Sufficiency of Cut-Generating Functions

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joint work with

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Outline

- Model
- Motivation
- Problem & Known Results
- Our Result & Proof
- Conclusion

Model

We consider sets of the form

$$X = X(R, S) := \{x \in \mathbb{R}^n_+ : Rx \in S\}$$

$$\text{where } \begin{cases} R = [r_1 \dots r_n] \text{ is a real, } q \times n \text{ matrix,} \\ S \subset \mathbb{R}^q \text{ is a nonempty, closed set with } 0 \notin S. \end{cases}$$

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Observation

$0 \notin S \;\Rightarrow\; 0 \notin \overline{\operatorname{conv}}\, X.$

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Such cuts were studied by Johnson (1981) and Conforti et al. (2013).

Motivation: Integer Programming

Let $P := \{(x, y) \in \mathbb{R}^n_+ \times \mathbb{R}^m : Ax + y = b\}$ and $b \notin \mathbb{Z}^m$.

$P \cap (\mathbb{Z}^n_+ \times \mathbb{Z}^m)$ is the set considered by Gomory (1969).
Its convex hull is the well-known corner polyhedron:

$$\begin{bmatrix} x \\ y \end{bmatrix} \in P \cap (\mathbb{Z}^n_+ \times \mathbb{Z}^m) \Leftrightarrow \begin{bmatrix} x \\ y = b - Ax \end{bmatrix} \in \mathbb{Z}^n_+ \times \mathbb{Z}^m$$

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Motivation: Complementarity Problems

Let $E \subseteq \{1, \ldots, m\}^2$ and $C := \{y \in \mathbb{R}^m_+ : y_i y_j = 0 \, \forall \, (i,j) \in E\}$.
As before, $P := \{(x,y) \in \mathbb{R}^n_+ \times \mathbb{R}^m : Ax + y = b\}$ and $b \notin C$.
The set $P \cap (\mathbb{R}^n_+ \times C)$ appears in complementarity problems:

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- R is an arbitrary matrix which defines the problem instance, and
- we want to generate cuts to obtain a better description of the set X that corresponds to a given pair (R, S).

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$$\sum_{j=1}^{n} \rho(r_j) x_j \geq 1$$

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Well-known example from integer programming: Gomory function.

The Geometric Perspective: S-Free Sets

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Theorem (Conforti et al. 2013)

Let the sublinear function $\rho : \mathbb{R}^q \mapsto \mathbb{R}$ and the closed, convex neighborhood V satisfy $V = \{r : \rho(r) \leq 1\}$. Then ρ is a CGF if and only if V is S-free.

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Conforti et al. (2013) also show that the answer to this question is "No" in general.

Our Main Result

Can we find sufficient conditions under which CGFs can generate all cuts of the form $c^\top x \geq 1$ for general S?

A cut $d^\top x \geq 1$ dominates another cut $c^\top x \geq 1$ if $d_j \leq c_j$ for all j (again, recall $X \subset \mathbb{R}^n_+$).

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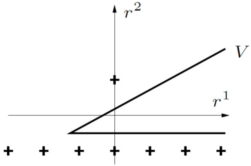
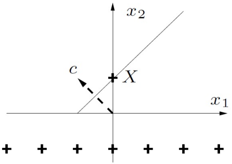
Theorem (Cornuéjols, Wolsey, Y.)

Suppose $S \subset \text{cone}\, R$. Then any cut separating the origin from X is dominated by a cut obtained from a CGF.

Our Main Result

Consider the following example from Conforti et al. (2013):





$S := \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \cup (\mathbb{Z} \times \{-1\})$, $r_1 := \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $r_2 := \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Then $X = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$.

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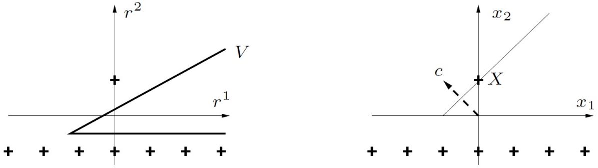

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Our result: There is a CGF which yields a cut that dominates $-x_1 + x_2 \geq 1$ when S is replaced with $S' := S \cap \operatorname{cone} R = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$. (Note $X(R, S) = X(R, S')$.)

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- *ρ is a sublinear function which is finite on* cone R.
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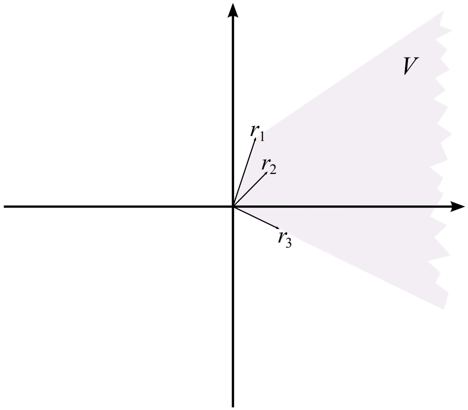

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Because $\rho'(r_j) = \rho(r_j) \leq c_j$ for all j, we are done.

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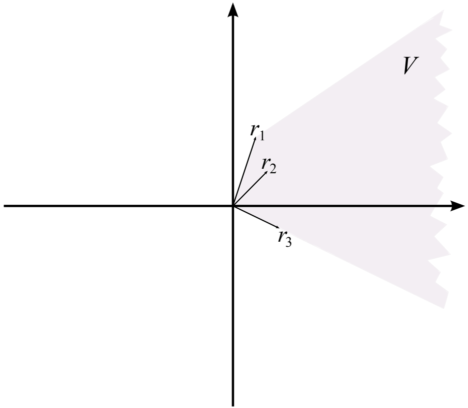
Example: $R := \begin{bmatrix} 1 & 1.5 & 2 \\ 3 & 1.5 & -1 \end{bmatrix}$ and $c := \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$.

Let $r_0 := \begin{bmatrix} -5 \\ -1 \end{bmatrix}$. Then $c_0 = 1$.



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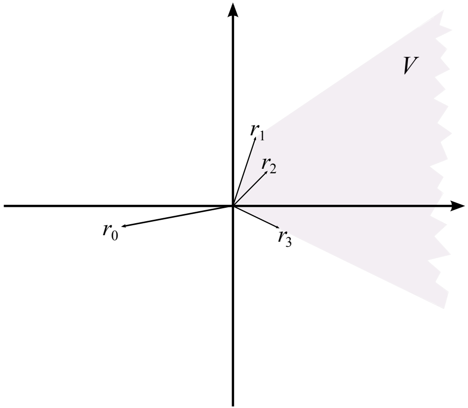

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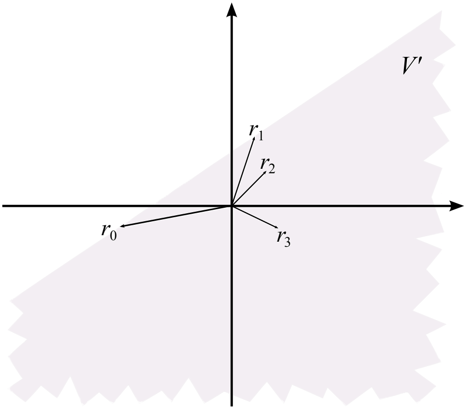

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Questions / Comments?

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