Mixed Integer Reformulations of Robust b-Matching Problems - Using Few Integer Variables

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In the talk we consider robust two-stage (bipartite) b-matching problems with one or several knapsack constraints that link the first stage solution to the second stage solution. Applications of such robust two-stage bipartite b-matching problems are for example in Air Traffic Management (ATM). In ATM an efficient planning of runway utilization is one of the main challenges. Thereby, uncertainty and inaccuracy often lead to deviations from the actual plan or schedule. By using ideas from robust optimisation this uncertainty can be incorporated into the initial computation of the plans. More precisely, each aircraft needs to be assigned to a time slot at the first stage before the uncertainty reveals. If an uncertain scenario occurs the plan might become infeasible and the aircraft need to be replanned in the second stage. To this end, second stage solutions are desired, that do not differ too much from the first stage plan. The restriction of the replanning action may be modelled by one or several special knapsack constraints. A straight forward way is to formulate the resulting two-stage bipartite b-matching problem with one or several knapsack constraints as a binary linear integer program (IP).
Following the idea of Bader et al.* we show how to reformulate the IP formulations of the two-stage bipartite b-matching problems as mixed integer programming problems that only use few integer variables. To preserve integrality, an appropriate so called affine TU decomposition of the constraint matrix needs to be found. In the case of the two-stage bipartite b-matching problem with only one knapsack constraint, we give such an affine TU decomposition and show that a MIP reformulation can be given where the number of integral variables only depends on the number of times slots. For the two-stage bipartite b-matching problem with several knapsack constraints an appropriate affine TU decomposition cannot be derived in a straight forward way. We give a similar decomposition of the constraint matrix and prove that this decomposition preserves integrality and thus allows a MIP reformulation with less integer variables.
In a computational study we show that when compared to solving the IP formulation that only uses binary variables, root bounds and cpu times can significantly be improved, when the reformulated MIPs are solved. Moreover, several instances could only be solved when the MIP reformulations are used.

*Jörg Bader, Robert Hildebrand, Robert Weismantel, Rico Zenklusen. *Mixed Integer Reformulations of Integer Programs and the Affine TU-dimension of a Matrix. Preprint, (2016)*.
