteria (op. costs, welfare)
- Derivation of a powerful Benders decomposition procedure

Would also work when considering linearly interpolated orders:

- Strong duality for convex quadratic programs (Dorn's Dual)
- One dense convex quadratic constraint (eq. of objective functions)
- Not tractable 'as is', but ...
- Benders decomposition with *locally strengthened cuts* works very well in this setting (see below)

Welfare max.: new cuts with a Benders decomposition ?

Branch-and-cut for the primal program + 'on-the-fly' lazy constraints for new incumbents without 'good prices' (European rules)

New formulation → new *locally valid* cuts (i.e. in subtrees of the B & B):

$$\text{Cut: } \sum_{j|y_j^*=1} (1 - y_j) \geq 0$$

improves on Martin-Muller-Pokutta 2013 ("no-good") cuts:

$$\sum_{j|y_j^*=1} (1 - y_j) + \sum_{j|y_j^*=0} y_j \geq 0$$

Presentation here in the linear case,
same locally valid cuts in the quadratic setting

Maximizing Welfare, new formulation:

$$\max_{x_i, y_j, p_m, s_i, s_j} \quad \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j$$

subject to:

$$x_i \leq 1 \qquad \forall i \in I \quad [s_i] \tag{61}$$

$$y_j \leq 1 \qquad \forall j \in J \quad "[s_j]" \tag{62}$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0 \qquad [p_m] \tag{63}$$

$$x_i, y_j \geq 0, \quad y_j \in \mathbb{Z} \tag{64}$$

$$s_i + Q^i p_m \geq Q^i P^i \qquad \forall i \in I \quad [\ldots u_i] \tag{65}$$

$$s_j + Q^j p_m \geq Q^j P^j - M(1 - y_j) \qquad \forall j \in J \quad [\ldots u_j] \tag{66}$$

$$\sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j \geq \sum_i s_i + \sum_j s_j \qquad [\ldots u_\sigma] \tag{67}$$

$$s_i, s_j \geq 0, \quad M >> 0 \tag{68}$$

reminder: Farkas lemma: $\exists s^* \geq 0, As^* \leq b \Leftrightarrow \forall u \geq 0, (uA \geq 0 \Rightarrow ub \geq 0)$
Given a 'primal incumbent' x^*, y^*, a 'good price' exists $\Leftrightarrow$ ... $\Leftrightarrow$:

$$\sum_i Q^i P^i u_i + \sum_j Q^j P^j u_j - \sum_j M(1 - y_j^*) u_j \leq obj^*$$

$\forall (u_i, u_j)$ in P defined by the constraints:

$$u_i \leq 1 \tag{69}$$
$$u_j \leq 1 \tag{70}$$
$$\sum_i Q^i u_i + \sum_j Q^j u_j = 0 \tag{71}$$
$$u_i, u_j \geq 0, \tag{72}$$

That is $\Leftrightarrow$:

$$\max_{(u_i,u_j)\in P} \sum_i Q^i P^i u_i + \sum_j Q^j P^j u_j - \sum_j M(1 - y_j^*) u_j \leq obj^*$$

big-M: $y_j^* = 0 \Rightarrow u_j^* = 0$: cont. relax. of the primal with $u_j = 0$ if $y_j^* = 0$

Suppose there are no prices for the feasible dispatch:

$$\max_{(u_i,u_j)\in P} \sum_i Q^i P^i u_i + \sum_j Q^j P^j u_j - \sum_j M(1-y_j^*)u_j > obj^*.$$

So, since $y_j^* = 0 \Rightarrow u_j^* = 0$:

$$\max_{(u_i,u_j)\in P} \sum_i Q^i P^i u_i + \sum_j Q^j P^j u_j > obj^*.$$

optimal $(u_i^*, u_j^*) \rightarrow$ new cut:

$$\sum_i u_i^* Q^i P^i + \sum_j u_j^* Q^j P^j - \sum_j M(1-y_j)u_j^* \leq obj.$$

Finite number of cuts to add...($\leq$ # vertices of the polytope P).

Not a strong cut as such (big-M), but...*locally in the B & B tree*:

$$\text{Cut: } \sum_{j|y_j^*=1} (1-y_j) \geq 0$$

Martin-Muller-Pokutta ("no-good") cuts: $\displaystyle\sum_{j|y_j^*=1} (1-y_j) + \sum_{j|y_j^*=0} y_j \geq 0$

Results:

- Real data of 2011 (Oracle dump with ATC data as well), thanks to Apx-Endex and Epex Spot!
- Belgium, France, Germany and the Nederlands, 24 time slots
- time limit: 10 min., about 60 000 cont. vars, 600/700 bin. vars
- Branch-and-cut in AIMMS, using
  Cplex 12.5 with *locally valid lazy constraints* callbacks
  platform: windows 7 64, i5 with 4 cores @ 3.10 GHz, 4 GB RAM

Stepwise preference curves (linearisation):

	Solved instances	Running time (solved instances, sec)	Final abs. gap (unsolved instances)	Nodes (solved - unsolved) instances	Cuts (solved - unsolved) instances
New MILP formulation	84%	104.42	418.16	43 - 33584	/
Decomposition Procedure	72.78%	6.47	402.05	16 - 1430	8 - 3492

Quadratic setting:

	Solved instances	Running time (solved instances, sec)	Final abs. gap (unsolved instances)	Nodes (solved - unsolved) instances	Cuts (solved - unsolved) instances
Decomposition Procedure	70.41%	16.70	370.91	11 - 619	7 - 1382

Many blocks (almost binary orders only):

	Solved instances	Running time (solved instances, sec)	Final abs. gap (unsolved instances)	Nodes (solved - unsolved) instances	Cuts (solved - unsolved) instances
New MILP formulation	100%	4.17	/	40797 - /	-
Decomposition Procedure	78%	13.82	9303.16	64564 / 937172	1662 / 82497

Linearised instances

		New MILP formulation			Decomposition approach			
Instance	# Block orders	Run. Time	Final Gap	Nodes	Run. Time	Final Gap	Nodes	Cuts
1	766	600.37	495.89	27478	600.276	1463.35	842	3442
2	477	64.912			2.106			0
3	731	277.042		18331	600.401	106.75	2204	3594
4	566	64.819		12	2.527		17	2
5	683	57.097			2.168			0
6	513	47.283		84	6.537		46	27
7	658	79.577		473	99.7		458	706
8	604	51.028		3	2.371			0
9	571	36.348		2	1.685		2	0
10	655	136.891		5292	600.651	179.12	993	3625
11	686	54.335		77	6.708		85	29
12	692	69.156		2	2.73		2	0
13	640	93.773		9	3.369		21	3
14	618	85.692		7	3.635		6	5
15	550	92.368		57	6.567		75	16
16	591	59.857		9	3.885		7	7
17	685	117.781		91	3.37		15	4
18	699	600.339	252.83	27679	600.588	268.65	1408	3042
19	578	71.32		10	122.27		133	570
20	703	600.308	235.22	39225	600.604	517.47	1046	3173

Quadratic instances (interpolated orders)

Instance	# Block orders	Run. Time	Final Gap	Nodes	Cuts
1	766	600.21	1160.50	565	1343
2	477	5.27			0
3	731	600.51	145.21	961	1382
4	566	8.47		13	2
5	683	8.30		1	0
6	513	10.95		32	7
7	658	13.01		59	16
8	604	5.82			0
9	571	4.31		1	0
10	655	601.10	256.69	357	1478
11	686	14.12		63	15
12	692	7.66			0
13	640	20.03		34	21
14	618	600.48	202.48	388	1291
15	550	180.29		247	366
16	591	42.67		40	58
17	685	44.43		82	66
18	699	600.56	204.42	753	1138
19	578	15.23		7	7
20	703	600.42	1740.25	457	1280

Instances where most orders are block orders

		New MILP Formulation			Decomposition approach			
Instance	Block orders	Run. Time	Final Gap	Nodes	Run. Time	Final Gap	Nodes	Cuts
1	526	7.18		75561	600.14	28497.24	493464	132118
2	508	12.18		168467	540.00		1777391	121336
3	612	1.34		15348	2.32		8784	367
4	594	9.95		114400	15.41		81721	2756
5	671	4.74		53026	4.88		18312	847
6	766	8.80		90938	129.04		1156506	17312
7	714	1.82		17111	10.25		70038	1517
8	497	1.16		16210	459.08		1090631	106874
9	460	0.53		6216	0.56		4219	84
10	579	0.31		2474	1.01		2437	199
11	668	0.16		725	0.19		473	15
12	684	0.70		6733	2.45		29995	310
13	650	1.84		19433	7.58		71328	988
14	682	1.48		13224	2.43		10835	374
15	487	14.68		192265	600.01	6099.59	794340	142957
16	477	1.09		15481	302.75		699328	69114
17	597	0.16		792	0.47		5716	20
18	740	3.12		28904	28.44		105697	4312
19	794	5.91		57537	113.37		366836	14008
20	823	1.01		9677	600.03	209922.61	155204	63899

Conclusion - Discussion

New primal-dual 'framework'

Useful...

- to deal with market design issues
  $\rightarrow$ Minimizing opportunity costs, # PRBs, etc
  $\rightarrow$ Considering any constraints on deviations, any objective function
- from an algorithmic point of view
  $\rightarrow$ no complementarity (non-linear) constraints
  $\rightarrow$ no more binary vars. than block orders
  $\rightarrow$ New efficient *locally valid* cuts for the Benders decomposition
- Best two approaches (full model vs. decomposition) available so far in the literature