

# Models for traffic engineering with multiple spanning tree protocols

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- 1 Problem rationale : Data centers and Ethernet networks
- 2 Problem description
- 3 Problem modelling
  - Multi-commodity Flow Formulation
  - Rooted Directed Formulation
  - Comparing the LP relaxations
- 4 Results
- 5 Binary search algorithm
- 6 Conclusions and Future Developments

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- Increasing need for large scale data centers
- Data centers support simultaneously multiple applications
- Critical need to improve the performance of telecommunications networks (**Traffic Engineering**)

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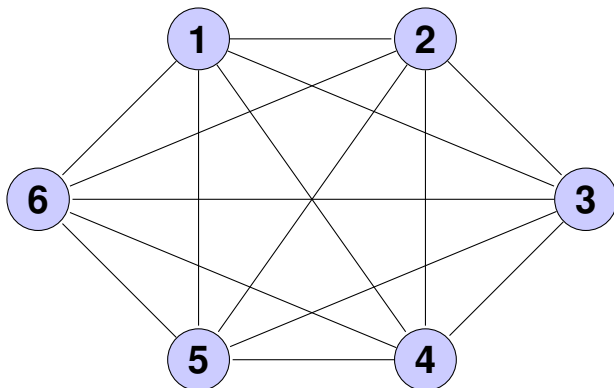
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# Ethernet networks

An Ethernet network can be represented by a graph  $G = (V,A)$ .

$V$  is the set of switches.

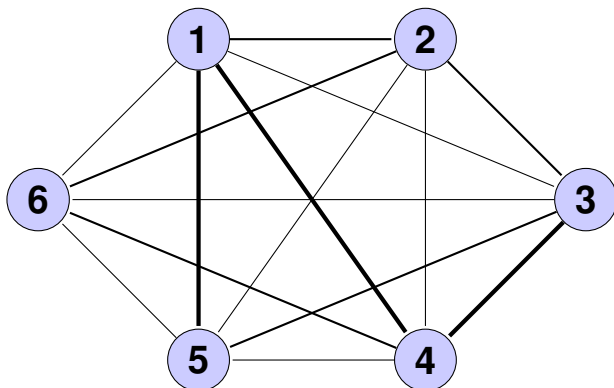
$A$  is the set of links connecting the switches.





# Ethernet Networks - Bandwidth

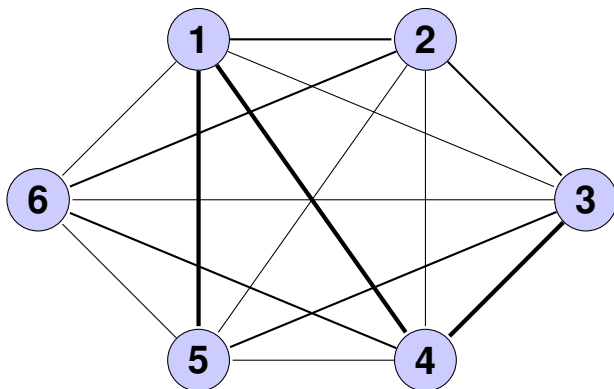
Links have different traffic capacity (Mb/s).



# Ethernet network - Traffic demands

Set of traffic demands between switches, to be routed in the network.

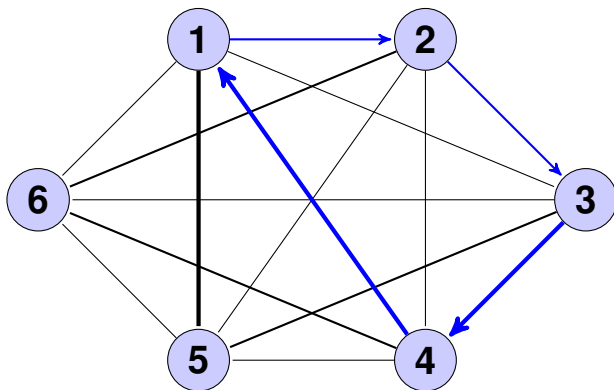
$d\{1,2\} = 10Mb$ ,  $d\{2,3\} = 7Mb$ ,  $d\{4,3\} = 13Mb$ ,  $d\{4,1\} = 5Mb$ , ...



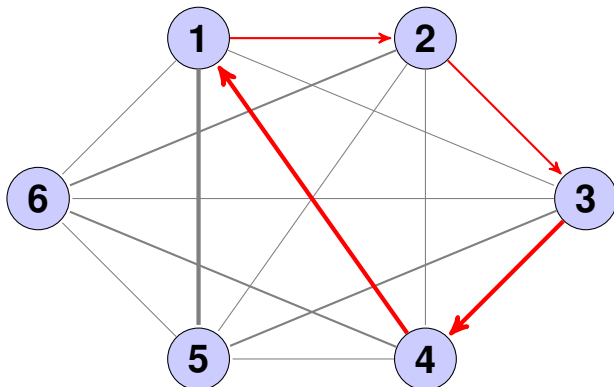
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Cycles in the network create **broadcast radiation** !



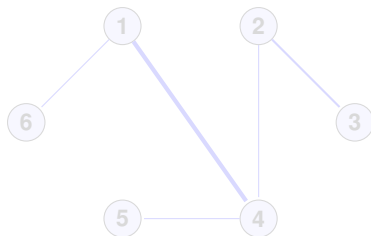
# Subnetwork of activated links

## IEEE 802.1D : Spanning Tree Protocol

The topology of the subnetwork of activated links must be cycle-free. That is to say, it must form a **spanning tree**.

### Definition

Graph S is a spanning tree of G if : i) includes all the nodes of G ; ii) it's connected and contains no cycles.



$$\#edges = \#nodes - 1$$

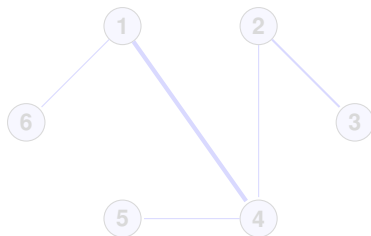
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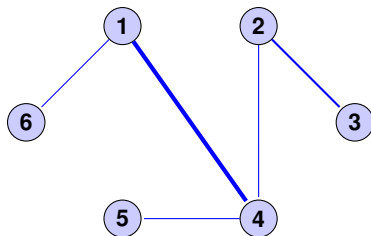
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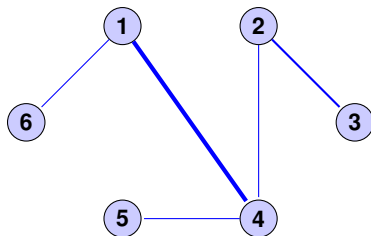
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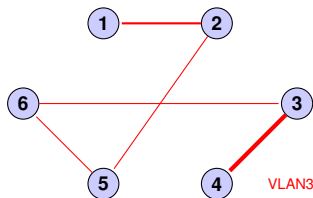
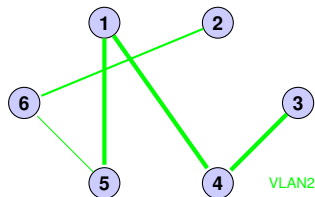
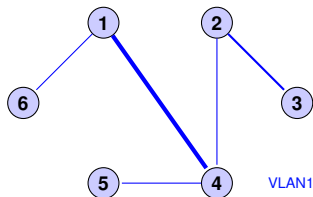


$$\#edges = \#nodes - 1$$



## IEEE 802.1q : Virtual LANs

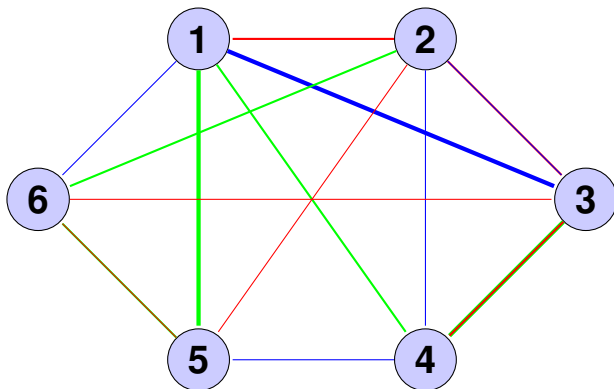
The Ethernet network can be partitioned in many Virtual Local Area Networks (**VLANs**). This way it's possible to isolate different applications.



# Multiple Spanning Tree Protocol (MSTP)

## IEEE 802.1s : Multiple Spanning Tree Protocol

Multiple VLANs may occupy a single physical topology.



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# Problem description

Given a network,  $G$ , characterized by :

- a set of nodes,  $N$ , and a set of links,  $A$  ;
- traffic capacity in each link,  $C_{\{i,j\}}$  ;

partitioned in the set of VLANs,  $S$ , characterized by

- traffic demand between nodes,  $d_s\{u, v\}$  ;

what is the **best design** for each VLAN, such that :

- traffic demands are routed ;
- each VLAN forms a spanning tree ;
- minimize worst-case link utilization (ratio between a link's load, and its capacity)

Literature has been focusing exclusively on heuristic approaches (no lower bounds).

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# Multi-commodity Flow Formulation (MFF)

- $x_{(i,j)}^{\{u,v\},s} = 1$  if node  $i$  is the predecessor of node  $j$  ( $\{i,j\} \in E$ ) on the unique path from node  $u$  to node  $v$ , on VLAN  $s \in S$ ; 0 otherwise;
- $w_{\{i,j\}}^s = 1$  if link  $\{i,j\} \in E$  is used in VLAN  $s \in S$ ; 0 otherwise;
- $U^{max}$  = maximum value of link utilization.

$$\min_{x,w} U^{max} \quad (1a)$$

s.t

$$\sum_{j:(u,j) \in E} x_{(u,j)}^{\{u,v\},s} = 1, \quad u, v \in V : u < v, s \in S, \quad (1b)$$

$$\sum_{j:(i,j) \in E} x_{(i,j)}^{\{u,v\},s} - \sum_{j:(j,i) \in E} x_{(j,i)}^{\{u,v\},s} = 0, \quad u, v, i \in V : u < v, i \neq \{u,v\}, s \in S, \quad (1c)$$

$$x_{(i,j)}^{\{u,v\},s} + x_{(j,i)}^{\{u,v\},s} \leq w_{\{i,j\}}^s, \quad u, v \in V : u < v, \{i,j\} \in E, s \in S, \quad (1d)$$

$$\sum_{\{i,j\} \in E} w_{\{i,j\}}^s = n - 1, \quad s \in S, \quad (1e)$$

$$\sum_{s \in S} \sum_{\{u,v\}: u < v} d_s\{u,v\} (x_{(i,j)}^{\{u,v\},s} + x_{(j,i)}^{\{u,v\},s}) \leq C_{\{i,j\}} U^{max}, \quad \{i,j\} \in E, \quad (1f)$$

$$x_{(i,j)}^{\{u,v\},s} \in \{0, 1\}, \quad (i,j) \in E, u, v \in V : u < v, j \neq u, s \in S, \quad (1g)$$

$$w_{\{i,j\}}^s \in \{0, 1\}, \quad \{i,j\} \in E, s \in S, \quad (1h)$$

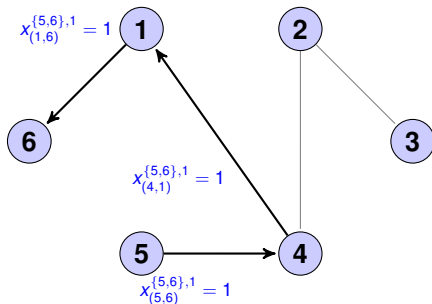
$$0 \leq U^{max} \leq 1 \quad (1i)$$

$$\sum_{j:(u,j) \in E} x_{(u,j)}^{\{u,v\},s} = 1,$$

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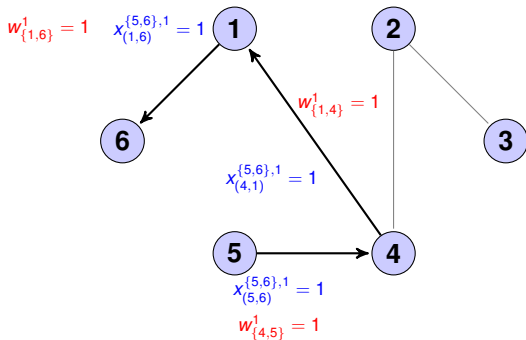
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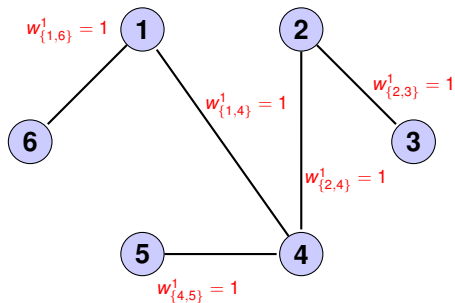


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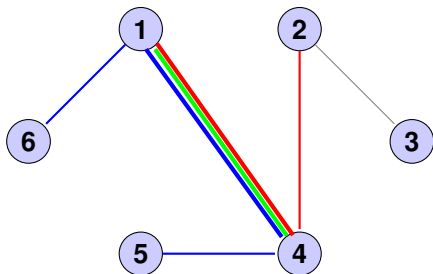
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$$d_1\{5,6\} = 0.1$$

$$d_1\{1,4\} = 0.3$$

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load in  $\{1,4\}$  @  $VLAN_1 = 0.5$

(...)

load in  $\{1,4\}$  @  $VLAN_2 = 0.3$

total load in  $\{1,4\} = 0.8$

$$C_{\{1,4\}} = 1$$

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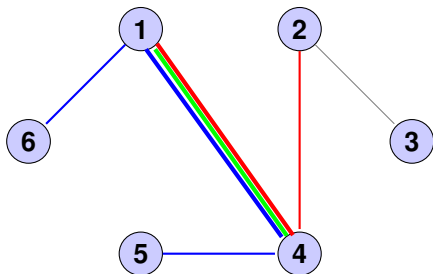
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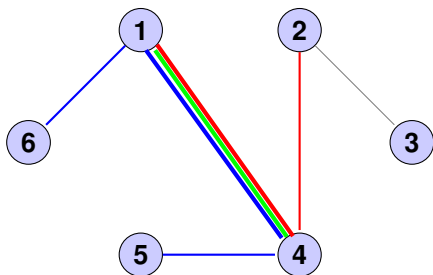
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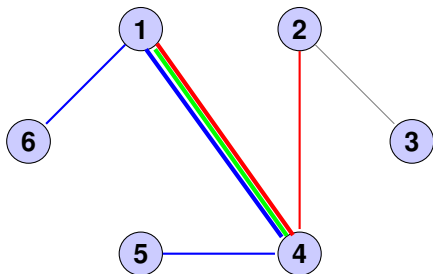
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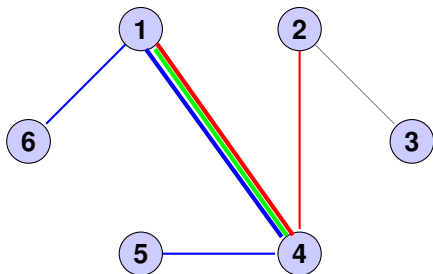


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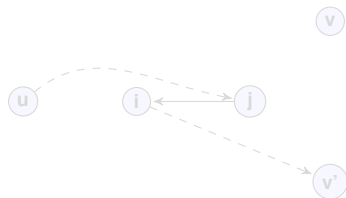
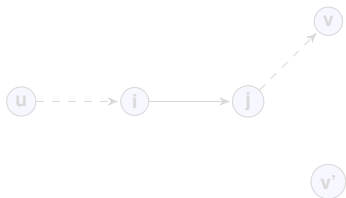
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# Valid inequalities for MFF

Balakrishnan *et al.* (*Operations Research*, 1989), have suggested for the SST some valid inequalities that can be extended to :

$$x_{(i,j)}^{\{u,v\},s} + x_{(j,i)}^{\{u,v'\},s} \leq w_{\{i,j\}}^s, \quad u, v, v' \in V : v \neq v', \{i, j\} \in E, s \in S, \quad (2a)$$

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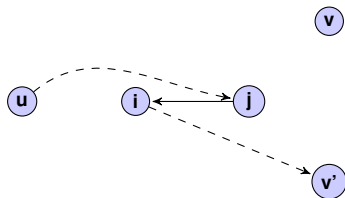
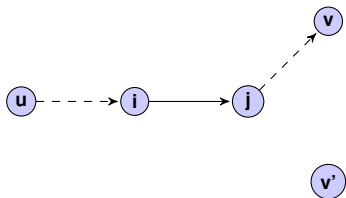


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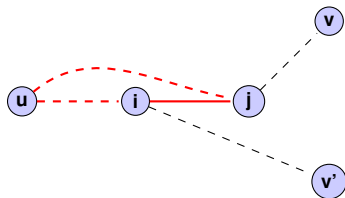
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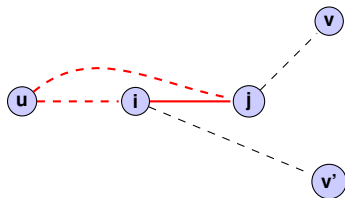
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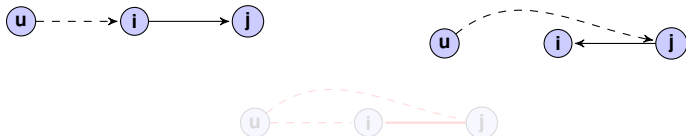
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Tests indicate that using  $MFF \cup (2, 3)$  strengthens the linear programming (LP) relaxation.

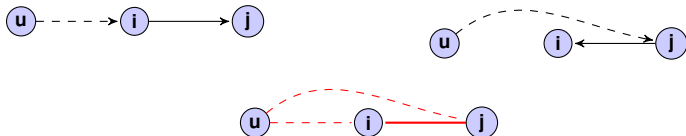
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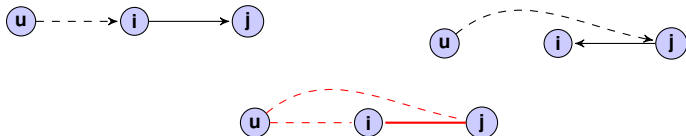
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$$x_{(i,j)}^{\{i,v\},s} + x_{(j,i)}^{\{j,v\},s} = w_{\{i,j\}}^s, \quad v \in V, \{i,j\} \in E : i < v, j < v, s \in S, \quad (3b)$$



Tests indicate that using  $MFF \cup (2, 3)$  strengthens the linear programming (LP) relaxation.

$MFF_+ := MFF \cup (2, 3)$

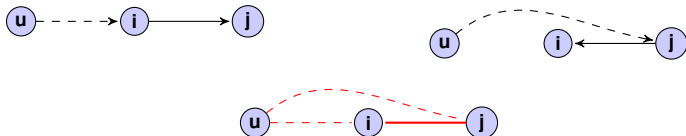


# Valid inequalities for MFF (III)

However, in some cases, the equality can be ensured :

$$x_{(i,j)}^{\{u,j\},s} + x_{(j,i)}^{\{u,i\},s} = w_{\{i,j\}}^s, \quad u \in V, \{i,j\} \in E : u < j, u < i, s \in S, \quad (3a)$$

$$x_{(i,j)}^{\{i,v\},s} + x_{(j,i)}^{\{j,v\},s} = w_{\{i,j\}}^s, \quad v \in V, \{i,j\} \in E : i < v, j < v, s \in S, \quad (3b)$$



Tests indicate that using  $MFF \cup (2, 3)$  strengthens the linear programming (LP) relaxation.

**MFF+** :=  $MFF \cup (2, 3)$

# Rooted Directed Formulation (RDF)

- $z_{i,j}^{u,s} = 1$  if node  $i$  is the "father" of node  $j$ , in an arborescence rooted at node  $u$ , in VLAN  $s \in S$ ; 0 otherwise;
- $f_{(i,j)}^{u,s}$  = traffic quantity, originated from node  $u$ , going through link  $(i, j) \in E$ , in VLAN  $s \in S$ ;
- $w_{\{i,j\}}^s = 1$  if link  $\{i, j\} \in E$  is used in VLAN  $s \in S$ ; 0 otherwise;
- $U^{max}$  = maximum value of link utilization.

$$\min_{z,f,w} U^{max} \quad (4a)$$

s.t

$$\sum_{i:(i,j) \in E} z_{i,j}^{u,s} = 1, \quad u, j \in V : u \neq j, s \in S, \quad (4b)$$

$$z_{i,j}^{u,s} + z_{j,i}^{u,s} = w_{\{i,j\}}^s, \quad u \in V, \{i,j\} \in E, \forall s \in S, \quad (4c)$$

$$\sum_{\{i,j\} \in E} w_{\{i,j\}}^s = n - 1, \quad \forall s \in S, \quad (4d)$$

$$\sum_{j:(j,i) \in E} f_{(j,i)}^{u,s} - \sum_{j:(i,j) \in E \wedge j \neq u} f_{(i,j)}^{u,s} = d_s\{u, i\}, \quad u, i \in V : u \neq i, s \in S, \quad (4e)$$

$$\sum_{i:(u,i) \in E} f_{(u,i)}^{u,s} = \sum_{v \in V: v \neq u} d_s\{u, v\}, \quad u \in V : u, s \in S, \quad (4f)$$

$$f_{(i,j)}^{u,s} \leq \sum_{v \in V \setminus \{u,i\}} d_s\{u, v\} z_{i,j}^{u,s}, \quad i, j, u \in V : u \neq j, \{i,j\} \in E, s \in S, \quad (4g)$$

$$\sum_{s \in S} \sum_{u \in V} (f_{(i,j)}^{u,s} + f_{(j,i)}^{u,s}) \leq C_{\{i,j\}} U^{max}, \quad \{i,j\} \in E, \quad (4h)$$

$$z_{i,j}^{u,s} \in \{0, 1\}, \quad u \in V, (i,j) \in E : j \neq u, s \in S, \quad (4i)$$

$$w_{\{i,j\}}^s \in \{0, 1\}, \quad \{i,j\} \in E, s \in S, \quad (4j)$$

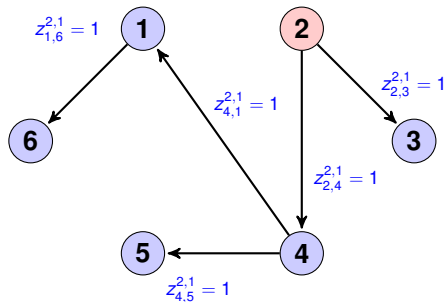
$$f_{(i,j)}^{u,s} \geq 0, \quad u \in V, (i,j) \in E : j \neq u, s \in S, \quad (4k)$$

$$0 \leq U^{max} \leq 1 \quad (4l)$$

$$\sum_{i:(i,j) \in E} z_{i,j}^{u,s} = 1,$$

$$u, j \in V : u \neq j, s \in S$$

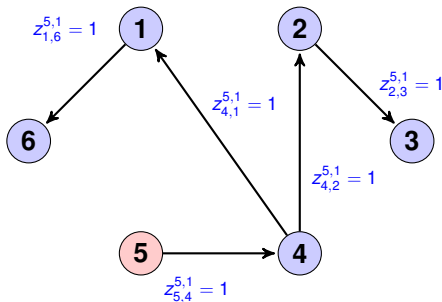
Arborescence rooted at node **2**.



$$\sum_{i:(i,j) \in E} z_{i,j}^{u,s} = 1,$$

$$u, j \in V : u \neq j, s \in S$$

Arborescence rooted at node **5**.

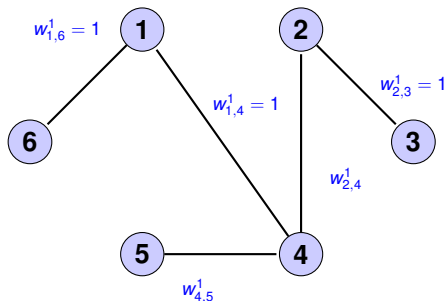


$$z_{i,j}^{u,s} + z_{j,i}^{u,s} = w_{\{i,j\}}^s,$$

$$\sum_{\{i,j\} \in E} w_{\{i,j\}}^s = n - 1,$$

$$u \in V, \{i,j\} \in E, \forall s \in S$$

$$\forall s \in S$$



$$\sum_{j:(j,i) \in E} f_{(j,i)}^{u,s} - \sum_{j:(i,j) \in E \wedge j \neq u} f_{(i,j)}^{u,s} = d_s\{u, i\},$$

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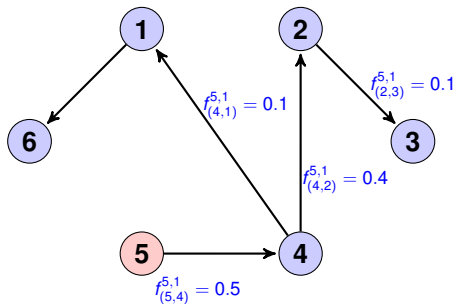
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$$u \in V : u, s \in S,$$

$$f_{(i,j)}^{u,s} \leq \sum_{v \in V \setminus \{u,i\}} d_s\{u, v\} z_{ij}^{u,s},$$

$$i, j, u \in V : u \neq j, \{i, j\} \in E, s \in S,$$

Arborescence rooted at node 5.



$$d_1\{5, 1\} = 0.1$$

$$d_1\{5, 2\} = 0.3$$

$$d_1\{5, 3\} = 0.1$$



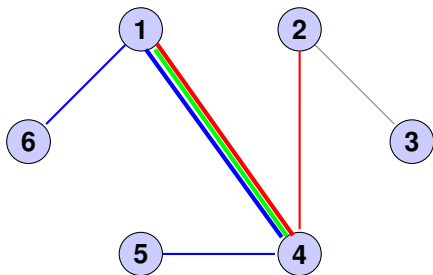
# RDF : 4a,4h,4l

$$\min_{z,f,w} U^{max}$$

$$\sum_{s \in S} \sum_{u \in V} (f_{(i,j)}^{u,s} + f_{(j,i)}^{u,s}) \leq C_{\{i,j\}} U^{max},$$

$$\{i,j\} \in E$$

$$0 \leq U^{max} \leq 1$$



$$d_1\{5,6\} = 0.1$$

$$f_{(1,4)}^{5,1} = 0.1$$

$$d_1\{1,4\} = 0.3$$

$$d_1\{1,2\} = 0.1$$

$$f_{(1,4)}^{1,1} = 0.4$$

load in  $\{1,4\}$  @  $VLAN_1 = 0.5$

(...)

load in  $\{1,4\}$  @  $VLAN_2 = 0.3$

total load in  $\{1,4\} = 0.8$

$$C_{\{1,4\}} = 1$$

utilization in  $\{1,4\} = 0.8$

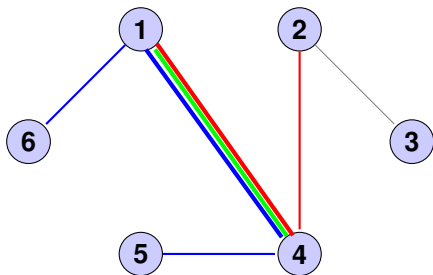


$$\min_{z,f,w} U^{max}$$

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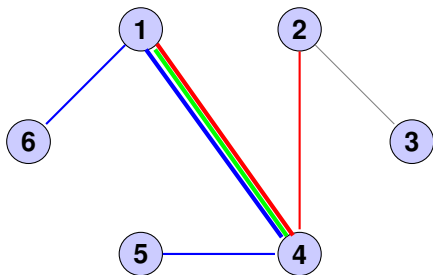
utilization in  $\{1,4\} = 0.8$

$$\min_{z,f,w} U^{max}$$

$$\sum_{s \in S} \sum_{u \in V} (f_{(i,j)}^{u,s} + f_{(j,i)}^{u,s}) \leq C_{\{i,j\}} U^{max},$$

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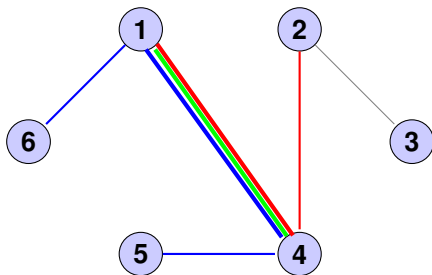
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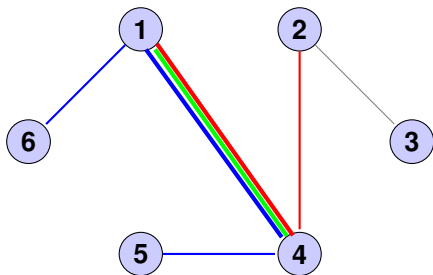
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## Definition

LP relaxation of formulation  $A$  is said to be **as strong as (stronger than)** the LP relaxation of formulation  $B$ , if the polyhedra defined by the set of feasible solutions of the LP relaxation of  $A$  is (strictly) included in the polyhedra of feasible solutions of the LP relaxation of  $B$ .

$$\mathcal{P}(A) \subseteq \mathcal{P}(B) \quad (\mathcal{P}(A) \subset \mathcal{P}(B))$$

## Theorem

$$\mathcal{P}(MFF+) \subseteq \mathcal{P}(RDF)$$

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- 1 Problem rationale : Data centers and Ethernet networks
- 2 Problem description
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  - Multi-commodity Flow Formulation
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- 4 Results**
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- 6 Conclusions and Future Developments

# Instances generation

Input :

- Number of nodes
- Number of VLANs
- Network density  $\sim$  Number of links per VLAN
- Parameter  $\alpha$

Capacity of each existing link :

$$C[i, j] \in \{0.5, 0.75, 1\}$$

Demand between switches :

$$d_s\{u, v\} = \alpha O_u D_v R_{(u,v)} e^{\frac{-L_2(u,v)}{2\Delta}} \quad (\text{Fortz \& Thorup, C.O.A., 2004})$$
$$O_u, D_u, R_{(u,v)} \in [0, 1]$$



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$$O_u, D_u, R_{(u,v)} \in [0, 1]$$

# Instances generated

Class ID	#nodes	density	#VLANs
1	4	1	1
2	4	1	5
3	6	0.8	2
4	6	0.8	4
5	8	0.4	3
6	8	0.4	6
7	8	0.6	2
8	8	0.8	2
9	8	0.8	3
10	10	0.3	2
11	10	0.3	4
12	10	0.5	2
13	10	0.5	3
14	12	0.3	4
15	14	0.3	2
16	14	0.5	3
17	16	0.4	4
18	16	0.4	5
19	20	0.2	1
20	20	0.2	2

# Gap values

Class ID	#ISF	MFF+		RDF	
		Gap avg	Gap std	Gap avg	Gap std
1	5	51.80	7.08	54.85	9.01
2	5	30.69	13.34	30.82	13.46
3	5	46.34	5.88	47.37	6.63
4	5	37.50	11.94	38.00	12.30
5	5	12.74	9.45	13.12	9.85
6	5	1.03	1.14	1.03	1.14
7	5	49.40	3.36	49.56	3.15
8	5	57.96	5.30	58.06	5.28
9	5	58.00	21.10	58.83	20.35
10	5	5.74	12.84	5.96	13.34
11	5	0.26	0.56	0.26	0.56
12	5	32.44	20.65	32.91	21.35
13	5	23.31	15.18	23.31	15.18
14	5	3.12	2.18	3.12	2.18
15	5	16.88	17.49	16.88	17.49
16	5	64.05	12.00	64.05	12.00
17	5	67.26	11.70	67.57	12.07
18	5	40.25	30.62	40.25	30.62
19	2	73.70	14.34	74.65	14.76
20	5	43.86	27.10	43.86	27.10

# CPU time (seconds) for CPLEX to solve IP

Class ID	MFF+			RDF		
	#solved	Time avg	Time std	#solved	Time avg	Time std
1	5	0.06	0.05	5	0.04	0.05
2	5	0.07	0.01	5	0.02	0.01
3	5	2.74	1.32	5	0.43	0.16
4	5	10.84	7.10	5	0.85	0.42
5	5	3.70	2.08	5	0.27	0.17
6	5	8.33	7.05	5	0.25	0.25
7	5	498.75	368.59	5	6.26	3.80
8	5	979.84	736.79	5	48.20	53.21
9	3	307.40	173.07	5	50.61	31.36
10	5	41.41	53.71	5	0.72	1.00
11	5	126.62	138.78	5	2.65	3.09
12	2	478.17	579.58	5	140.5498	182.36
13	3	3733.48	3097.41	4	544.68	785.88
14	0	-	-	3	236.40	356.00
15	1	45.70	0.00	3	1957.82	1801.02
16	0	-	-	0	-	-
17	0	-	-	0	-	-
18	0	-	-	1	1307.49	0
19	0	-	-	0	-	-
20	0	-	-	1	227.30	0

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## Idea

Max-min objective function produces unpredictable gaps. Therefore, we shall fix  $U^{max}$  in RDF to a value iteratively given by a binary search algorithm. Use *faux* objective function.

$$\begin{aligned}
 \min_{z,f,w} \quad & \sum_{s \in S, \{i,j\} \in E} w_{\{i,j\}}^s \\
 \text{s.t} \quad & (\dots) \\
 & \sum_{s \in S} \sum_{u \in V} (f_{(i,j)}^{u,s} + f_{(j,i)}^{u,s}) \leq C[i,j] \cdot U^{fix}, \quad \forall \{i,j\} \in E, \\
 & z_{i,j}^{u,s} \in \{0,1\}, \quad \forall u \in V, \forall \{i,j\} \in E : j \neq u, \forall s \in S, \\
 & w_{\{i,j\}}^s \in \{0,1\}, \quad \forall \{i,j\} \in E, \forall s \in S, \\
 & f_{(i,j)}^{u,s} \geq 0, \quad \forall u \in V, \forall \{i,j\} \in E : j \neq u, \forall s \in S,
 \end{aligned}$$

## Idea

Max-min objective function produces unpredictable gaps. Therefore, we shall fix  $U^{max}$  in RDF to a value iteratively given by a binary search algorithm. Use *faux* objective function.

min  
 $z, f, w$

$$\sum_{s \in S, \{i,j\} \in E} w_{\{i,j\}}^s$$

s.t

(...)

$$\sum_{s \in S} \sum_{u \in V} (f_{(i,j)}^{u,s} + f_{(j,i)}^{u,s}) \leq C[i,j] \cdot U^{fix}, \quad \forall \{i,j\} \in E,$$

$$z_{i,j}^{u,s} \in \{0, 1\}, \quad \forall u \in V, \forall \{i,j\} \in E : j \neq u, \forall s \in S,$$

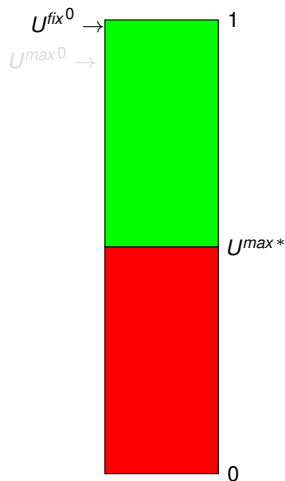
$$w_{\{i,j\}}^s \in \{0, 1\}, \quad \forall \{i,j\} \in E, \forall s \in S,$$

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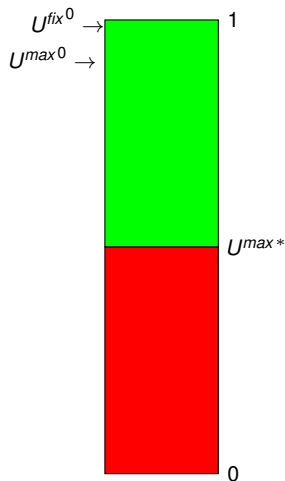


# BSA : Test feasibility

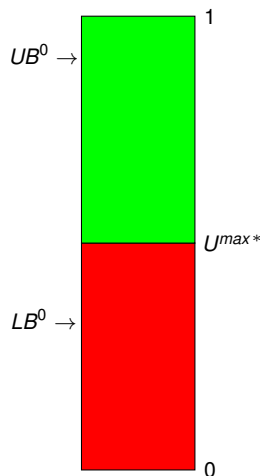
- 
- 
- 1  $U^{fix} := 1$
  - 2  $U^{max*} := \text{Solve\_RDF}(U^{fix})$
  - 3 if *Optimal solution is found* then
  - 4  $UB := U^{max*}$
- 



- 
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  - 5      $U^{max*} := Solve\_MFF +_{LR}$
  - 6      $LB := U^{max*}$
- 
- 



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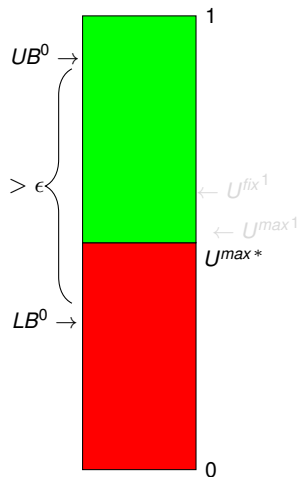
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```

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3 if Optimal solution is found then
4    $UB := U^{max*}$ 
5    $U^{max*} := \text{Solve\_MFF} + LR$ 
6    $LB := U^{max*}$ 
7   while  $UB - LB > \epsilon$  do
8      $U^{fix} := LB + \frac{UB - LB}{2}$ 
9      $U^{max*} := \text{Solve\_RDF}(U^{fix})$ 

```

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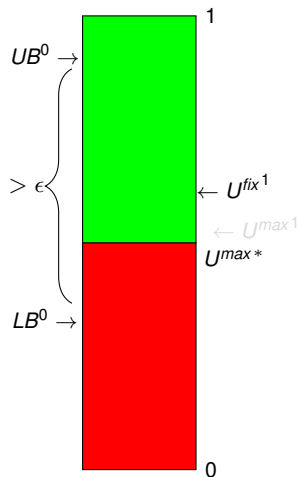
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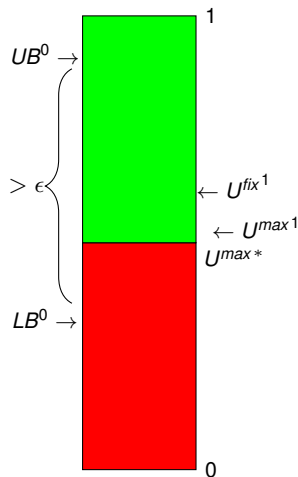
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```

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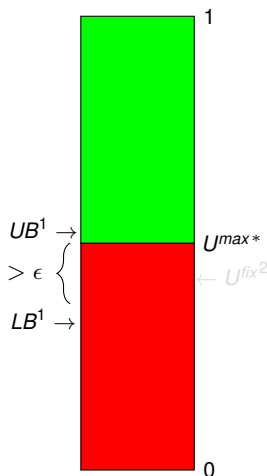
```

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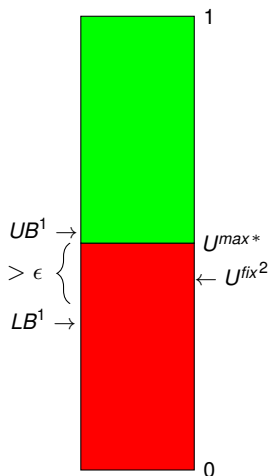
# BSA : Updating UB

```
1  $U^{fix} := 1$ 
2  $U^{max*} := Solve\_RDF(U^{fix})$ 
3 if Optimal solution is found then
4    $UB := U^{max*}$ 
5    $U^{max*} := Solve\_MFF+LR$ 
6    $LB := U^{max*}$ 
7   while  $UB - LB > \epsilon$  do
8      $U^{fix} := LB + \frac{UB-LB}{2}$ 
9      $U^{max*} := Solve\_RDF(U^{fix})$ 
10    if Optimal solution is found
11      then
12         $UB := U^{max*}$ 
```



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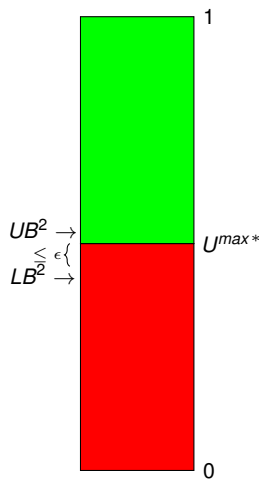
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# BSA : Updating LB

```
1  $U^{fix} := 1$ 
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3 if Optimal solution is found then
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5    $U^{max*} := Solve\_MFF+LR$ 
6    $LB := U^{max*}$ 
7   while  $UB - LB > \epsilon$  do
8      $U^{fix} := LB + \frac{UB-LB}{2}$ 
9      $U^{max*} := Solve\_RDF(U^{fix})$ 
10    if Optimal solution is found
11      then
12         $UB := U^{max*}$ 
13      else
14         $LB := U^{fix}$ 
```



# Binary Search Algorithm CPU Time ( $\epsilon = 0.01$ )

Class ID	RDF			BSA		
	#solved	Time avg	Time std	#solved	Time avg	Time std
1	5	0.04	0.05	5	0.04	0.05
2	5	0.02	0.01	5	0.06	0.01
3	5	0.43	0.16	5	0.52	0.43
4	5	0.85	0.42	5	0.48	0.40
5	5	0.27	0.17	5	0.12	0.03
6	5	0.25	0.25	5	0.25	0.05
7	5	6.26	3.80	5	5.56	3.62
8	5	48.20	53.21	5	13.39	10.66
9	5	50.61	31.36	5	33.47	58.03
10	5	0.72	1.00	5	1.29	2.08
11	5	2.65	3.09	5	0.53	0.23
12	5	140.5498	182.36	5	53.62	59.05
13	4	544.68	785.88	5	103.91	122.97
14	3	236.40	356.00	5	556.02	798.25
15	3	1957.82	1801.02	4	940.53	1094.78
16	0	-	-	0	-	-
17	0	-	-	0	-	-
18	1	1307.49	0	2	221.59	40.93
19	0	-	-	0	-	-
20	1	227.30	0	1	174.48	0

- 1 Problem rationale : Data centers and Ethernet networks
- 2 Problem description
- 3 Problem modelling
  - Multi-commodity Flow Formulation
  - Rooted Directed Formulation
  - Comparing the LP relaxations
- 4 Results
- 5 Binary search algorithm
- 6 Conclusions and Future Developments

# Conclusions

- Two models were developed for the problem, MFF and RDF ;
- LP relaxation of MFF was strengthened : MFF+ ;
- We are able to prove that  $LP(MFF+) \subseteq LP(RDF)$  ;
  
- Gap values fluctuate (a lot !)
- For some instances  $LP(MFF+) \subset LP(RDF)$  ;
- In those instances, gap difference is little ;
- RDF is much faster than MFF+ ;
  
- Binary search algorithm proposed, that is faster than RDF for most instances.

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# Future developments

## Short-term :

- Develop valid inequalities that can further strengthen the models (So far, no luck ☹) ;
- Develop other models (*e.g.*, single-commodity flows).

## Medium-term :

- Tackle larger problems by using Benders decomposition in a branch-and-cut framework.

## Long-term :

- Make the assignment of traffic commodities to VLANs part of the decision process ;
- Optimize other objectives (*e.g.* total load, number of used links, *etc.*) ;
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**Thank you ! Questions ?**