The data arrangement problem on binary trees

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Short overview

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- lower bound

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- $\mathcal{NP}$-hardness for a special case

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$$\sum_{(i,j)\in E(G)} d\big(\phi(i), \phi(j)\big), \tag{1}$$

  where $d(x, y)$ denotes the length of the shortest path between x and y in T.

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- In our case T is a d-regular tree and B is the set of its leaves.
- We will call this problem **data arrangement problem on regular trees (DAPT)** and denote the objective value $OV(G, d, \phi)$.

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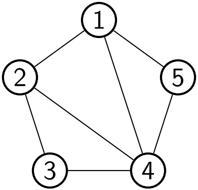

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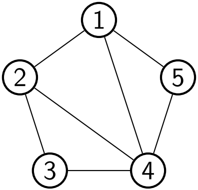

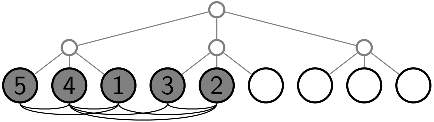

$$OV(G, 3, \phi) = 20$$

General properties and our special case

- DAPT is $\mathcal{NP}$-hard for every fixed $d \geq 2$ [LUCZAK, NOBLE 2002[3]].

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- The question about the computational complexity of the DAPT in the case where the guest graph is a tree was open in [Luzcak, Noble 2002[4]].
- We deal with the special case where G and T are both binary regular trees.

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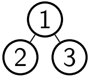
Solution algorithm



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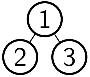
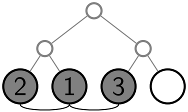
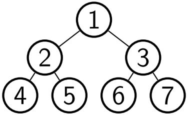




$$OV(G, 2, \phi^*) = 6$$

Solution algorithm



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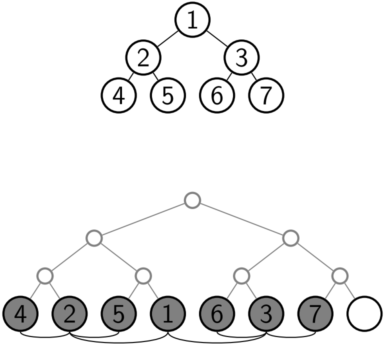
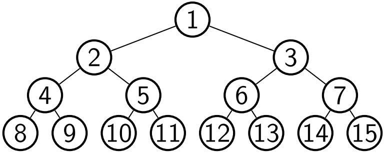


$$OV(G, 2, \phi^*) = 22$$

Solution algorithm



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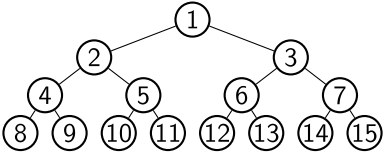

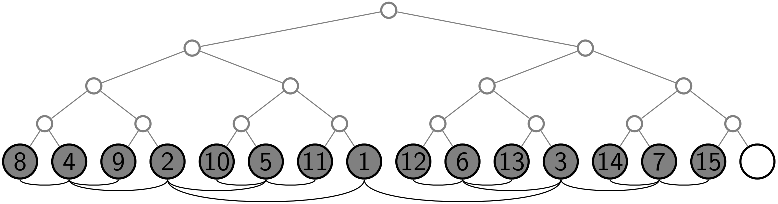

$$OV(G, 2, \phi^*) = 58$$

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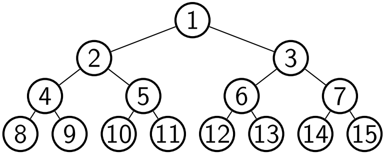

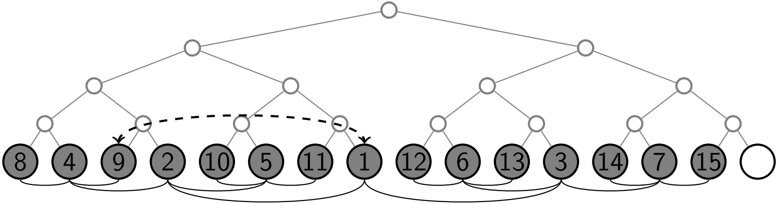

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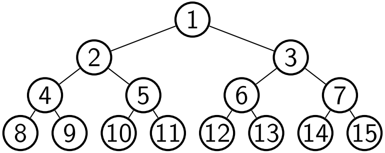
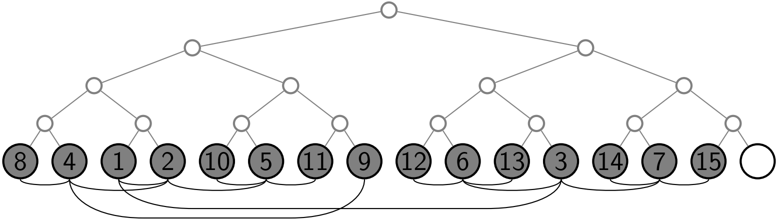




$$OV(G, 2, \phi^*) = 56$$

Solution algorithm

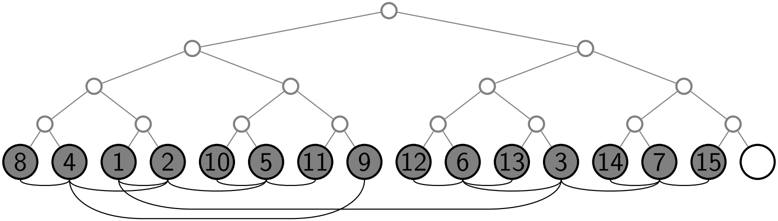
Theorem

Given the binary regular trees $G = (V, E)$ and T with heights h_G and $h = h_G + 1$, let G be the guest graph and T the host graph and let ϕ^* be the arrangement obtained from the described algorithm. Then

$$OV(G, 2, \phi^*) = \begin{cases} 0 & \text{for } h_G = 0 \\ \frac{29}{3} \cdot 2^{h_G} - 4h_G - 9 + \frac{1}{3}(-1)^{h_G} & \text{for } h_G \geq 1 \end{cases} \tag{2}$$

holds.

Lower bound – problem transformation



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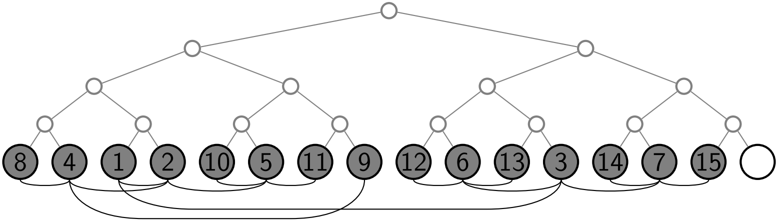

$$OV(G, 2, \phi^*) = 56$$

- $OV(G, 2, \phi) = 2(1 \cdot 4 + 3 \cdot 3 + 5 \cdot 2 + 5 \cdot 1) = 56$

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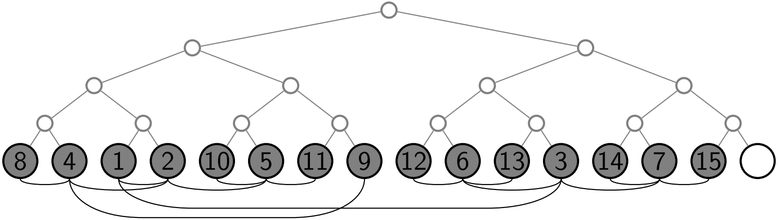

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- $OV(G, 2, \phi) = 2(1 \cdot 4 + 3 \cdot 3 + 5 \cdot 2 + 5 \cdot 1) = 56$
- $OV(G, 2, \phi) = 2\big(a_h(\phi) \cdot h + a_{h-1}(\phi) \cdot (h-1) + \ldots + a_1(\phi) \cdot 1\big)$

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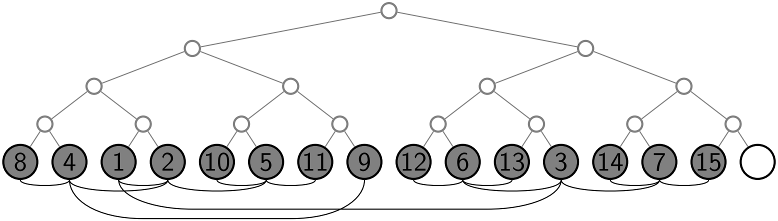

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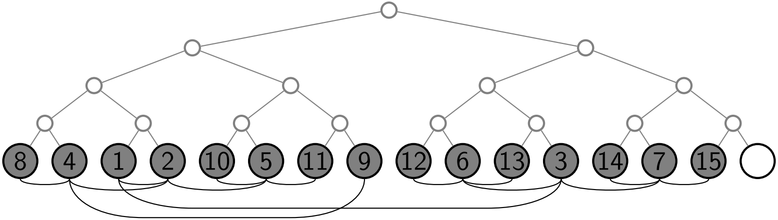

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- $OV(G, 2, \phi) = 2 \sum_{i=1}^{h} a_i(\phi) \cdot i$
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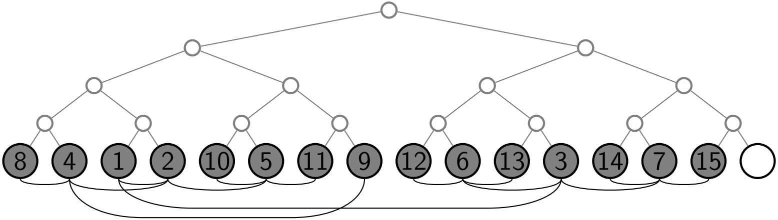

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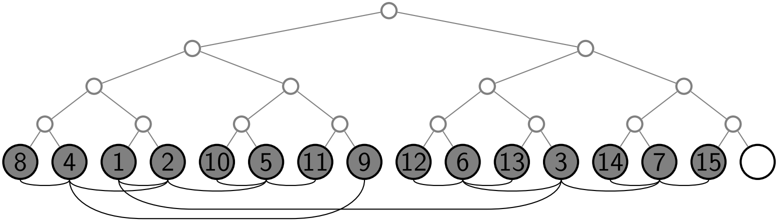

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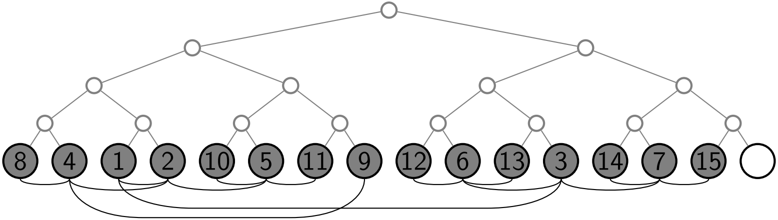

$$OV(G, 2, \phi^*) = 56$$

i	4	3	2	1
▶ a_i	1	3	5	5
s_i	1	4	9	14

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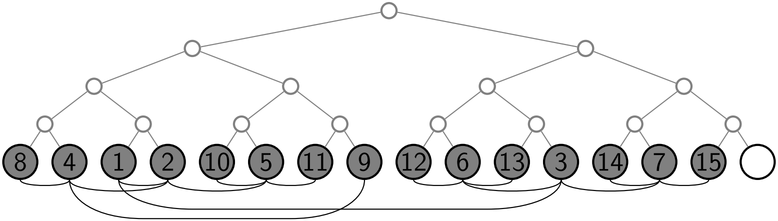

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the **k-balanced partitioning problem (kBPP)** asks for a partition of the vertex set V into k non-empty vertex sets

- $V_1 \neq \emptyset,\ V_2 \neq \emptyset,\ \ldots,\ V_k \neq \emptyset$, where
- $\cup_{i=1}^{k} V_k = V$, $V_i \cap V_j = \emptyset$ for every $i \neq j$ and
- $|V_i| \leq \left\lceil \frac{n}{k} \right\rceil$ for all $1 \leq i \leq k$,

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such that the number of edges connecting these vertex sets

$$c(G, \mathscr{V}) := \left| \left\{ (u, v) \in E | u \in V_i,\ v \in V_j,\ i \neq j \right\} \right|, \tag{3}$$

where $\mathscr{V} = \{V_i | 1 \leq i \leq k\}$, is minimized.

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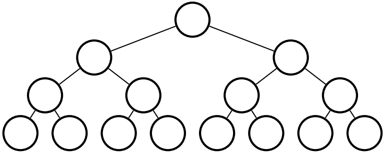

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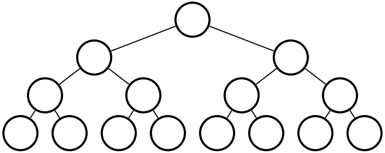

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- It is obvious that $s_i \geq c(G, \mathscr{V})$, where $k = 2^{h-i+2}$ for all $2 \leq i \leq h$ and that $s_1 = \left|E(G)\right|$.

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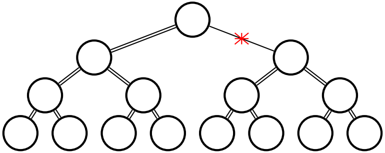

i	4	3	2	1
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  , $k = 2^{3-4+2} = 2$
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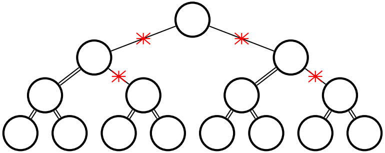

i	4	3	2	1
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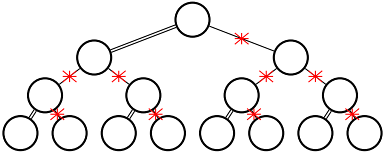
  , $k = 2^{3-3+2} = 4$
- It is obvious that $s_i \geq c(G, \mathscr{V})$, where $k = 2^{h-i+2}$ for all $2 \leq i \leq h$ and that $s_1 = \left|E(G)\right|$.

Lower bound – problem transformation



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i	4	3	2	1
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  , $k = 2^{3-2+2} = 8$

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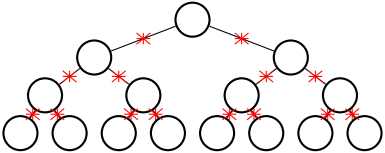

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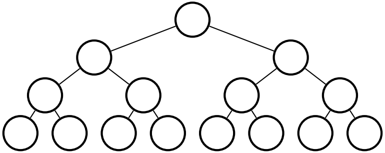

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- It is obvious that $s_i \geq c(G, \mathscr{V})$, where $k = 2^{h-i+2}$ for all $2 \leq i \leq h$ and that $s_1 = \big|E(G)\big|$.
- All but one components have the size $\frac{|V|+1}{k}$.

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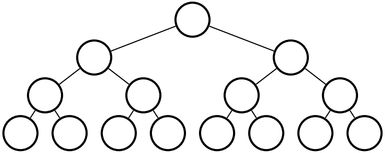

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- It is obvious that $s_i \geq c(G, \mathscr{V})$, where $k = 2^{h-i+2}$ for all $2 \leq i \leq h$ and that $s_1 = \left|E(G)\right|$.
- All but one components have the size $\frac{|V|+1}{k}$.
- One component has the size $\frac{|V|+1}{k} - 1$.

Lower bound – problem transformation

- kBPP is $\mathcal{NP}$-hard (we get the *minimum bisection problem* which is $\mathcal{NP}$-hard for $k = 2$ [Garey, Johnson 2002[5]]).

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[8] R. Krauthgamer, J. Naor and R. Schwartz, Partitioning graphs into balanced components, *Proceeding SODA '09 Proceedings of the twentieth Annual ACM-SIAM Symposium on Discrete Algorithms*, 942–949, 2009.

Lower bound – problem transformation

- kBPP remains $\mathcal{APX}$-hard even if the graph is an unweighted tree with constant maximum degree [FELDMANN, FOSCHINI 2013[8]].

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Lower bound – problem transformation

- kBPP remains $\mathcal{APX}$-hard even if the graph is an unweighted tree with constant maximum degree [FELDMANN, FOSCHINI 2013[8]].

Theorem

Let $G = (V, E)$ be a binary regular tree of height $h \geq 1$ and let $k = 2^{k'}$, where $1 \leq k' \leq h$, and $\mathscr{V}^*$ an optimal k-balanced partition. Then

$$c(G, \mathscr{V}^*) = \left(3 \cdot 2^{h+1} - 2^{k'+1}\right)\left(\frac{1}{2^s - 1} - \frac{1}{(1 - 2^{-s})\, 2^{sl}}\right) + 3 \cdot 2^{h-sl+1} - 2, \tag{4}$$

where $s = h - k' + 2$ and $l = \left\lfloor \frac{h+1}{s} \right\rfloor$.

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a_i	1	3	5	5
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  $\Rightarrow$ optimality in this case ✓
- $OV(G, 2, \phi^*) = 56$

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Lower bound – problem transformation

i	6	5	4	3	2	1
a_i	1	3	6	12	19	21
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- $278 \leq OV(G, 2, \phi^*) \leq 280$

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- In fact, the lower bound is tight for all $\left\lfloor \frac{h}{2} \right\rfloor + 1 \leq i \leq h$ and for $i = 1$ and $i = 2$.

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- A straightforward analysis yields an approximation ratio 2.

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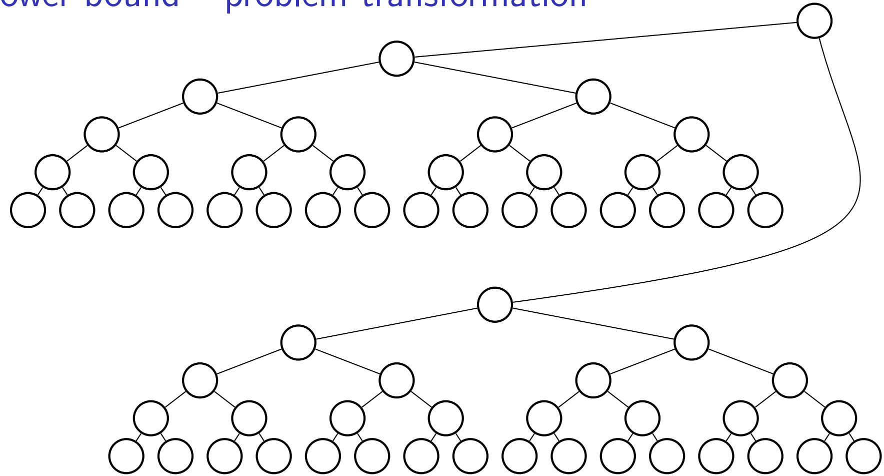
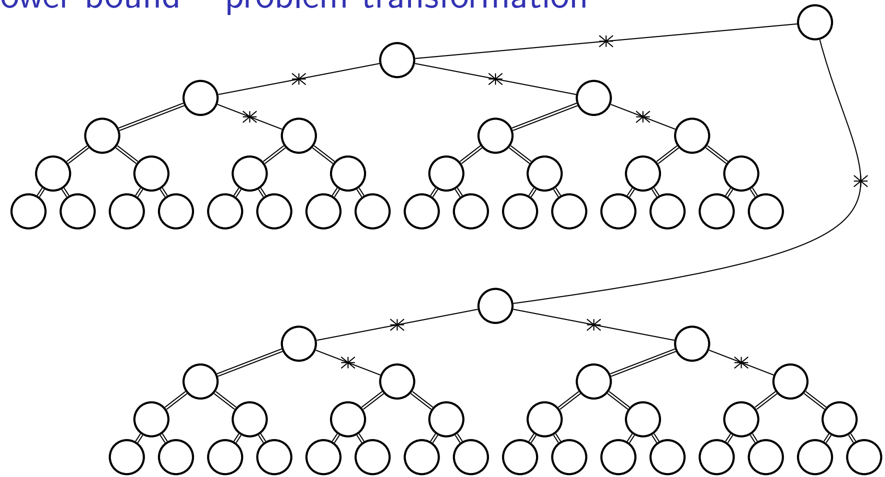
- In fact, the lower bound is tight for all $\left\lfloor \frac{h}{2} \right\rfloor + 1 \leq i \leq h$ and for $i = 1$ and $i = 2$.
- A straightforward analysis yields an approximation ratio 2.
- A $\frac{203}{200}$-approximation ratio follows by a deeper analysis.

Lower bound – problem transformation

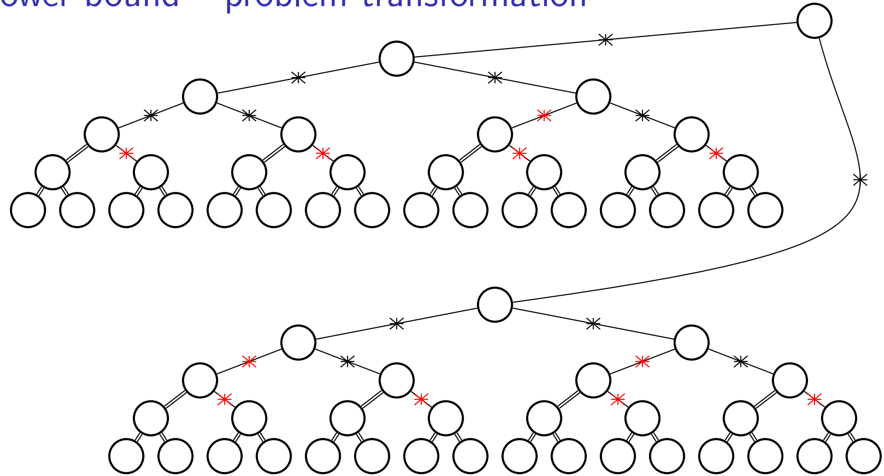


Lower bound – problem transformation



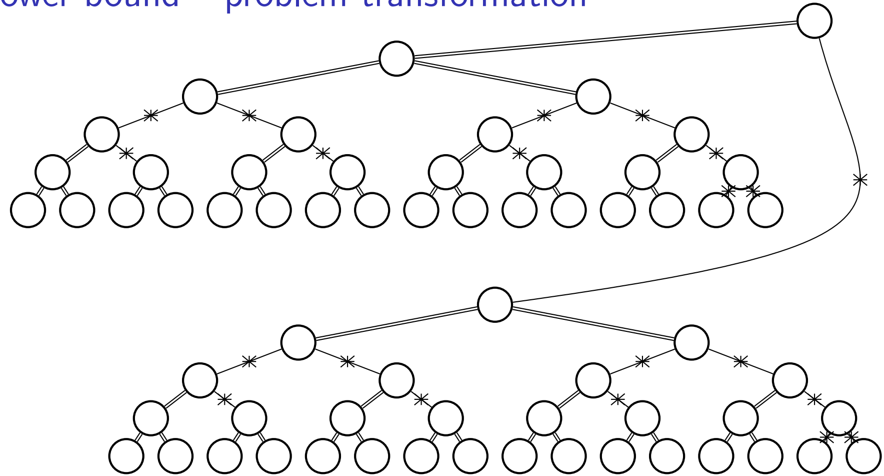
$$c(G, k, \mathscr{V}) = 10$$

Lower bound – problem transformation



$$c(G, k, \mathscr{V}) = 10 + 12 = 22$$

Lower bound – problem transformation



$$c(G, k, \mathscr{V}) = 21$$

Lower bound – problem transformation

- The presented algorithm does not yield an optimal solution for larger guest graphs (there exists a counterexample e.g. for $h_G = 6$ and $h = 7$).

Data arrangement problem

$\mathcal{NP}$-hardness result for trees
proof sketch

Open problem from [LUCZAK, NOBLE 2002]:

Computational complexity when Guest graph is a tree and host is a complete d-regular tree?

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Open problem from [LUCZAK, NOBLE 2002]:

Computational complexity when Guest graph is a tree and host is a complete *d*-regular tree?

Theorem

The DAPT with a host graph T being a complete d-regular tree is $\mathcal{NP}$-hard for every fixed $d \geq 2$ even if the guest graph G is a tree.

Data arrangement problem

$\mathcal{NP}$-hardness result for trees
proof sketch

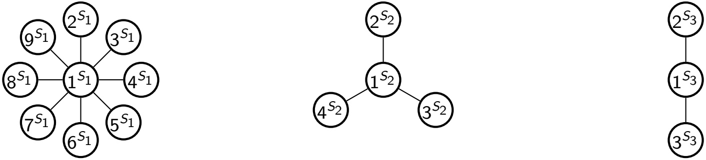
- The $\mathcal{NP}$-hardness is proven by a reduction from the strongly $\mathcal{NP}$-hard **numerical matching with target sums (NMTS)** problem:
  - Given three sets of positive integers
    - $X = \{x_1, x_2, \dots, x_n\}$, $Y = \{y_1, y_2, \dots, y_n\}$ and $Z = \{z_1, z_2, \dots, z_n\}$,

    such that
    - $\sum_{i=1}^{n} z_i = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$.
  - The NMTS asks whether there exist two permutations
    - $(j_1, j_2, \dots, j_n)$ and
    - $(k_1, k_2, \dots, k_n)$

    of the indices $\{1, 2, \dots, n\}$, such that
    - $z_i = x_{j_i} + y_{k_i}$ for all $i = 1, 2, \dots, n$.

Data arrangement problem

$\mathcal{NP}$-hardness result for trees
proof sketch



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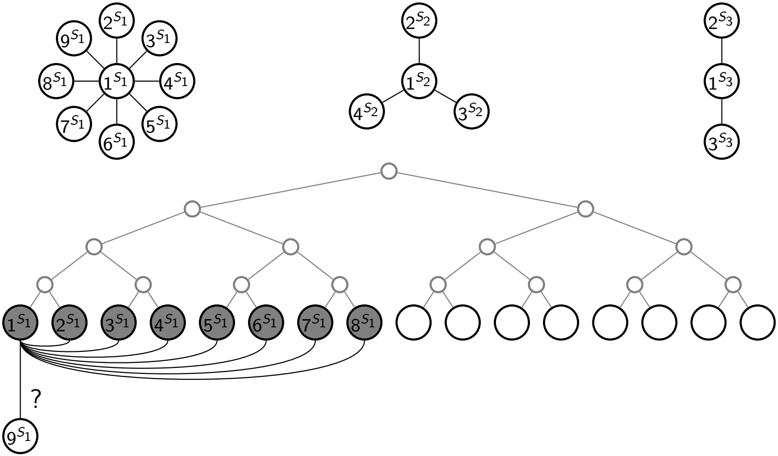

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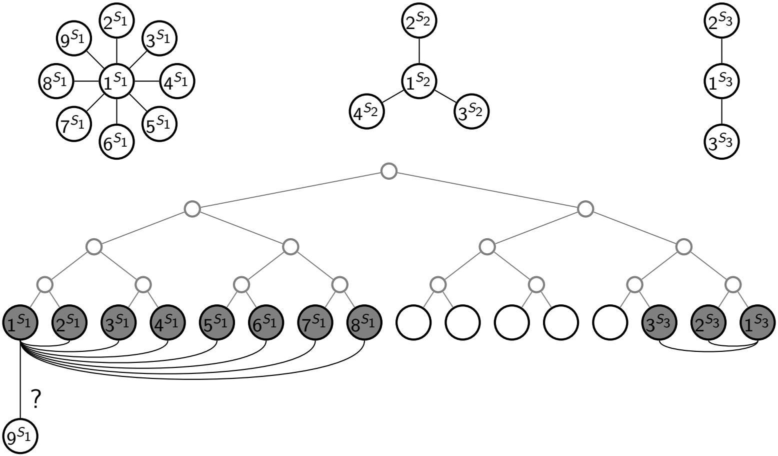

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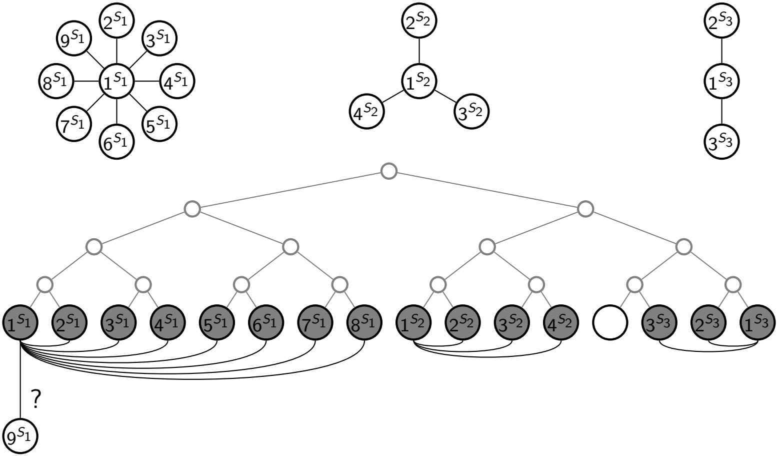

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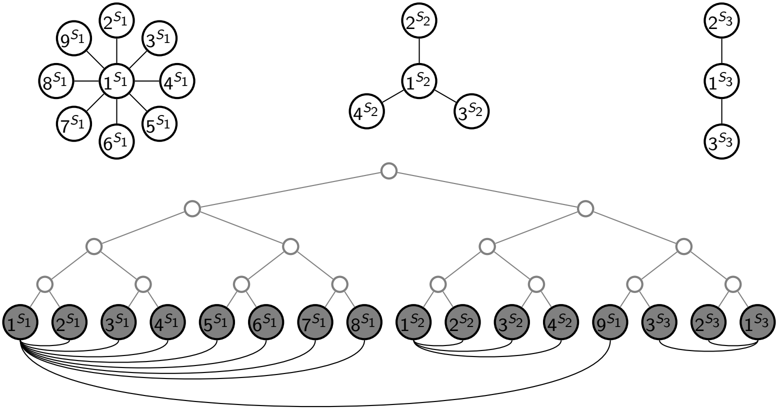

Recapitulation, future research and open questions

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Thank you for your attention!