Models for traffic engineering with multiple spanning tree protocols

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9th January 2014
18th Combinatorial Optimization Workshop, Aussois

Outline

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Data centers

- Increasing demand for Internet and cloud services
- Increasing need for large scale data centers
- Data centers support simultaneously multiple applications
- Critical need to improve the performance of telecommunications networks (**Traffic Engineering**)

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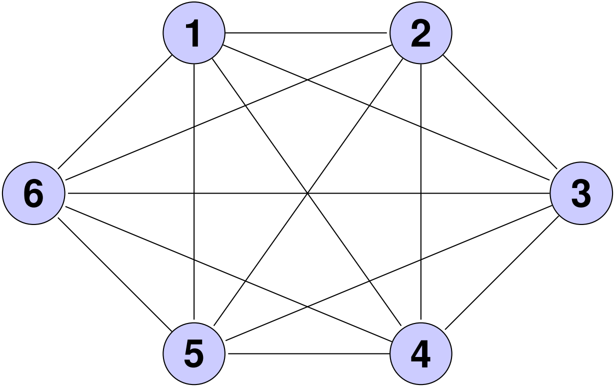
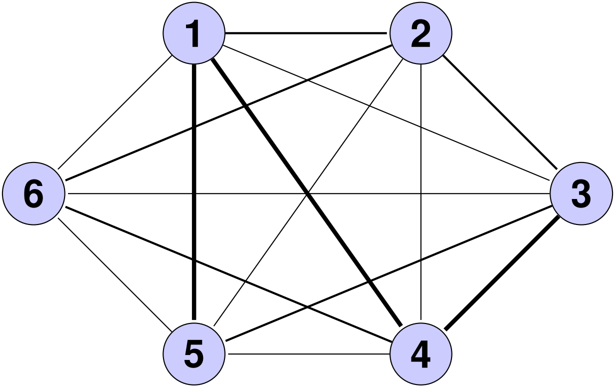
Ethernet networks

An Ethernet network can be represented by a graph **G = (V,A).**
V is the set of switches.
A is the set of links connecting the switches.



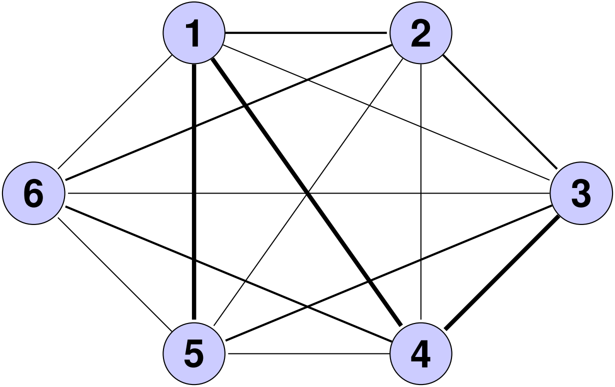
Ethernet Networks - Bandwidth

Links have different traffic capacity (Mb/s).



Ethernet network - Traffic demands

Set of traffic demands between switches, to be routed in the network.
$d\{1,2\} = 10Mb, d\{2,3\} = 7Mb, d\{4,3\} = 13Mb, d\{4,1\} = 5Mb, ...$



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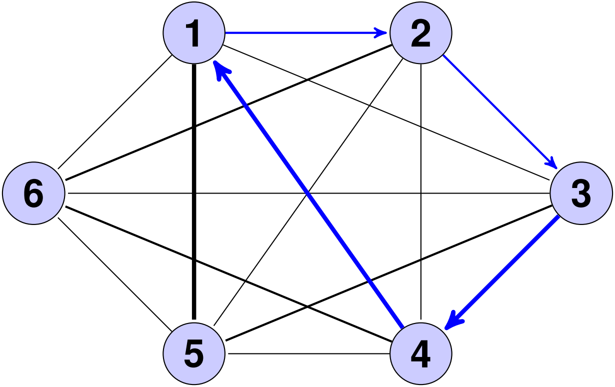
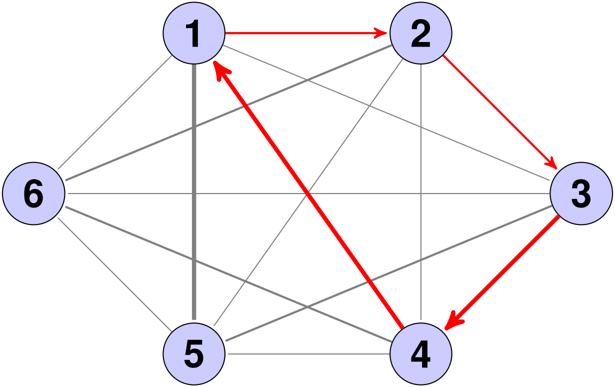


Broadcast Radiation

Cycles in the network create broadcast radiation !



Subnetwork of activated links

IEEE 802.1D : Spanning Tree Protocol

The topology of the subnetwork of activated links must be cycle-free. That is to say, it most form a **spanning tree**.

Definition

Graph S is a spanning tree of G if : i) includes all the nodes of G ; ii) it's connected and contains no cycles.

#edges = #nodes-1

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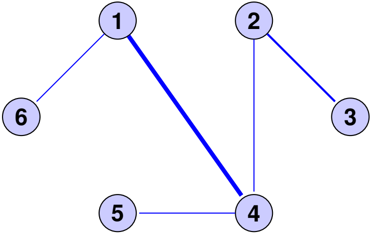

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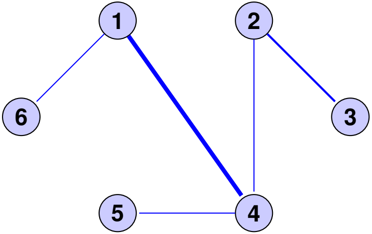

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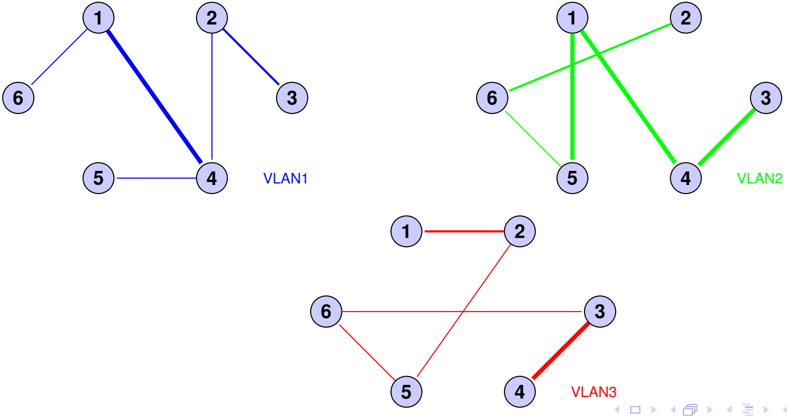
VLANs

IEEE 802.1q : Virtual LANs

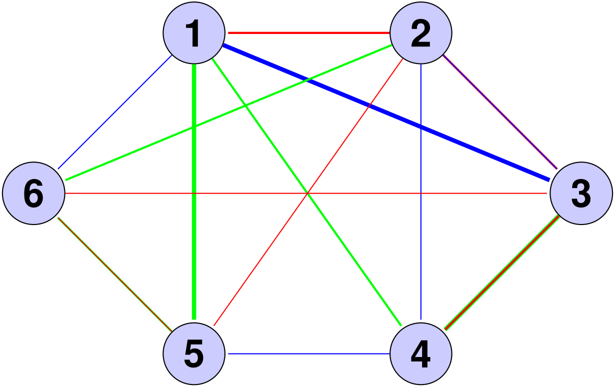
The Ethernet network can be partitioned in many Virtual Local Area Networks (**VLANs**). This way it's possible to isolate different applications.



Multiple Spanning Tree Protocol (MSTP)

IEEE 802.1s : Multiple Spanning Tree Protocol

Multiple VLANs may occupy a single physical topology.



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Problem description

Given a network, G, characterized by :

- a set of nodes, N, and a set of links, A ;
- traffic capacity in each link, $C_{\{i,j\}}$;

partitioned in the set of VLANs, S, characterized by

- traffic demand between nodes, $d_s\{u, v\}$;

what is the **best design** for each VLAN, such that :

- traffic demands are routed ;
- each VLAN forms a spanning tree ;
- minimize worst-case link utilization (ratio between a link's load, and its capacity)

Literature has been focusing exclusively on heuristic approaches (no lower bounds).

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Multi-commodity Flow Formulation (MFF)

- $x_{(i,j)}^{\{u,v\},s} = 1$ if node i is the predecessor of node j ($\{i,j\} \in E$) on the unique path from node u to node v, on VLAN $s \in S$; 0 otherwise ;
- $w_{\{i,j\}}^{s} = 1$ if link $\{i,j\} \in E$ is used in VLAN $s \in S$; 0 otherwise ;
- U^{max} = maximum value of link utilization.

$$\min_{x,w} \quad U^{max} \tag{1a}$$

s.t

$$\sum_{j:(u,j)\in E} x_{(u,j)}^{\{u,v\},s} = 1, \qquad u,v \in V : u < v, s \in S, \tag{1b}$$

$$\sum_{j:(i,j)\in E} x_{(i,j)}^{\{u,v\},s} - \sum_{j:(j,i)\in E} x_{(j,i)}^{\{u,v\},s} = 0, \qquad u,v,i \in V : u < v, i \neq \{u,v\}, s \in S, \tag{1c}$$

$$x_{(i,j)}^{\{u,v\},s} + x_{(j,i)}^{\{u,v\},s} \leq w_{\{i,j\}}^{s}, \qquad u,v \in V : u < v, \{i,j\} \in E, s \in S, \tag{1d}$$

$$\sum_{\{i,j\}\in E} w_{\{i,j\}}^{s} = n - 1, \qquad s \in S, \tag{1e}$$

$$\sum_{s\in S} \sum_{\{u,v\}:u<v} d_s\{u,v\} \, (x_{(i,j)}^{\{u,v\},s} + x_{(j,i)}^{\{u,v\},s}) \leq C_{\{i,j\}} \, U^{max}, \qquad \{i,j\} \in E, \tag{1f}$$

$$x_{(i,j)}^{\{u,v\},s} \in \{0,1\}, \qquad (i,j) \in E, u,v \in V : u < v, j \neq u, s \in S, \tag{1g}$$

$$w_{\{i,j\}}^{s} \in \{0,1\}, \qquad \{i,j\} \in E, s \in S, \tag{1h}$$

$$0 \leq U^{max} \leq 1 \tag{1i}$$

MFF : 1b,1c

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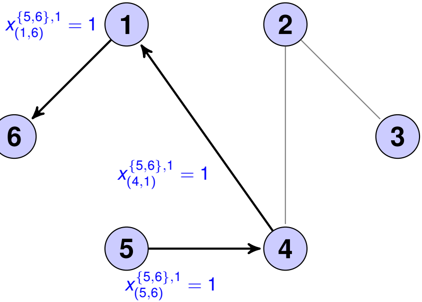

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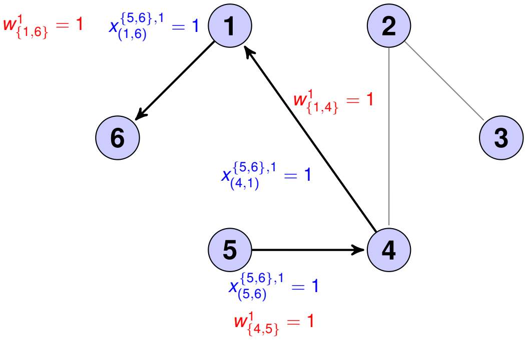

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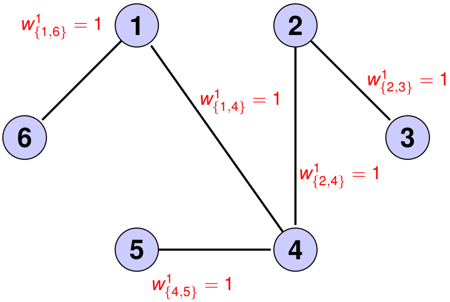

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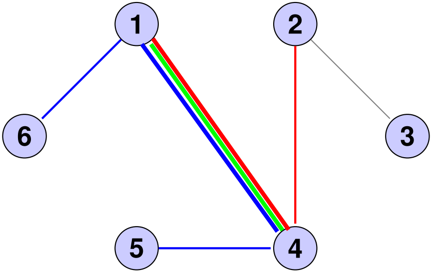

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(...)

load in $\{1,4\}$ @ $VLAN_2$ = 0.3

total load in $\{1,4\}$ = 0.8

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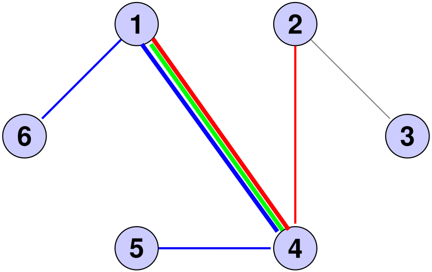

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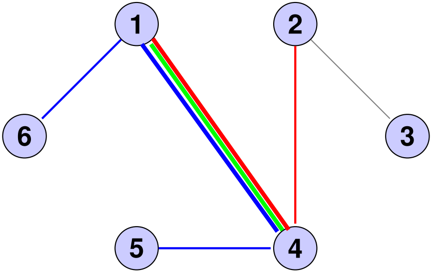

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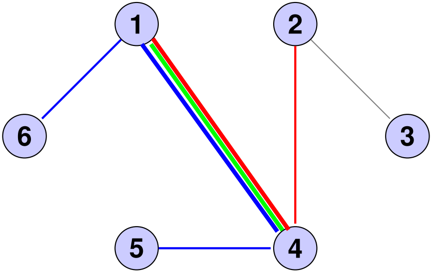

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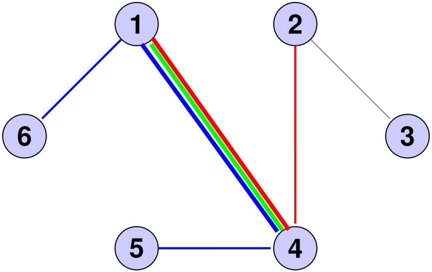

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Valid inequalities for MFF

Balakrishnan *et al.* (*Operations Research*, 1989), have suggested for the SST some valid inequalities that can be extended to :

$$x_{(i,j)}^{\{u,v\},s} + x_{(j,i)}^{\{u,v'\},s} \leq w_{\{i,j\}}^{s}, \qquad u, v, v' \in V : v \neq v', \{i,j\} \in E, s \in S, \tag{2a}$$

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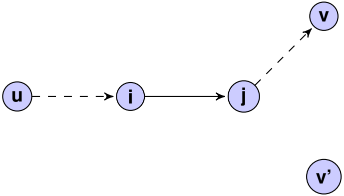

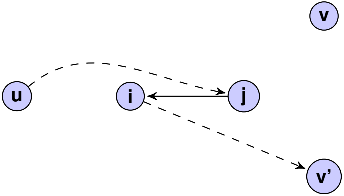

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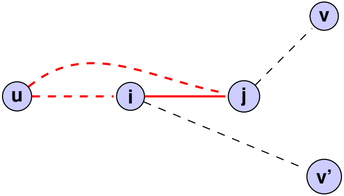

Subtour elimination constraints.

Valid inequalities for MFF (II)

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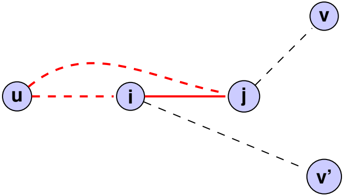

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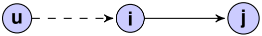

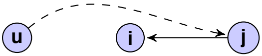

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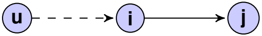

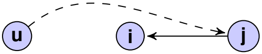

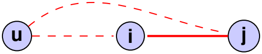

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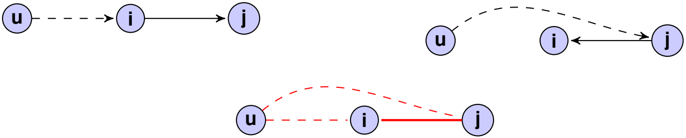

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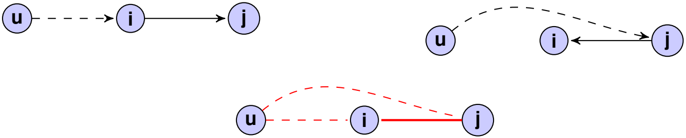
$$x^{\{i,v\},s}_{(i,j)} + x^{\{j,v\},s}_{(j,i)} = w^s_{\{i,j\}}, \qquad v \in V, \{i,j\} \in E : i < v, j < v, s \in S, \tag{3b}$$



Tests indicate that using $MFF \bigcup (2,3)$ strengthens the linear programming (LP) relaxation.

MFF+ $:= MFF \bigcup (2,3)$

Rooted Directed Formulation (RDF)

- $z_{i,j}^{u,s} = 1$ if node i is the "father" of node j, in an arborescence rooted at node u, in VLAN $s \in S$; 0 otherwise;
- $f_{(i,j)}^{u,s}$ = traffic quantity, originated from node u, going through link $(i,j) \in E$, in VLAN $s \in S$;
- $w_{\{i,j\}}^{s} = 1$ if link $\{i,j\} \in E$ is used in VLAN $s \in S$; 0 otherwise;
- U^{max} = maximum value of link utilization.

$$\min_{z,f,w} \quad U^{max} \tag{4a}$$

s.t

$$\sum_{i:(i,j)\in E} z_{i,j}^{u,s} = 1, \qquad u,j \in V : u \neq j, s \in S, \tag{4b}$$

$$z_{i,j}^{u,s} + z_{j,i}^{u,s} = w_{\{i,j\}}^{s}, \qquad u \in V, \{i,j\} \in E, \forall s \in S, \tag{4c}$$

$$\sum_{\{i,j\}\in E} w_{\{i,j\}}^{s} = n - 1, \qquad \forall s \in S, \tag{4d}$$

$$\sum_{j:(j,i)\in E} f_{(j,i)}^{u,s} - \sum_{j:(i,j)\in E \wedge j\neq u} f_{(i,j)}^{u,s} = d_s\{u,i\}, \qquad u,i \in V : u \neq i, s \in S, \tag{4e}$$

$$\sum_{i:(u,i)\in E} f_{(u,i)}^{u,s} = \sum_{v\in V:v\neq u} d_s\{u,v\}, \qquad u \in V : u, s \in S, \tag{4f}$$

$$f_{(i,j)}^{u,s} \leq \sum_{v\in V\setminus\{u,i\}} d_s\{u,v\}\, z_{i,j}^{u,s}, \qquad i,j,u \in V : u \neq j, \{i,j\} \in E, s \in S, \tag{4g}$$

$$\sum_{s\in S}\sum_{u\in V}(f_{(i,j)}^{u,s} + f_{(j,i)}^{u,s}) \leq C_{\{i,j\}} U^{max}, \qquad \{i,j\} \in E, \tag{4h}$$

$$z_{i,j}^{u,s} \in \{0,1\}, \qquad u \in V, (i,j) \in E : j \neq u, s \in S, \tag{4i}$$

$$w_{\{i,j\}}^{s} \in \{0,1\}, \qquad \{i,j\} \in E, s \in S, \tag{4j}$$

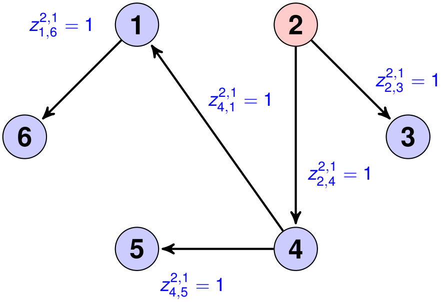
$$f_{(i,j)}^{u,s} \geq 0, \qquad u \in V, (i,j) \in E : j \neq u, s \in S, \tag{4k}$$

$$0 \leq U^{max} \leq 1 \tag{4l}$$

RDF : 4b

$$\sum_{i:(i,j)\in E} z_{i,j}^{u,s} = 1, \qquad u, j \in V : u \neq j, s \in S$$

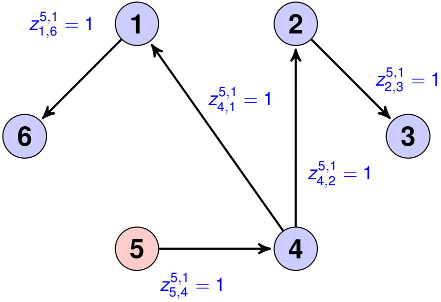
Arborescence rooted at node **2**.



RDF : 4b (II)

$$\sum_{i:(i,j)\in E} z_{i,j}^{u,s} = 1, \qquad u,j \in V : u \neq j, s \in S$$

Arborescence rooted at node **5**.



RDF : 4c,4d

$$z^{u,s}_{i,j} + z^{u,s}_{j,i} = w^s_{\{i,j\}}, \qquad u \in V, \{i,j\} \in E, \forall s \in S$$

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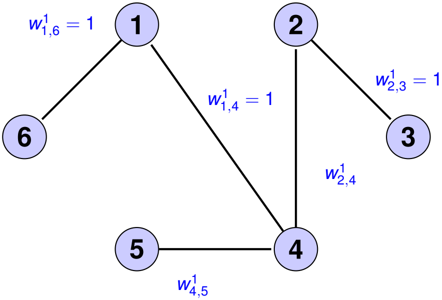

RDF : 4e,4f,4g

$$\sum_{j:(j,i)\in E} f^{u,s}_{(j,i)} - \sum_{j:(i,j)\in E \wedge j\neq u} f^{u,s}_{(i,j)} = d_s\{u,i\}, \qquad u,i \in V : u \neq i, s \in S,$$

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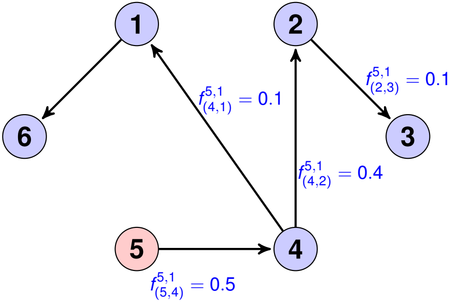

$$d_1\{5,1\} = 0.1$$
$$d_1\{5,2\} = 0.3$$
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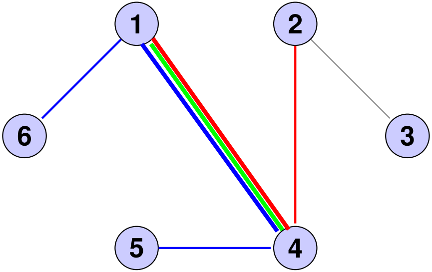
RDF : 4a,4h,4l

$$\min_{z,f,w} \quad U^{max}$$

$$\sum_{s \in S} \sum_{u \in V} (f^{u,s}_{(i,j)} + f^{u,s}_{(j,i)}) \leq C_{\{i,j\}} U^{max}, \qquad \{i,j\} \in E$$

$$0 \leq U^{max} \leq 1$$



$d_1\{5,6\} = 0.1$

$f^{5,1}_{(1,4)} = 0.1$

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load in $\{1,4\}$ @ $VLAN_1 = 0.5$

(...)

load in $\{1,4\}$ @ $VLAN_2 = 0.3$

total load in $\{1,4\} = 0.8$

$C_{\{1,4\}} = 1$

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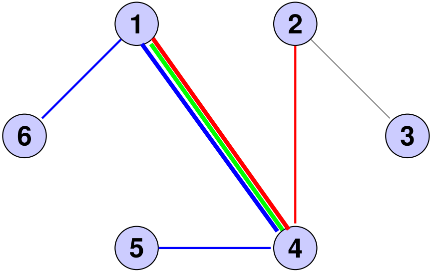

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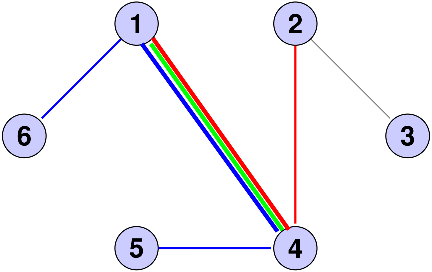

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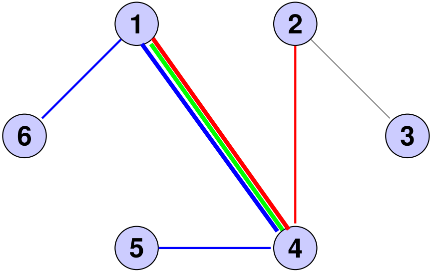

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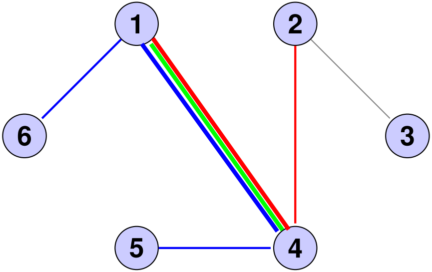

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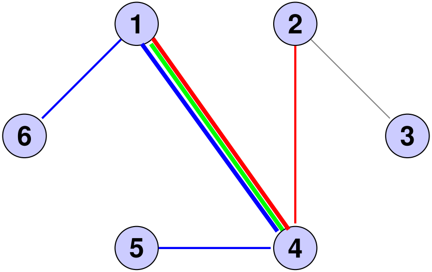

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LP strength

Definition

LP relaxation of formulation A is said to be **as strong as** (**stronger than**) the LP relaxation of formulation B, if the polyhedra defined by the set of feasible solutions of the LP relaxation of A is (strictly) included in the polyhedra of feasible solutions of the LP relaxation of B.
$\mathcal{P}(A) \subseteq \mathcal{P}(B)$ $(\mathcal{P}(A) \subset \mathcal{P}(B))$

Theorem

$\mathcal{P}(MFF+) \subseteq \mathcal{P}(RDF)$

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Outline

Input :

- Number of nodes
- Number of VLANs
- Network density $\sim$ Number of links per VLAN
- Parameter α

Capacity of each existing link :
$C[i,j] \in \{0.5, 0.75, 1\}$

Demand between switches :
$d_s\{u,v\} = \alpha O_u D_v R_{(u,v)} e^{\frac{-L_2(u,v)}{2\Delta}}$ (Fortz & Thorup, C.O.A., 2004)
$O_u, D_u, R_{(u,v)} \in [0,1]$

Instances generation

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Instances generated

Class ID	#nodes	density	#VLANs
1	4	1	1
2	4	1	5
3	6	0.8	2
4	6	0.8	4
5	8	0.4	3
6	8	0.4	6
7	8	0.6	2
8	8	0.8	2
9	8	0.8	3
10	10	0.3	2
11	10	0.3	4
12	10	0.5	2
13	10	0.5	3
14	12	0.3	4
15	14	0.3	2
16	14	0.5	3
17	16	0.4	4
18	16	0.4	5
19	20	0.2	1
20	20	0.2	2

Gap values

		MFF+		RDF	
Class ID	#ISF	Gap avg	Gap std	Gap avg	Gap std
1	5	51.80	7.08	54.85	9.01
2	5	30.69	13.34	30.82	13.46
3	5	46.34	5.88	47.37	6.63
4	5	37.50	11.94	38.00	12.30
5	5	12.74	9.45	13.12	9.85
6	5	1.03	1.14	1.03	1.14
7	5	49.40	3.36	49.56	3.15
8	5	57.96	5.30	58.06	5.28
9	5	58.00	21.10	58.83	20.35
10	5	5.74	12.84	5.96	13.34
11	5	0.26	0.56	0.26	0.56
12	5	32.44	20.65	32.91	21.35
13	5	23.31	15.18	23.31	15.18
14	5	3.12	2.18	3.12	2.18
15	5	16.88	17.49	16.88	17.49
16	5	64.05	12.00	64.05	12.00
17	5	67.26	11.70	67.57	12.07
18	5	40.25	30.62	40.25	30.62
19	2	73.70	14.34	74.65	14.76
20	5	43.86	27.10	43.86	27.10

CPU time (seconds) for CPLEX to solve IP

	MFF+			RDF		
Class ID	#solved	Time avg	Time std	#solved	Time avg	Time std
1	5	0.06	0.05	5	0.04	0.05
2	5	0.07	0.01	5	0.02	0.01
3	5	2.74	1.32	5	0.43	0.16
4	5	10.84	7.10	5	0.85	0.42
5	5	3.70	2.08	5	0.27	0.17
6	5	8.33	7.05	5	0.25	0.25
7	5	498.75	368.59	5	6.26	3.80
8	5	979.84	736.79	5	48.20	53.21
9	3	307.40	173.07	5	50.61	31.36
10	5	41.41	53.71	5	0.72	1.00
11	5	126.62	138.78	5	2.65	3.09
12	2	478.17	579.58	5	140.5498	182.36
13	3	3733.48	3097.41	4	544.68	785.88
14	0	-	-	3	236.40	356.00
15	1	45.70	0.00	3	1957.82	1801.02
16	0	-	-	0	-	-
17	0	-	-	0	-	-
18	0	-	-	1	1307.49	0
19	0	-	-	0	-	-
20	0	-	-	1	227.30	0

Outline

RDF with fixed U^{max}

Idea

Max-min objective function produces unpredictable gaps. Therefore, we shall fix U^{max} in RDF to a value iteratively given by a binary search algorithm. Use *faux* objective function.

$$\min_{z,f,w} \sum_{s\in S,\{i,j\}\in E} w^s_{\{i,j\}}$$

s.t

(...)

$$\sum_{s\in S}\sum_{u\in V}(f^{u,s}_{(i,j)} + f^{u,s}_{(j,i)}) \leq C[i,j]\cdot U^{fix} \quad ,\forall\{i,j\}\in E,$$

$$z^{u,s}_{i,j} \in \{0,1\} \quad ,\forall u\in V,\forall(i,j)\in E: j\neq u,\forall s\in S,$$

$$w^s_{\{i,j\}} \in \{0,1\} \quad ,\forall\{i,j\}\in E,\forall s\in S,$$

$$f^{u,s}_{(i,j)} \geq 0 \quad ,\forall u\in V,\forall(i,j)\in E: j\neq u,\forall s\in S,$$

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$$w^s_{\{i,j\}} \in \{0,1\} \qquad ,\forall\{i,j\}\in E,\forall s\in S,$$

$$f^{u,s}_{(i,j)} \geq 0 \qquad ,\forall u\in V,\forall(i,j)\in E : j\neq u,\forall s\in S,$$

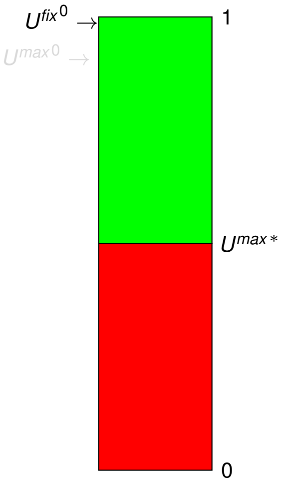
BSA : Test feasibility

$U^{fix} := 1$

$U^{max*} := Solve\_RDF(U^{fix})$

if *Optimal solution is found* **then**

$\quad UB := U^{max*}$



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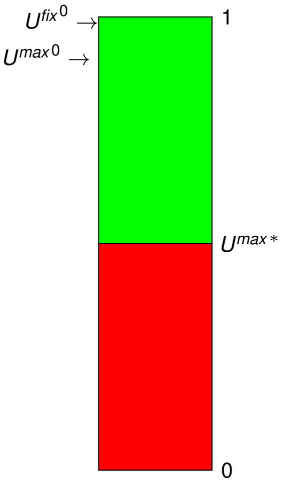

BSA : Obtain LB

$U^{fix} := 1$

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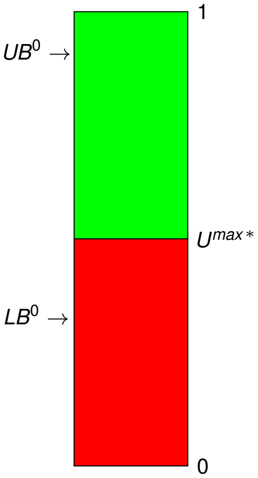
if *Optimal solution is found* **then**

  $UB := U^{max*}$

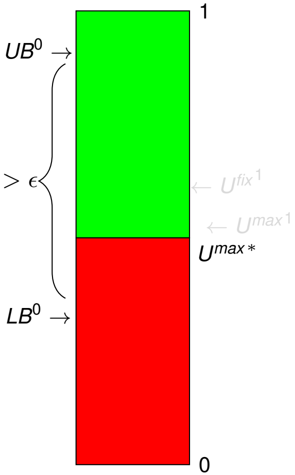
  $U^{max*} := Solve\_MFF+_{LR}$

  $LB := U^{max*}$



BSA : New U^{fix}

$U^{fix} := 1$
$U^{max*} := Solve\_RDF(U^{fix})$
if *Optimal solution is found* **then**
    $UB := U^{max*}$
    $U^{max*} := Solve\_MFF+_{LR}$
    $LB := U^{max*}$
    while $UB - LB > \epsilon$ **do**
        $U^{fix} := LB + \frac{UB-LB}{2}$
        $U^{max*} := Solve\_RDF(U^{fix})$



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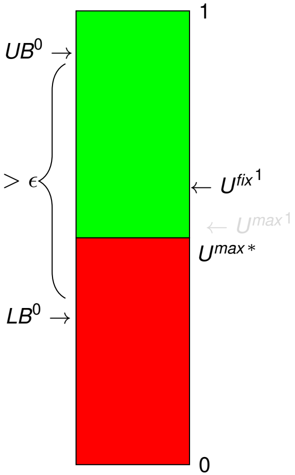

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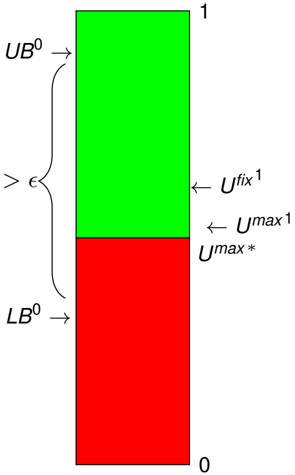

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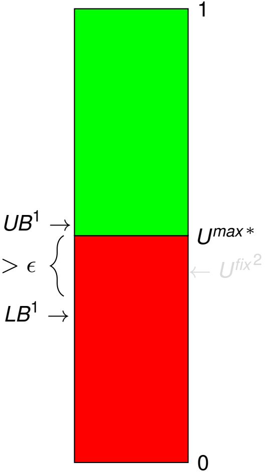
  $LB := U^{max*}$
  while $UB - LB > \epsilon$ **do**
    $U^{fix} := LB + \frac{UB-LB}{2}$
    $U^{max*} := Solve\_RDF(U^{fix})$
    if *Optimal solution is found* **then**
      $UB := U^{max*}$



BSA : Updating UB

$U^{fix} := 1$
$U^{max*} := Solve\_RDF(U^{fix})$
if *Optimal solution is found* **then**
    $UB := U^{max*}$
    $U^{max*} := Solve\_MFF+_{LR}$
    $LB := U^{max*}$
    while $UB - LB > \epsilon$ **do**
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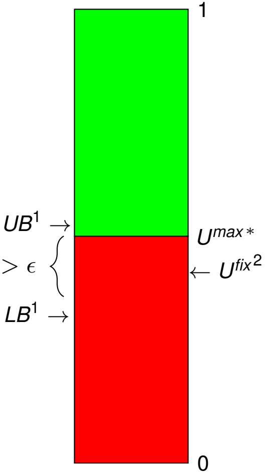
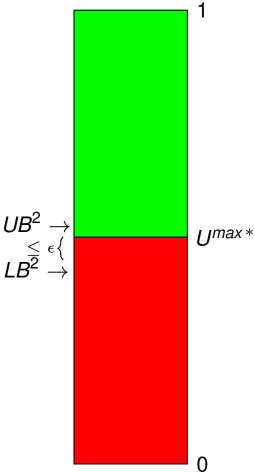


BSA : Updating LB

$U^{fix} := 1$
$U^{max*} := Solve\_RDF(U^{fix})$
if *Optimal solution is found* **then**
  $UB := U^{max*}$
  $U^{max*} := Solve\_MFF{+}_{LR}$
  $LB := U^{max*}$
  while $UB - LB > \epsilon$ **do**
    $U^{fix} := LB + \frac{UB-LB}{2}$
    $U^{max*} := Solve\_RDF(U^{fix})$
    if *Optimal solution is found* **then**
      $UB := U^{max*}$
    else
      $LB := U^{fix}$



Binary Search Algorithm CPU Time ($\epsilon = 0.01$)

	RDF			BSA		
Class ID	#solved	Time avg	Time std	#solved	Time avg	Time std
1	5	0.04	0.05	5	0.04	0.05
2	5	0.02	0.01	5	0.06	0.01
3	5	0.43	0.16	5	0.52	0.43
4	5	0.85	0.42	5	0.48	0.40
5	5	0.27	0.17	5	0.12	0.03
6	5	0.25	0.25	5	0.25	0.05
7	5	6.26	3.80	5	5.56	3.62
8	5	48.20	53.21	5	13.39	10.66
9	5	50.61	31.36	5	33.47	58.03
10	5	0.72	1.00	5	1.29	2.08
11	5	2.65	3.09	5	0.53	0.23
12	5	140.5498	182.36	5	53.62	59.05
13	4	544.68	785.88	5	103.91	122.97
14	3	236.40	356.00	5	556.02	798.25
15	3	1957.82	1801.02	4	940.53	1094.78
16	0	-	-	0	-	-
17	0	-	-	0	-	-
18	1	1307.49	0	2	221.59	40.93
19	0	-	-	0	-	-
20	1	227.30	0	1	174.48	0

Outline

Conclusions

- Two models were developed for the problem, MFF and RDF ;
- LP relaxation of MFF was strengthened : MFF+ ;
- We are able to prove that $LP(MFF+) \subseteq LP(RDF)$;

- Gap values fluctuate (a lot !) ;
- For some instances $LP(MFF+) \subset LP(RDF)$;
- In those instances, gap difference is little ;
- RDF is much faster than MFF+ ;

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Future developments

Short-term :

- Develop valid inequalities that can further strengthen the models (So far, no luck ☹) ;
- Develop other models (*e.g.*, single-commodity flows).

Medium-term :

- Tackle larger problems by using Benders decomposition in a branch-and-cut framework.

Long-term :

- Make the assignment of traffic commodities to VLANs part of the decision process ;
- Optimize other objectives (*e.g.* total load, number of used links, *etc.*) ;
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Thank you ! Questions ?