Staffing and scheduling flexible call centers by two-stage robust optimization

Sara Mattia
IASI-CNR
sara.mattia@iasi.cnr.it

joint work with

Fabrizio Rossi, Stefano Smriglio
DISIM, Università di L'Aquila

Mara Servilio
IASI-CNR

this talk

robust shift scheduling in call centers

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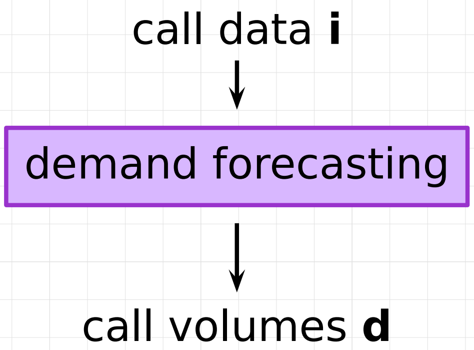

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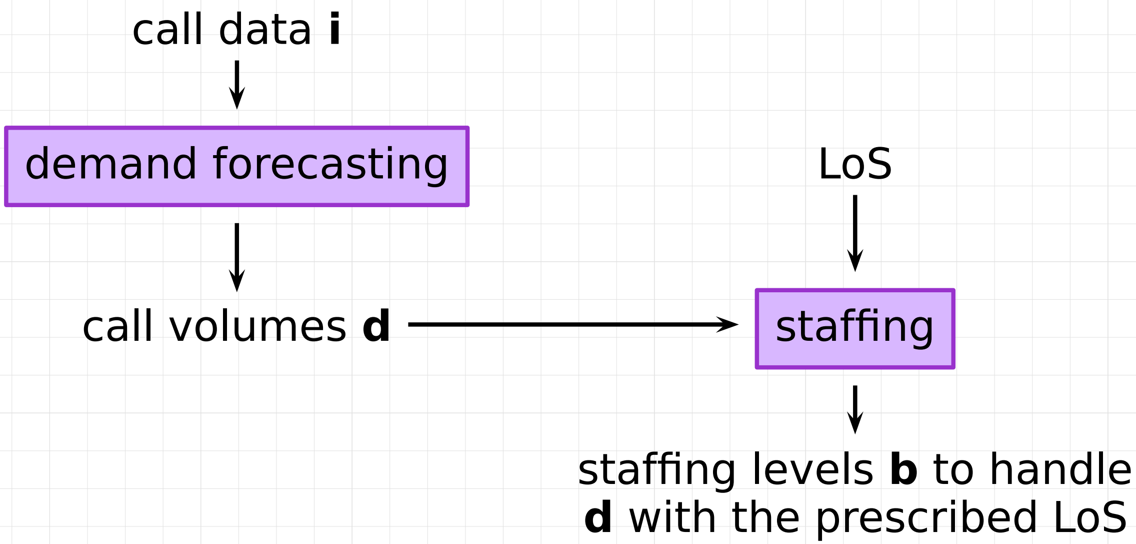

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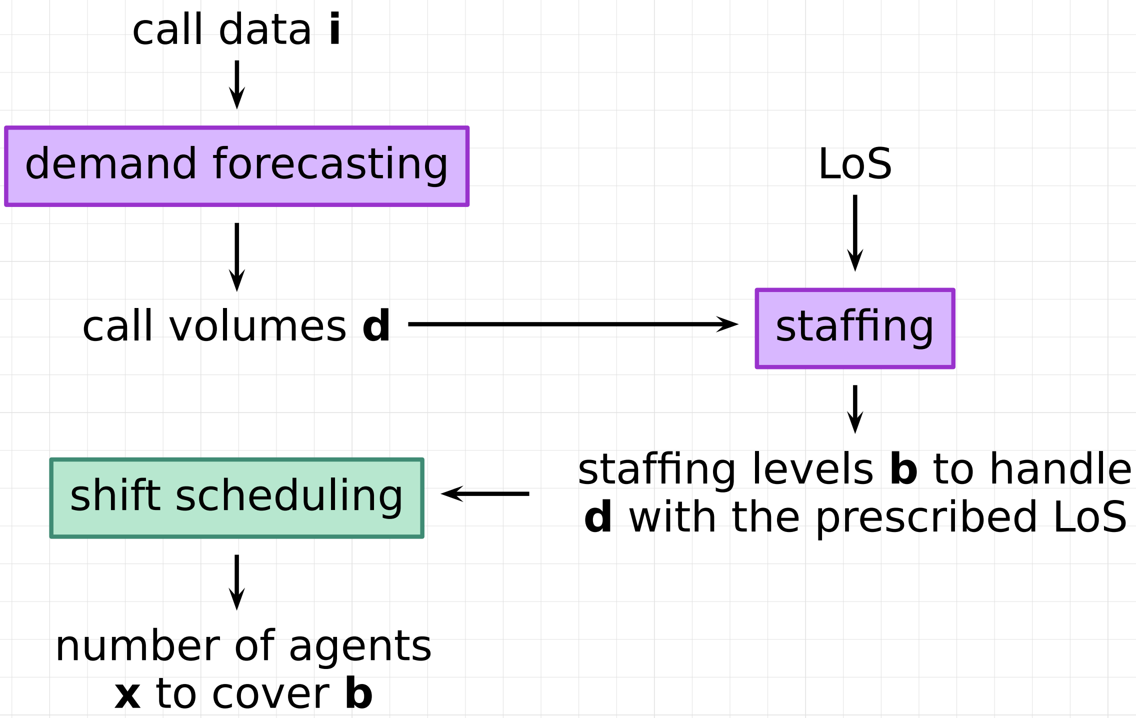

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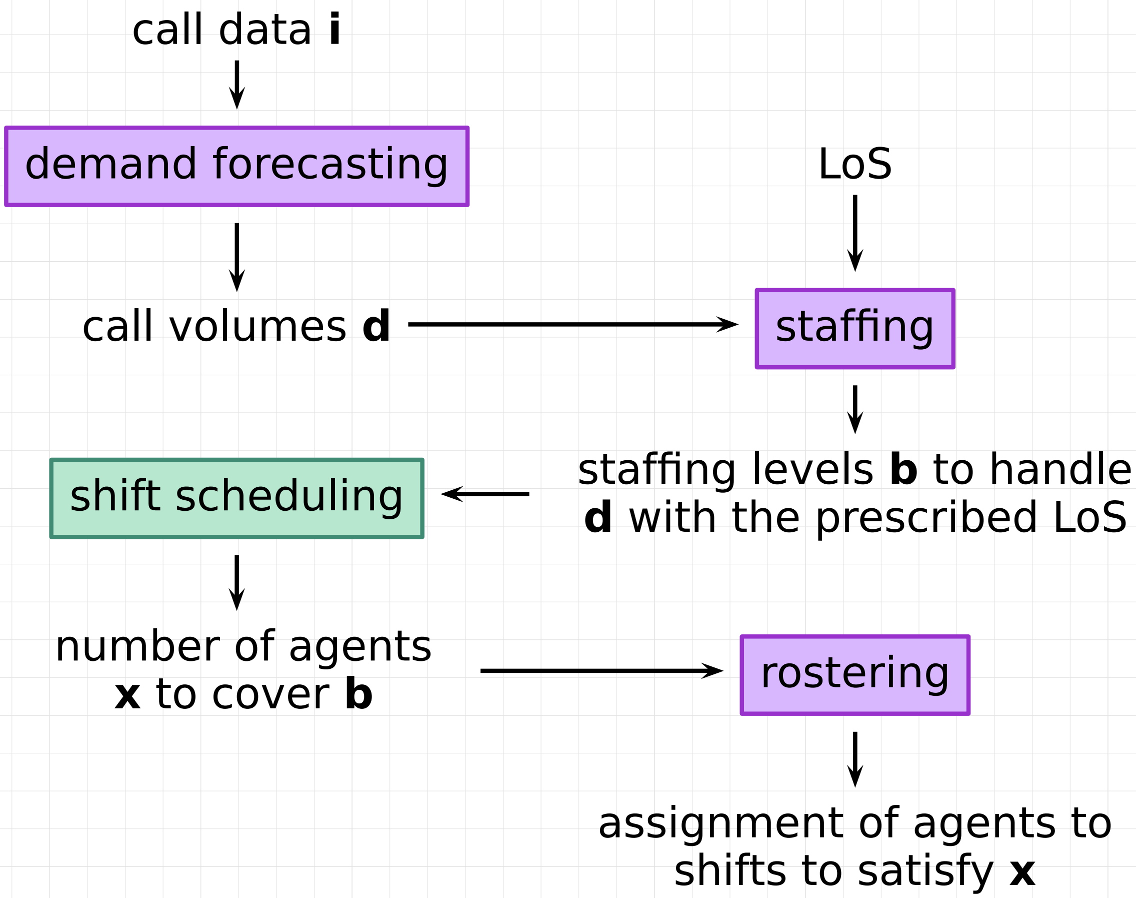

input

- time horizon T
- staffing levels b_t, $t \in T$
- shifts J

shift scheduling problem

choose the number of employees to be assigned to the shifts to cover the demand at minimum cost

input

- time horizon T
- staffing levels b_t, $t \in T$
- shifts J, with costs $\mathbf{c}$
- $A \in \{0,1\}^{|T| \times |J|}$, $a_{tj} = 1$ if j covers t, zero otherwise

classic model [Segal 1974]

$$\min \sum_{j \in J} c_j x_j$$

$$\sum_{j \in J} a_{tj} x_j \geq b_t \quad t \in T$$

$$\mathbf{x} \in \mathbb{Z}_+^n$$

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advantages

- well-studied model
- network structure (C1P)

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assumptions

- no understaffing and overstaffing costs
- **b** perfectly matches the real demand

flexible model

$$\min \sum_{j \in J} c_j x_j + \sum_{t \in T} (w_t^o o_t + w_t^u u_t)$$

$$\sum_{j \in J} a_{tj} x_j + u_t - o_t = b_t \qquad t \in T$$

$$\mathbf{x} \in \mathbb{Z}_+^{|J|}, \mathbf{u}, \mathbf{o} \in \mathbb{Z}_+^{|T|}$$

- u_t, o_t under and overstaffing in $t \in T$, with costs w_t^u and w_t^o

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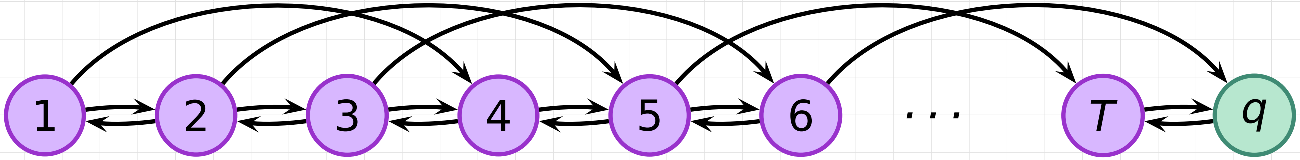

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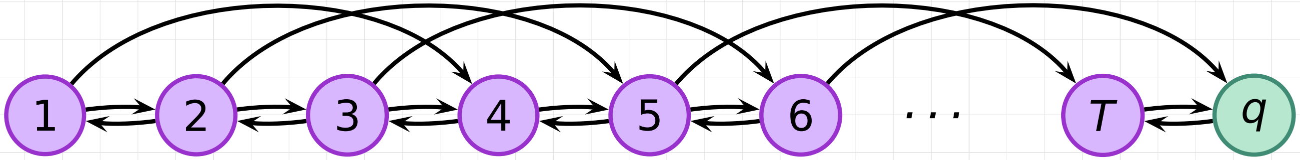

- $o_t u_t = 0$, $t \in T$ for any optimal solution

staffing levels and real demands

computed staffing levels may differ from the real demands

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how variations from real demands are handled

- simulation-based cutting plane methods [Atlason et al. 2008, Avramidis et al. 2004]
- stochastic programming [Bodur and Luedtke 2016, Liao et al. 2012, Thomas and Harrison 2010]
- managers react to front-end understaffing by moving people from/to the back-office

this talk

- an exact method matching the managers practice

this talk

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- an exact method matching the managers practice $\Rightarrow$ two-stage optimization
- **b** uncertain

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- **b** uncertain ⇒ merge staffing aspects and shift scheduling
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  - correlation

    deviations at consecutive time periods are often not independent

shift scheduling problem

$$\min \sum_{j \in J} c_j x_j + \sum_{t \in T} (w_t^o o_t + w_t^u u_t)$$

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robust optimization

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robust optimization

- single stage
  - single stage with RHS only uncertainty is easy [Minoux 2008]
  - no single stage with equality constraints

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robust optimization

- single stage
- two-stage
  - two stage with RHS only uncertainty is difficult [Minoux 2011]
  - branch-and-cut

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robust optimization

- single stage
- two-stage
- affinely adjustable
  - no second stage full flexibility
  - compact reformulation if U computationally tractable

robust problem

$$\min_{\mathbf{x}\in\mathbb{Z}_+^{|J|}} \left\{ \sum_{j\in J} c_j x_j + \max_{\mathbf{b}\in U} \begin{array}{ll} \min \sum_{t\in T}(w_t^o o_+ w_t^u u_t) & \\ u_t - o_t = b_t - \sum_{j\in J} a_{tj} x_j & t \in T \\ \mathbf{u}, \mathbf{o} \in \mathbb{R}^{|T|} & \end{array} \right\}$$

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personnel reallocation (PR) cost

$$\min \sum_{t\in T}(w_t^o o_t + w_t^u u_t)$$

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$$\mathbf{u}, \mathbf{o} \in \mathbb{R}^{|T|}$$

$$\max \sum_{t\in T}(\bar{b}_t - \sum_{j\in J} a_{jt}\bar{x}_j) y_t$$

$$-w_t^o \leq y_t \leq w_t^u \quad t \in T$$

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Benders-like reformulation

$$\min_{\mathbf{x}\in\mathbb{Z}_+^{|J|}} \left\{ \sum_{j\in J} c_j x_j + \max_{\mathbf{b}\in U, -w_t^o \le y_t \le w_t^u,\, t\in T} \sum_{t\in T}(b_t - \sum_{j\in J} a_{jt} x_j) y_t \right\}$$

Benders-like reformulation

$$\min \sum_{j \in J} c_j x_j + \lambda$$

$$\lambda \geq \sum_{t \in T} (b_t - \sum_{j \in J} a_{jt} x_j) y_t \quad \mathbf{b} \in U, \mathbf{y} \in dual(PR)$$

$$\mathbf{x} \in \mathbb{Z}_+^{|J|}$$

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the separation problem

$$\max_{\mathbf{b} \in U} \quad \min \sum_{t \in T} (w_t^o o_t + w_t^u u_t)$$

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$$\mathbf{b} \in U$$

the uncertainty set

- cardinality

  deviations typically occur only in a limited number of time periods

- correlation

  deviations at consecutive time periods are often not independent

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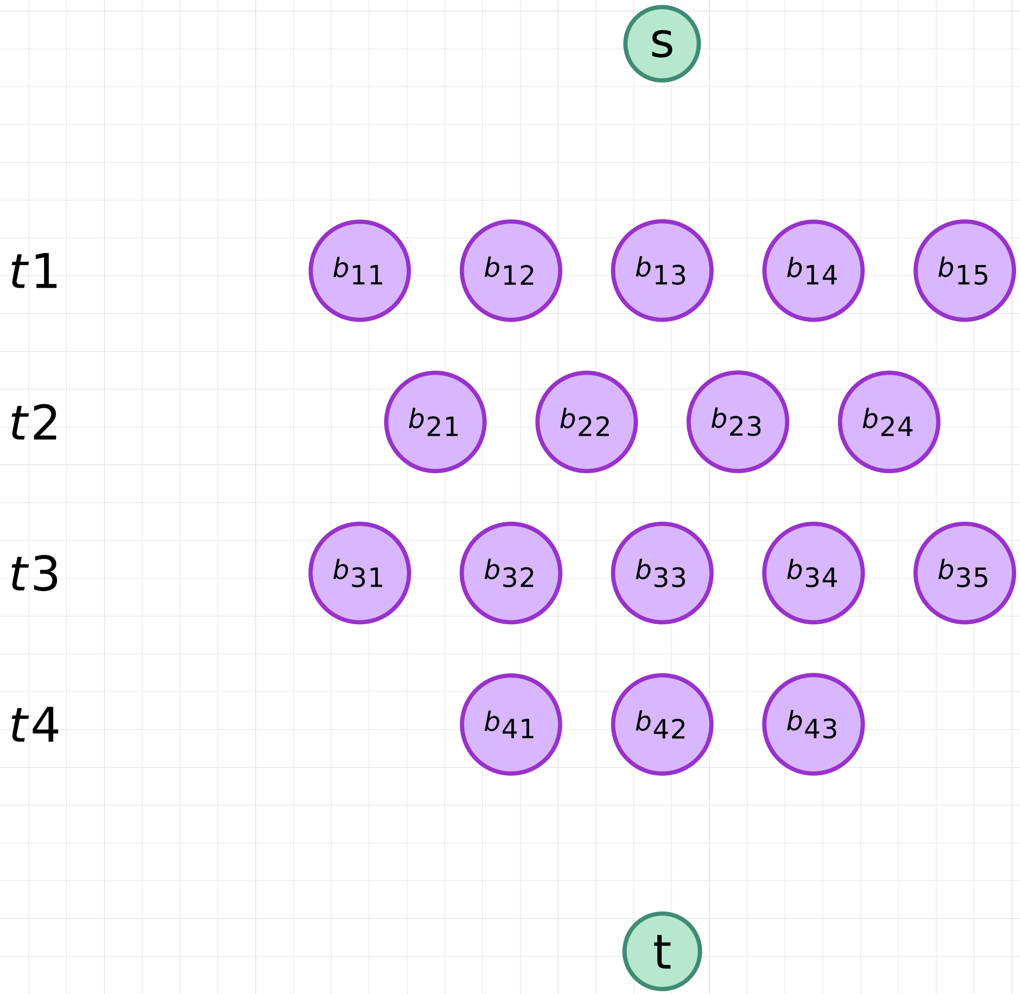
separation by (single) resource constrained longest path



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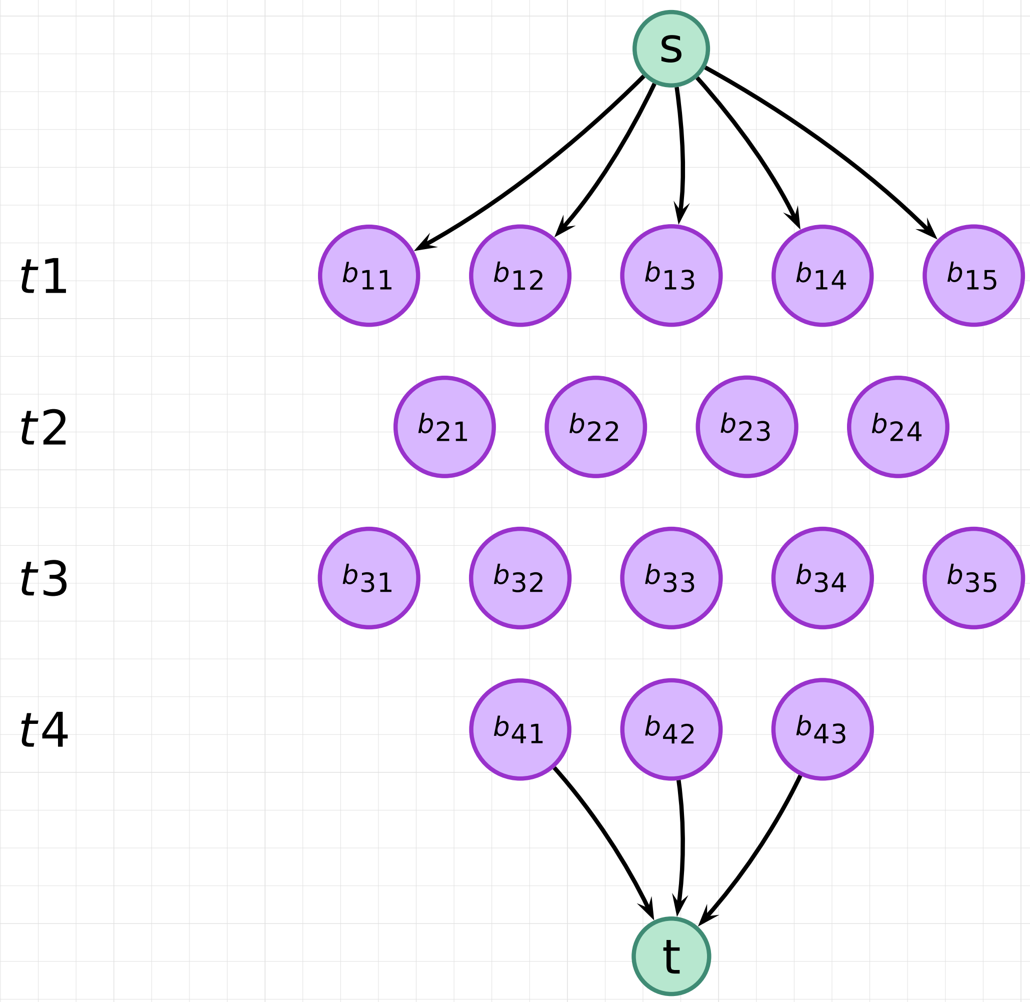

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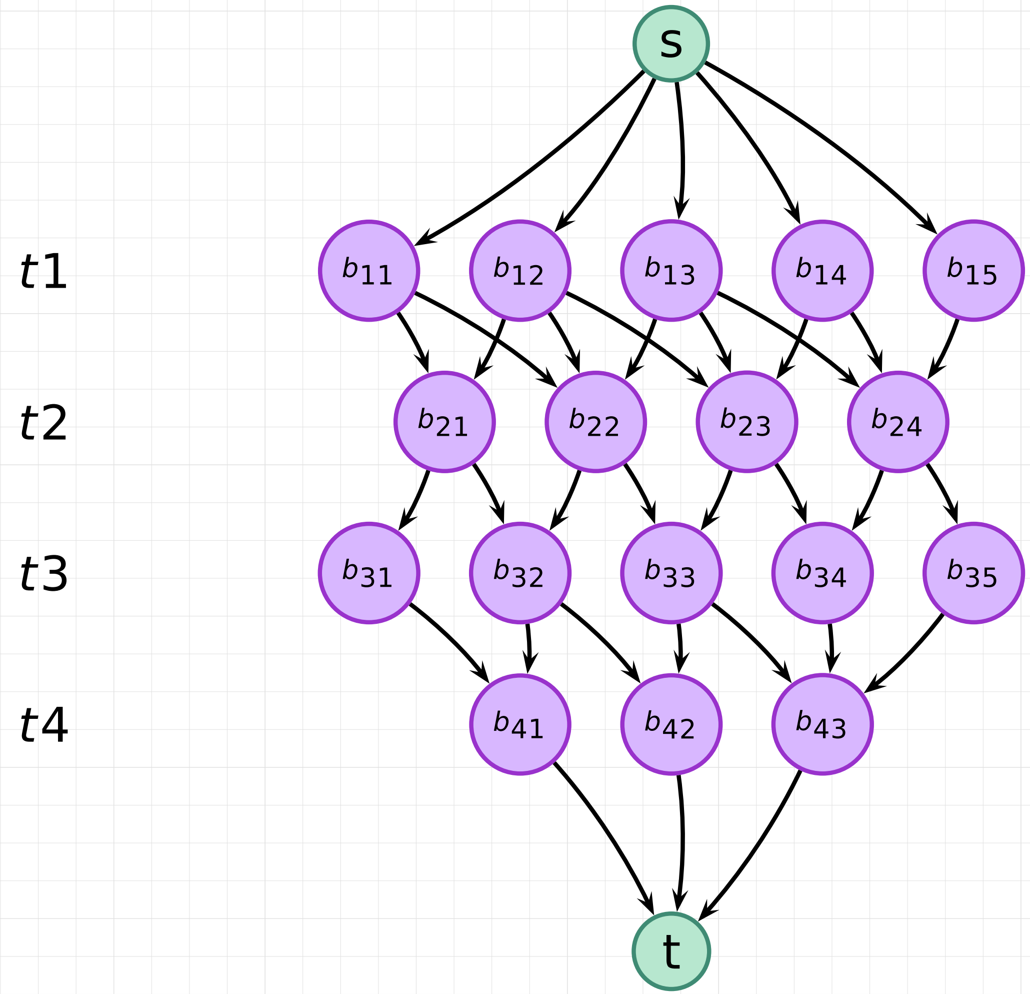

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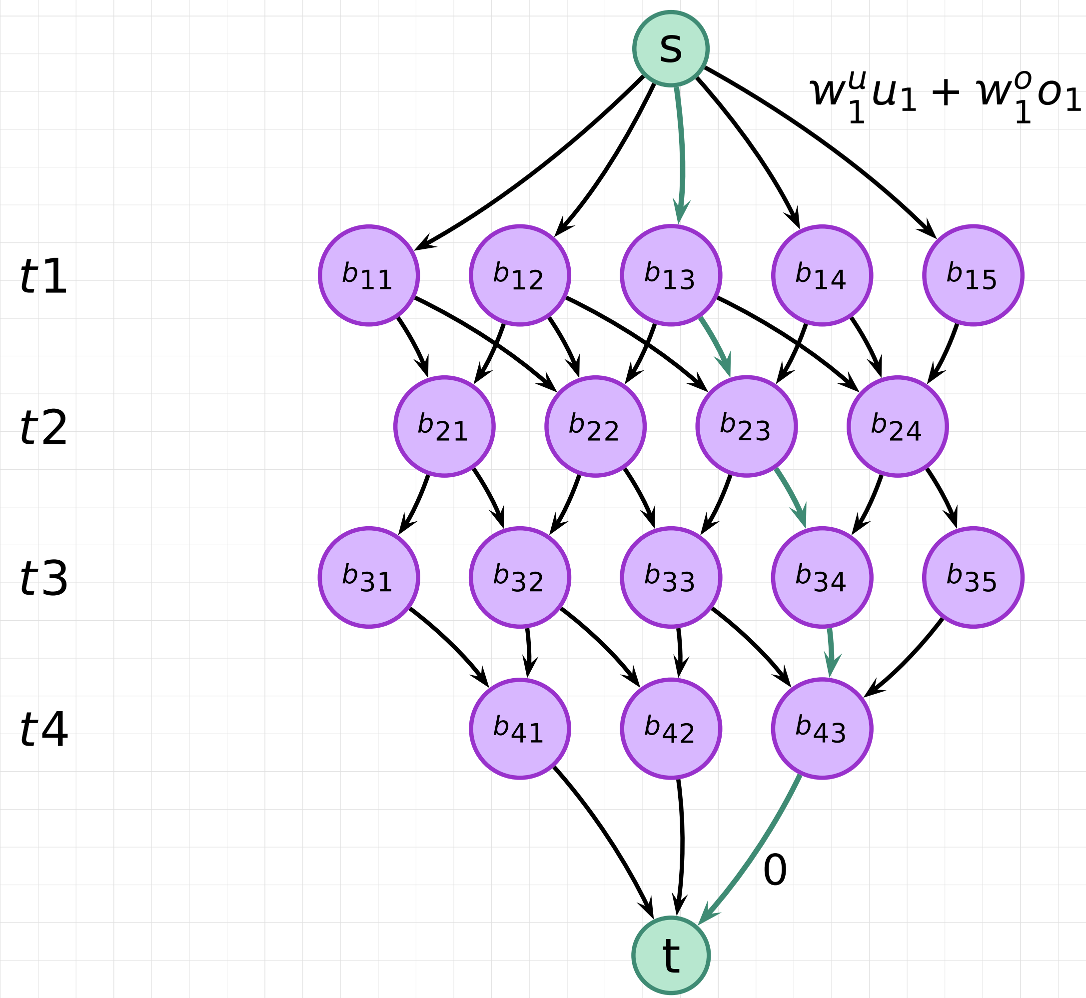

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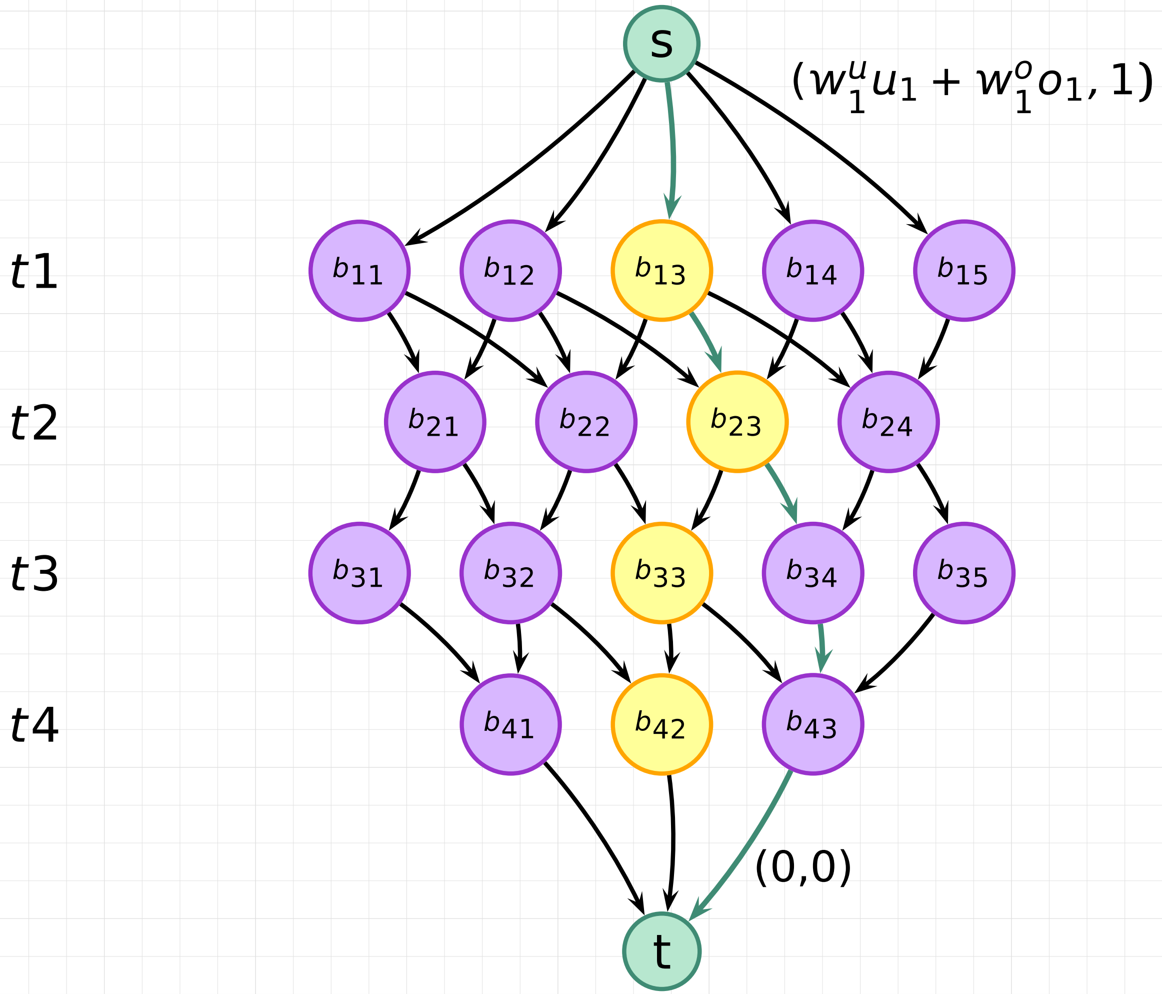

the MIP separation problem

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MIP vs RCSP

- MIP vs dynamic programming
- $O(|T|)$ right-hand-sides vs $O(\bar{D}^2|T|)$ edge costs to update
- MIP solver can be stopped before optimality

istances

- call center of an Italian Public Agency (2008)

istances

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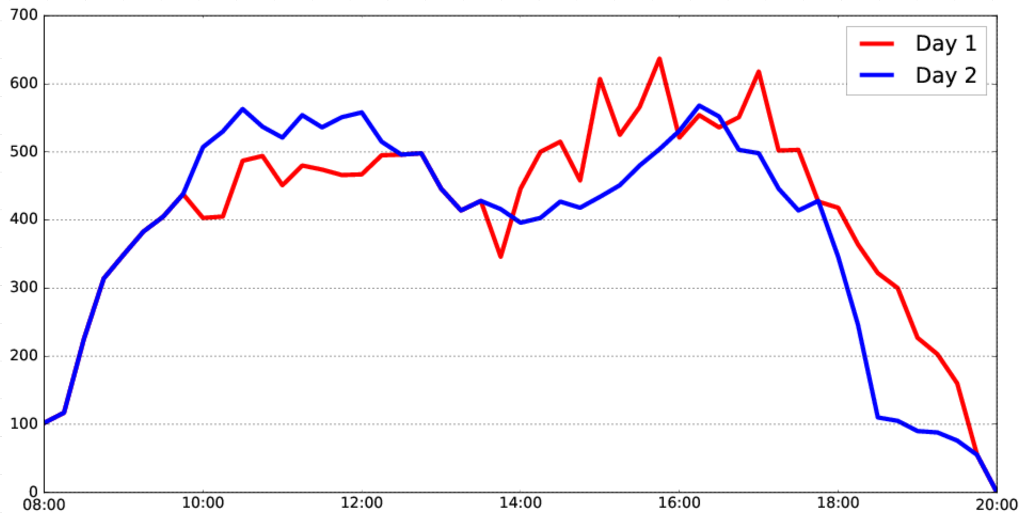
- 49 time slots of 15 minutes, 4-,6- and 8-hour shifts

istances

- call center of an Italian Public Agency (2008)
- 49 time slots of 15 minutes, 4-,6- and 8-hour shifts
- two sampling days
  - Day1: working day
  - Day2: day before a holiday



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- $\Gamma \Rightarrow$ cardinality
- $\Delta \Rightarrow$ correlation
- $D \Rightarrow$ confidence in staffing values

istances

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$$

- six Γ values $\Rightarrow$ cardinality
- two Δ values $\Rightarrow$ correlation
- three D values $\Rightarrow$ confidence

branch-and-cut statistics

	max	avg	min
time	1420	191	6
nodes	813	132	1

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gap at root	0.57%	0.12%	0.02%

branch-and-cut statistics

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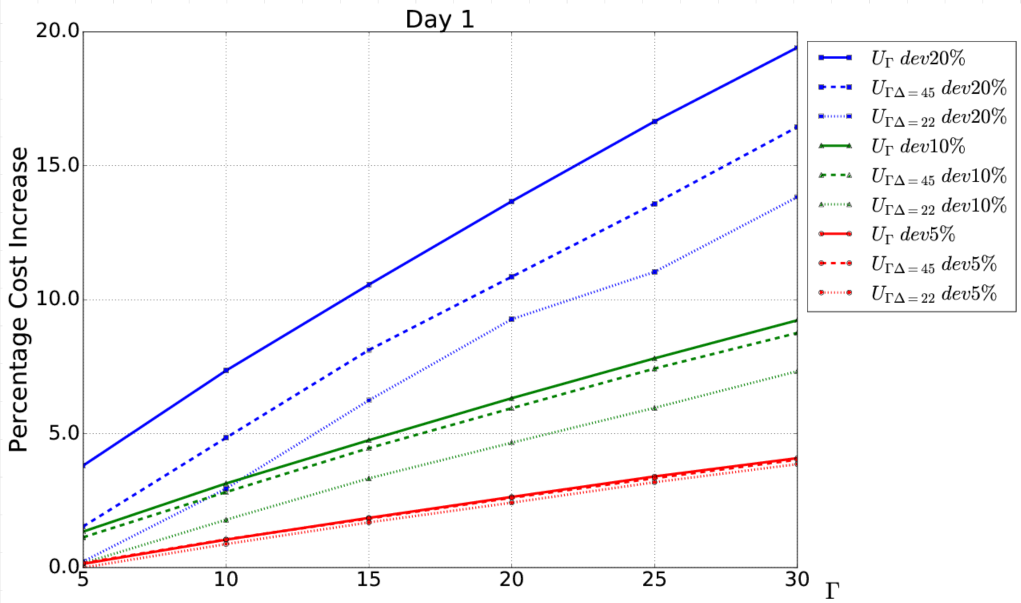
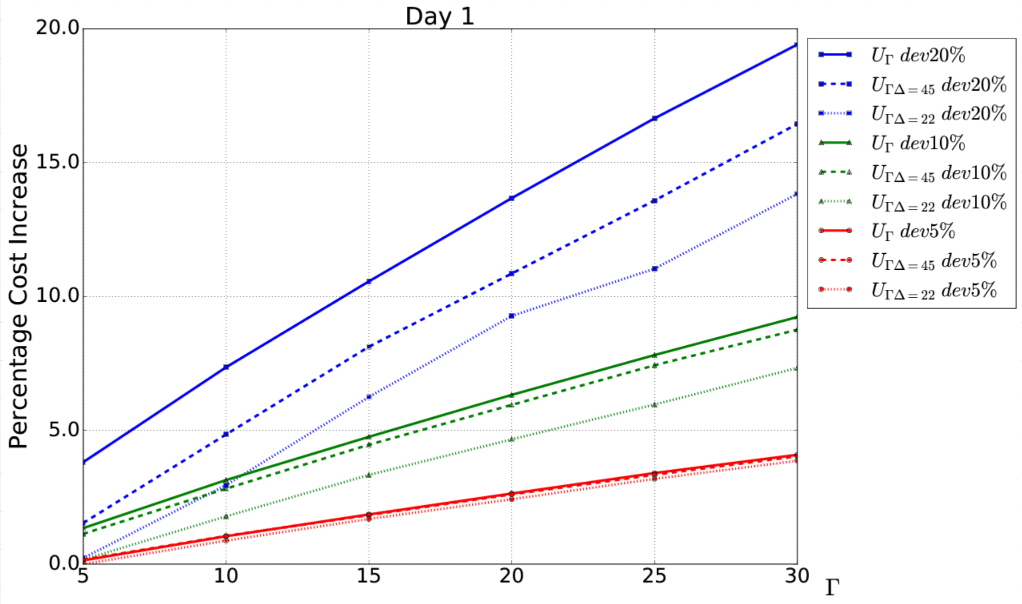
time	1420	191	6
nodes	813	132	1
gap at root	0.57%	0.12%	0.02%
sep time	1400	185	5
cuts	199	128	89

correlation vs SIPP model



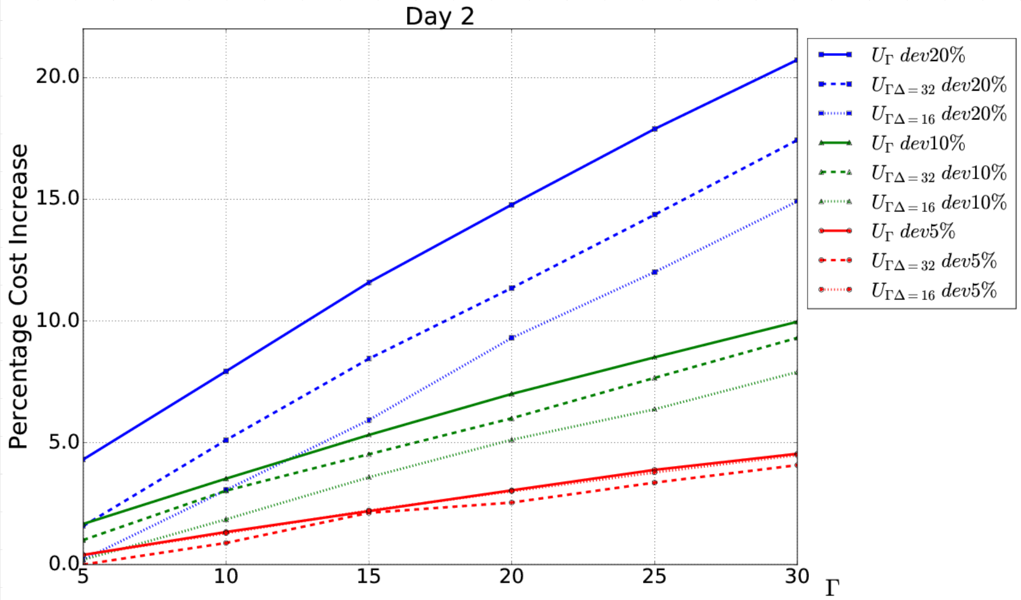
- solid lines ⇒ only cardinality
  dotted and dashed lines ⇒ cardinality plus correlation
- different line styles ⇒ different Δ values
- different colors ⇒ different D values

correlation vs SIPP model



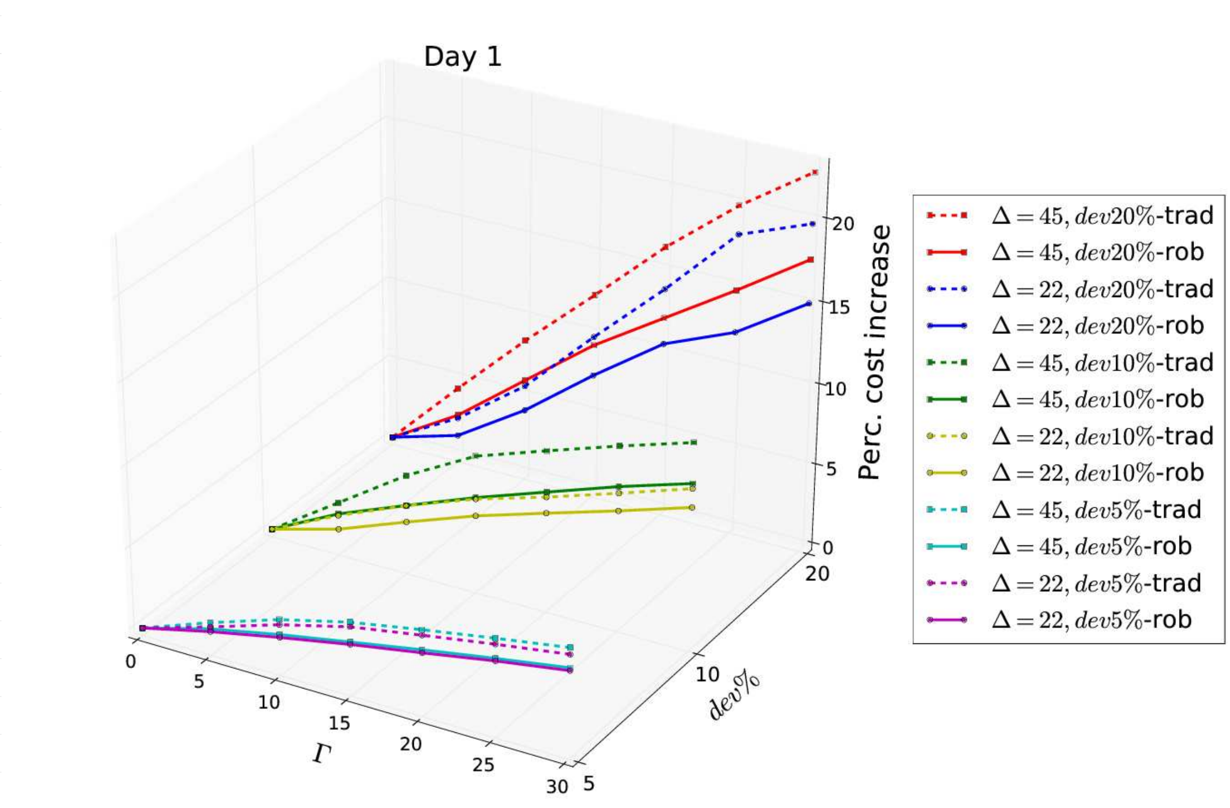
- no correlation ⇒ larger costs
- maximum difference is about 5% (5000 €, per day)

correlation vs SIPP model



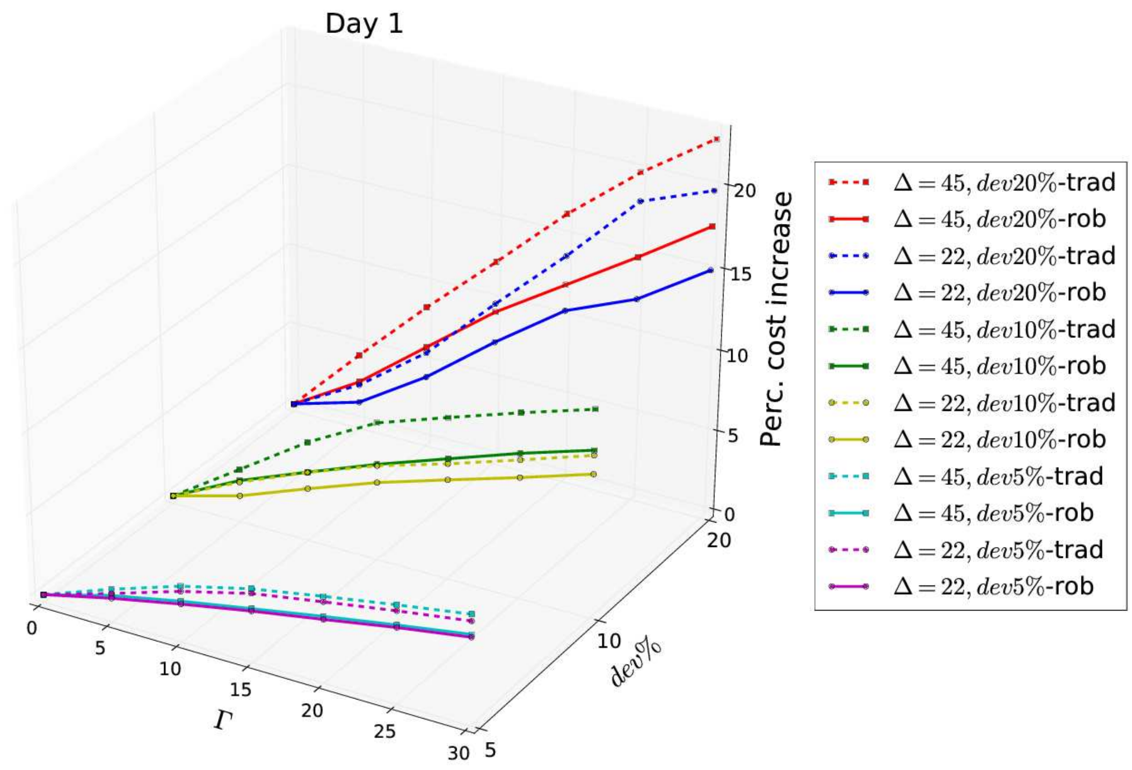
- no correlation ⇒ larger costs
- maximum difference is about 5% (5000 €, per day)

optimization vs managers: costs



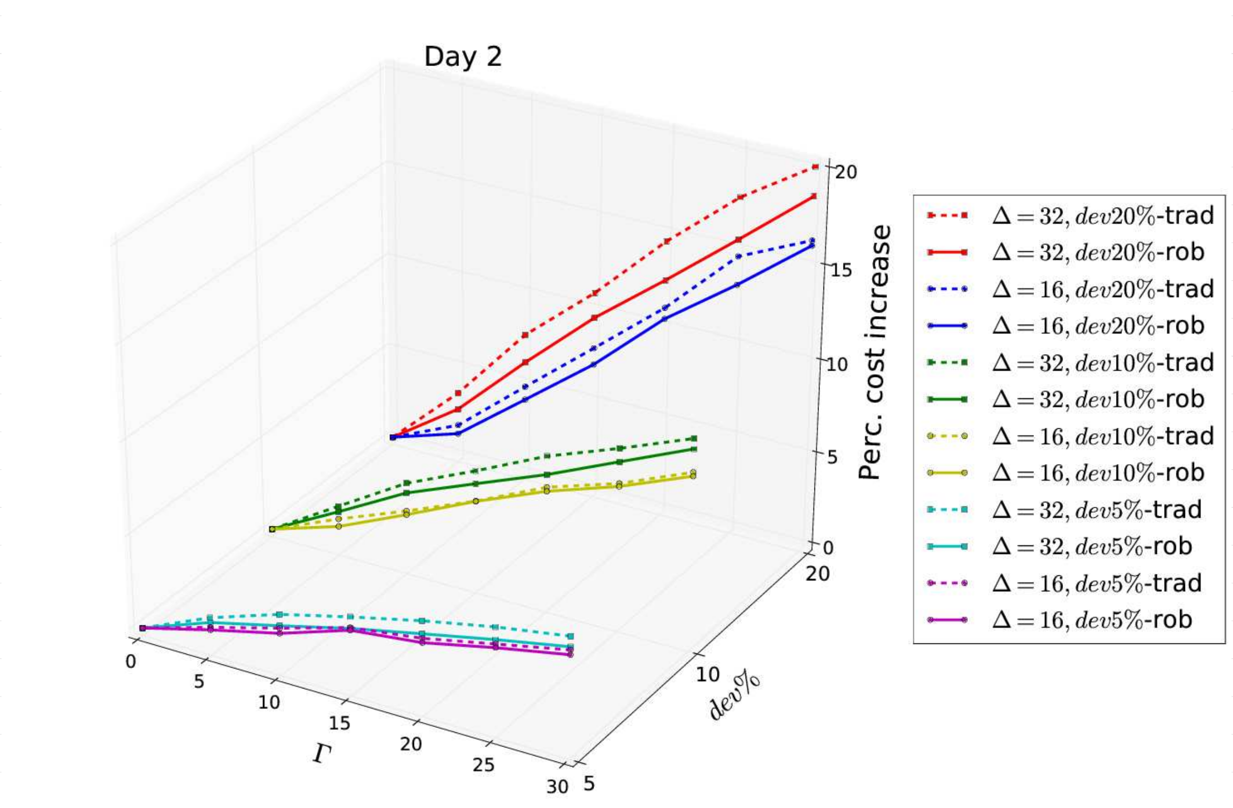
- solid lines ⇒ optimization
  dotted lines ⇒ managers
- same group ⇒ same D
  same color ⇒ same Δ

optimization vs managers: costs



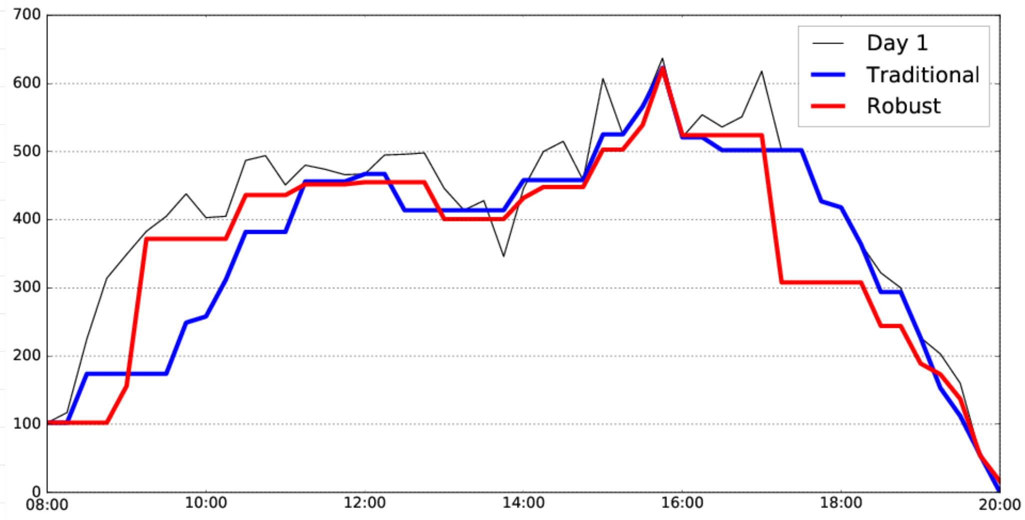
- optimization ⇒ smaller costs
- maximum difference is about 5% (5000 €, per day)

optimization vs managers: costs



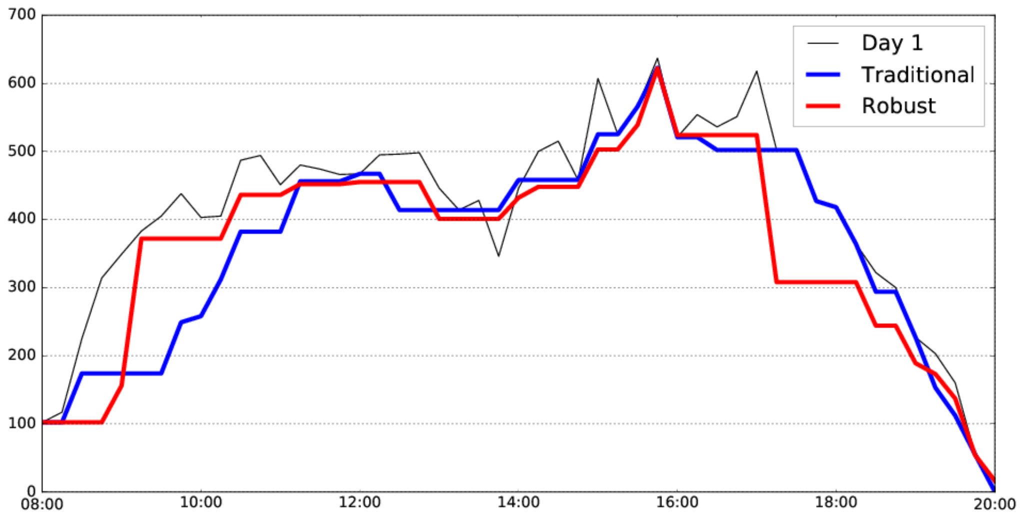
- optimization ⇒ smaller costs
- costs are closer (2% of difference)

optimization vs managers: staffing



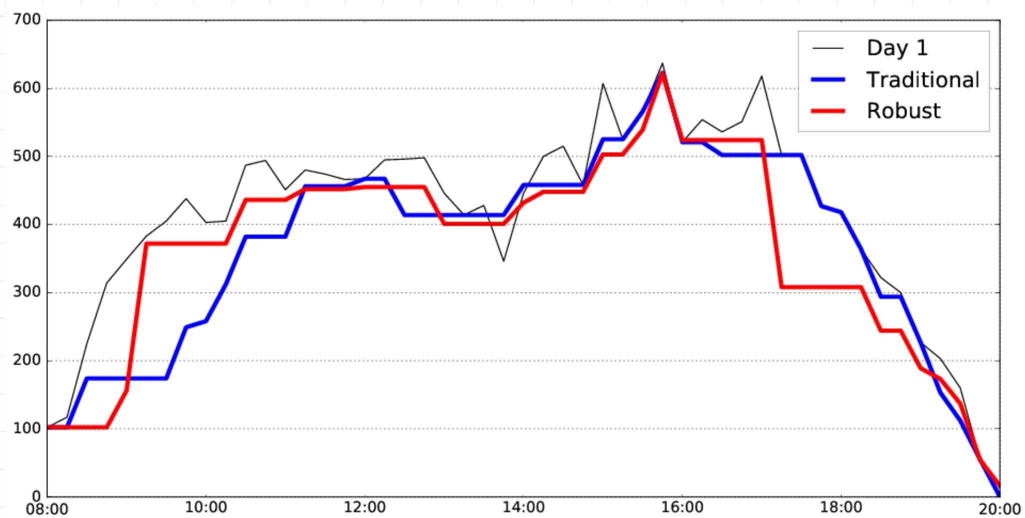
- black line ⇒ demand **b**
- blue line ⇒ staffing **x** according to managers practice
- red line ⇒ staffing **x** computed by optimization

optimization vs managers: staffing



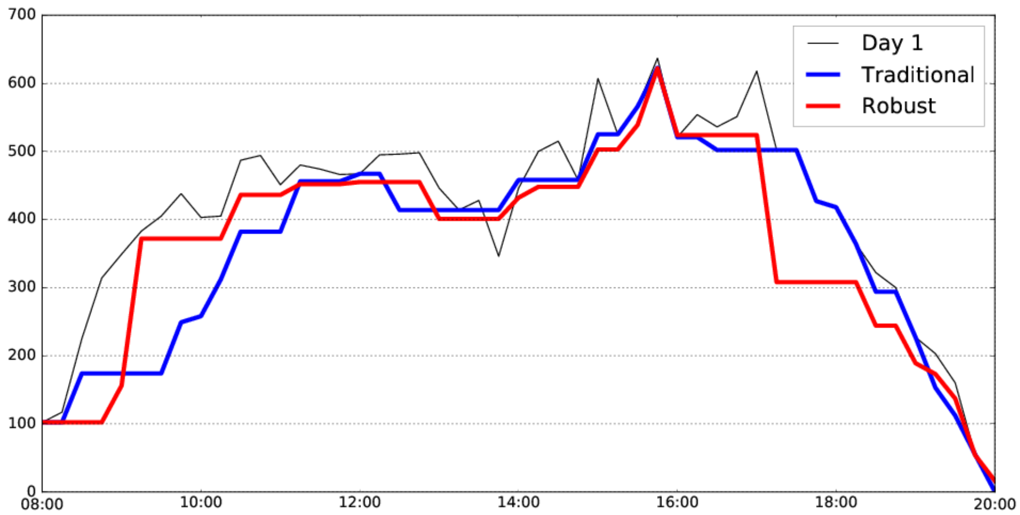
- many **x** with similar cost, but different robustness

optimization vs managers: staffing



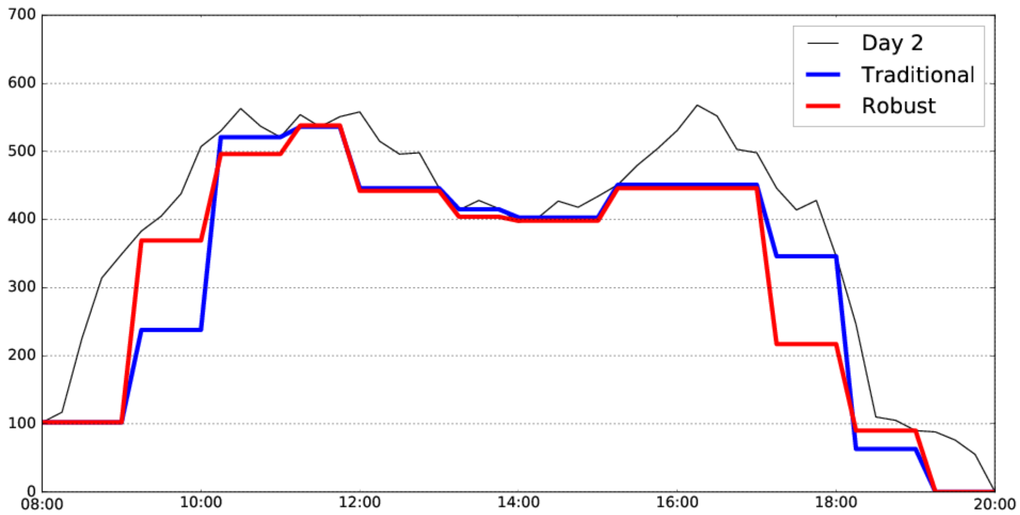
- many **x** with similar cost, but different robustness
- both methods prefer understaffing vs overstaffing

optimization vs managers: staffing



- many **x** with similar cost, but different robustness
- both methods prefer understaffing vs overstaffing
- managers follow the demand variation
- optimization anticipates the demand variation

optimization vs managers: staffing



- many **x** with similar cost, but different robustness
- both methods prefer understaffing vs overstaffing
- managers follow the demand variation
- optimization anticipates the demand

paper

S. Mattia, F. Rossi, M. Servilio, S. Smriglio.
Staffing and scheduling flexible call centers by two-stage robust optimization. Omega, in press.
`http://dx.doi.org/10.1016/j.omega.2016.11.001`

thank you