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Optimal Price Zones of Electricity Markets

A Mixed-Integer Multilevel Model and Global Solution Approaches

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WIRTSCHAFTS
MATHEMATIK

Outline

Motivation

A Mixed-Integer Multilevel Model

Solution Approaches

Computational Results

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Motivation

A Mixed-Integer Multilevel Model

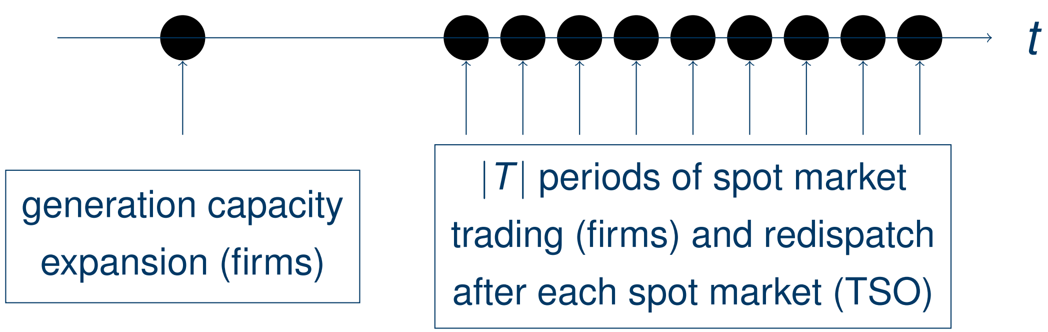
Solution Approaches

Computational Results

Liberalized Electricity Markets

Timing

1. Generation capacity investment by profit-maximizing firms
2. Spot-market trading
   - Energy-only market: no network considered
   - Sole requirement: market clearing
3. Cost-based redispatch (if required)



Cost-Based Redispatch

- Technically infeasible spot-market results $\rightarrow$ redispatch
- Modification of traded quantities
  - Redispatched electricity can be transported
  - Objective: minimum redispatch cost

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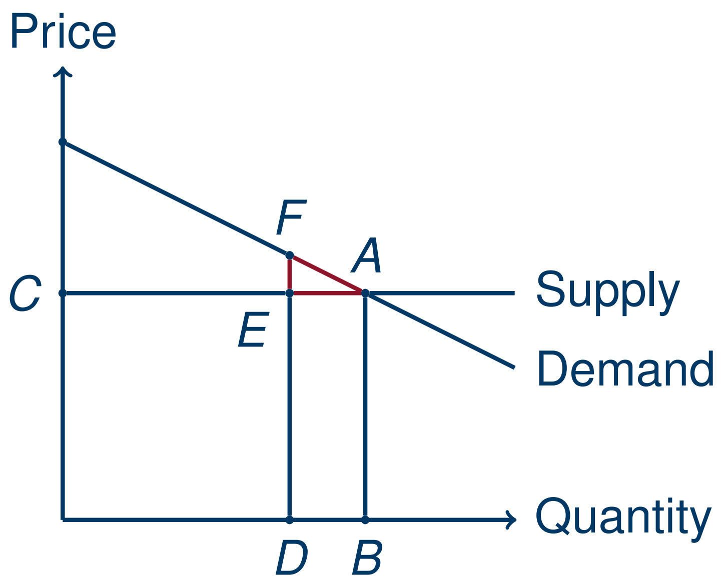


- Energy-only market:
  equilibrium quantity *B*
  equilibrium price *C*
- Transmission constraints:
  transportable capacity *D*
- Producer pays to TSO: *ABDE*
- TSO pays to consumer: *ABDF*
- TSO's cost: *AEF*

Zonal Pricing

- Implemented in parts of Europe, Australia, or Latin America

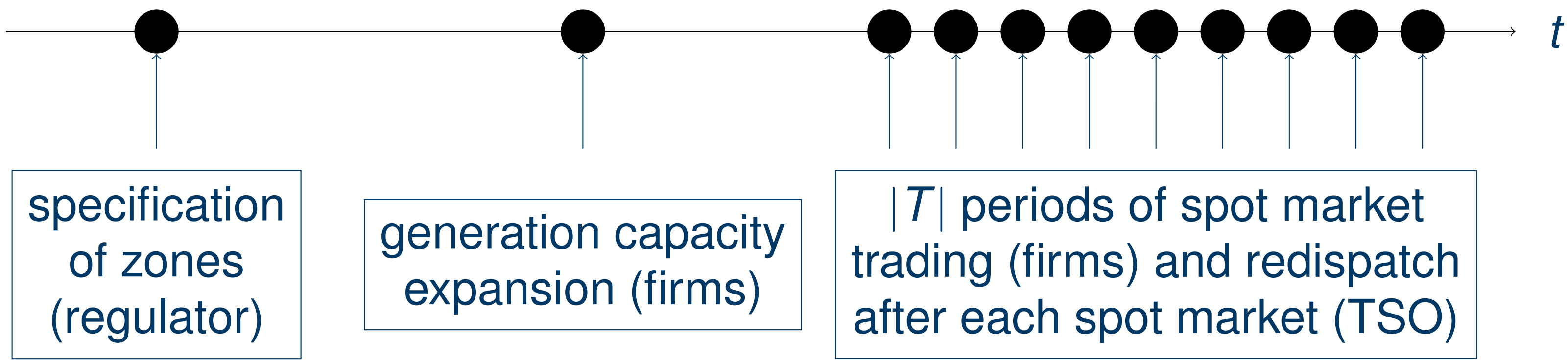
- Market area is divided into price zones
- Intra-zonal network constraints: ignored at the spot market
- Inter-zonal network constraints: (partly) respected at the spot market
- Bad zoning: distorted investment incentives for generation capacity leading to inefficiencies
- Good zoning: congestion issues are reflected (most appropriately) in spot-market trading
- Goal of the regulator: optimal configuration of price zones
    - Maximization of resulting social welfare

Outline

Some Notation

- Transmission network: directed graph $\mathcal{G} = (N, L)$
- Scenarios/time periods: $T = \{t_1, \dots, t_{|T|}\}$
- Node set: $n \in N$
  - Consumers $c \in C_n$ with demand $d_{t,c} \geq 0$
  - Elastic demand modeled by continuous and strictly decreasing function $p_{t,c}(d_{t,c})$
  - Generators g with production $q_{t,g} \in [0, \bar{q}_g]$
  - Some producers may invest in generation capacity $\bar{q}_g$
- Arc set: $l \in L$
  - Transmission lines with capacity $\bar{f}_l$
  - Lossless DC power flow model
- Price zones Z_i: parts of a partition $N = Z_1 \cup \dots \cup Z_k$
  - k is given as input

Trilevel Market Model: Timing



Trilevel Market Model: Model Structure

max social welfare (regulator)
s.t. graph partitioning with connectivity constraints

> max profits (competitive firms)
> s.t. generation capacity investment,
> production & demand constraints,
> Kirchhoff's 1st law (inter-zonal),
> flow restrictions (inter-zonal)
>
> > min redispatch costs (TSO)
> > s.t. production & demand constraints,
> > lossless DC power flow constraints

1st Level: Specification of Price Zones

Maximization of total social welfare

$$\psi_1 := \sum_{t\in T}\sum_{n\in N}\sum_{c\in C_n}\int_0^{d_{t,c}^{\text{red}}} p_{t,c}(\omega)\,\mathrm{d}\omega - \sum_{n\in N}\left(\sum_{g\in G_n^{\text{new}}} c_g^{\text{inv}}\bar{q}_g^{\text{new}} + \sum_{t\in T}\sum_{g\in G_n^{\text{all}}} c_g^{\text{var}} q_{t,g}^{\text{red}}\right)$$

subject to graph partitioning with multi-commodity flow connectivity constraints

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subject to graph partitioning with multi-commodity flow connectivity constraints

$$\begin{aligned}
\sum_{i\in[k]} x_{n,i} &= 1 && n \in N\\
\sum_{n\in N} z_{n,i} &= 1 && i \in [k]\\
z_{n,i} &\le x_{n,i} && n \in N,\ i \in [k]\\
\sum_{a\in\delta_n^{\text{out}}} m_a^i &\le M x_{n,i} && n \in N,\ i \in [k]\\
\sum_{a\in\delta_n^{\text{out}}} m_a^i - \sum_{a\in\delta_n^{\text{in}}} m_a^i &\ge x_{n,i} - M z_{n,i} && n \in N,\ i \in [k]
\end{aligned}$$

2nd Level: Capacity Investment & Spot Market

Economic Assumption: Perfect Competition

- No market power; otherwise multiple equilibria (Zöttl, 2010)
- Mathematically “necessary” assumption
- Commonly used in electricity market literature:
  Boucher, Smeers (2001), Daxhelet, Smeers (2007),
  Grimm, Martin, S., Weibelzahl, Zöttl (2016)

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Objective

Profit (= total social welfare) maximization

$$\psi_2 := \sum_{t \in T} \sum_{n \in N} \sum_{c \in C_n} \int_0^{d_{t,c}^{\text{spot}}} p_{t,c}(\omega)\,\mathrm{d}\omega - \sum_{n \in N} \left(\sum_{g \in G_n^{\text{new}}} c_g^{\text{inv}} \bar{q}_g^{\text{new}} + \sum_{t \in T} \sum_{g \in G_n^{\text{all}}} c_g^{\text{var}} q_{t,g}^{\text{spot}} \right)$$

2nd Level: Capacity Investment & Spot Market

Zonal version of Kirchhoff's first law

$$d_{t,n}^{\text{spot}} = \sum_{c \in C_n} d_{t,c}^{\text{spot}}, \quad q_{t,n}^{\text{spot}} = \sum_{g \in G_n^{\text{all}}} q_{t,g}^{\text{spot}} \qquad n \in N, \ t \in T$$

$$D_t^i = \sum_{n \in N} x_{n,i} d_{t,n}^{\text{spot}}, \quad Q_t^i = \sum_{n \in N} x_{n,i} q_{t,n}^{\text{spot}} \qquad i \in [k], \ t \in T$$

$$F_{i,t}^{\text{in}} = \sum_{l=(n,m) \in L} (1 - x_{n,i}) x_{m,i} f_{t,l}^{\text{spot}} \qquad i \in [k], \ t \in T$$

$$F_{i,t}^{\text{out}} = \sum_{l=(n,m) \in L} x_{n,i} (1 - x_{m,i}) f_{t,l}^{\text{spot}} \qquad i \in [k], \ t \in T$$

$$D_t^i + F_{i,t}^{\text{out}} = Q_t^i + F_{i,t}^{\text{in}} \qquad i \in [k], \ t \in T$$

Nonlinearities can be linearized

2nd Level: Capacity Investment & Spot Market

Flow restrictions on inter-zonal lines

$$-\bar{f}_l - (1 - y_l)M \le f^{\text{spot}}_{t,l} \le \bar{f}_l + (1 - y_l)M \qquad l \in L,\ t \in T$$

Demand and production bounds

$$0 \le d^{\text{spot}}_{t,c} \qquad t \in T,\ n \in N,\ c \in C_n$$

$$0 \le q^{\text{spot}}_{t,g} \le \tau \bar{q}^{\text{new}}_g \qquad t \in T,\ n \in N,\ g \in G^{\text{new}}_n$$

$$0 \le q^{\text{spot}}_{t,g} \le \tau \bar{q}^{\text{ex}}_g \qquad t \in T,\ n \in N,\ g \in G^{\text{ex}}_n$$

3rd Level: Cost-Based Redispatch

Minimize redispatch costs

$$\psi_3 := \sum_{t \in T} \sum_{n \in N} \sum_{c \in C_n} \int_{d_{t,c}^{\text{red}}}^{d_{t,c}^{\text{spot}}} p_{t,c}(\omega) \, \mathrm{d}\omega + \sum_{t \in T} \sum_{n \in N} \sum_{g \in G_n^{\text{all}}} c_g^{\text{var}} (q_{t,g}^{\text{red}} - q_{t,g}^{\text{spot}})$$

subject to lossless DC power flow model:

- Kirchhoff's 1st law

$$\sum_{c \in C_n} d_{t,c}^{\text{red}} + \sum_{l \in \delta_n^{\text{out}}} f_{t,l}^{\text{red}} = \sum_{g \in G_n^{\text{all}}} q_{t,g}^{\text{red}} + \sum_{l \in \delta_n^{\text{in}}} f_{t,l}^{\text{red}}, \quad n \in N, \ t \in T$$

- Kirchhoff's 2nd law

$$f_{t,l}^{\text{red}} = B_l(\theta_{t,n} - \theta_{t,m}), \quad l = (n,m) \in L, \ t \in T$$

$$\theta_{t,\hat{n}} = 0, \quad t \in T$$

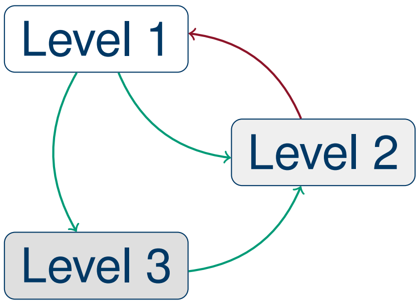
- Flow capacities

$$-\bar{f}_l \le f_{t,l}^{\text{red}} \le \bar{f}_l, \quad l \in L, \ t \in T$$

Model Discussion

1st Level	MIQP with graph partitioning and multi-commodity flow model
2nd Level	MIQP; no “genuine” 2nd level integers
3rd Level	QP with lossless DC power flow model

$$
\begin{aligned}
\max \quad & \psi_1(W_2, W_3) \\
\text{s.t.} \quad & (W_1, X_1) \in \Omega_1 \\
& \quad \max \quad \psi_2(W_2) \\
& \quad \text{s.t.} \quad (W_2, X_1) \in \Omega_2 \\
& \quad\quad \min \quad \psi_3(W_2, W_3) \\
& \quad\quad \text{s.t.} \quad (W_2, W_3) \in \Omega_3
\end{aligned}
$$



Outline

Approach #1: KKT Reformulation

Proposition

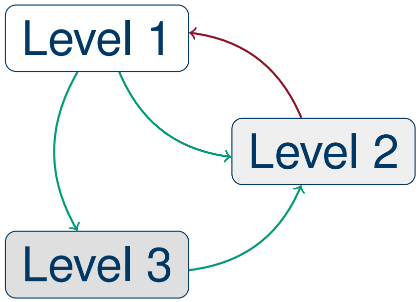
Let ψ_1, ψ_2, and ψ_3 be the objective functions of the trilevel market model. Then, $\psi_1 = \psi_2 - \psi_3$ holds.

- First used in Grimm, Martin, S., Weibelzahl, Zöttl (2016)
- Allows to reduce trilevel to bilevel problem

$$
\begin{aligned}
\max \quad & \psi_1(W_2, W_3) \\
\text{s.t.} \quad & (W_1, X_1) \in \Omega_1, \ (W_2, W_3) \in \Omega_3, \\
& W_2 \in \arg\max \{\psi_2(W_2) \colon (W_2, X_1) \in \Omega_2\} .
\end{aligned}
$$

- Lower level is a concave QP for fixed discrete first-level variables
- KKT reformulation + linearization of KKT complementarity conditions $\rightarrow$ single-level MIQP

Approach #2: Generalized Benders Decomposition



Model Decomposition

- Master problem
  - 1st level model
    price zone configuration
- Subproblem
  - 2nd level problem
    generation capacity &
    zonal spot market
  - 3rd level problem
    cost-based redispatch

Approach #2: Generalized Benders Decomposition

Master Problem

$$
\begin{aligned}
\max \quad & \tau \\
\text{s.t.} \quad & \tau \le a^\top x + b \quad \text{for all } (a, b) \in O, \\
& \text{graph partition with connectivity}
\end{aligned}
$$

Approach #2: Generalized Benders Decomposition

Master Problem

$$\begin{aligned} \max \quad & \tau \\ \text{s.t.} \quad & \tau \le a^\top x + b \quad \text{for all } (a, b) \in O, \\ & \text{graph partition with connectivity} \end{aligned}$$

Optimality cuts

$$\tau \le \psi_2(\hat{x}) - \psi_3(\hat{x}) + \psi^*_{\text{IGTC}} \left(\sum_{i \in [k]} \sum_{n \in N : \hat{x}_{n,i} = 0} x_{n,i} + \sum_{i \in [k]} \sum_{n \in N : \hat{x}_{n,i} = 1} (1 - x_{n,i}) \right)$$

Approach #2: Generalized Benders Decomposition

Master Problem

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No feasibility cuts needed!

- Connected graph $\implies$ feasible master problem
- 2nd and 3rd level are always feasible

Approach #2: Generalized Benders Decomposition

Set $O \leftarrow \{(0, \psi^*_{\text{IGTC}})\}$, $\Theta \leftarrow 0$, $\phi \leftarrow \infty$.
while $\Theta < \phi$ **do**
  Solve the master problem. Let $\hat{x}$ be its optimal solution, set ϕ to its optimal value.
  Solve the second-level problem with fixed $\hat{x}$. Let q^{spot}, d^{spot}, and $\bar{q}^{\text{new}}$ be part of its optimal solution and let $\psi_2(\hat{x})$ be its optimal value.
  Solve the third-level problem with fixed q^{spot}, d^{spot}, and $\bar{q}^{\text{new}}$. Let (q, d, f, θ) be the optimal solution and let $\psi_3(\hat{x})$ be its optimal value.
  if $\psi_2(\hat{x}) - \psi_3(\hat{x}) > \Theta$ **then**
    Set $\Theta \leftarrow \psi_2(\hat{x}) - \psi_3(\hat{x})$ and $(x^*, (q^{\text{spot}})^*, (d^{\text{spot}})^*, (\bar{q}^{\text{new}})^*, q^*, d^*, f^*, \theta^*) \leftarrow (\hat{x}, q^{\text{spot}}, d^{\text{spot}}, \bar{q}^{\text{new}}, q, d, f, \theta)$.
  Add optimality cut to O.
return $(x^*, (q^{\text{spot}})^*, (d^{\text{spot}})^*, (\bar{q}^{\text{new}})^*, q^*, d^*, f^*, \theta^*)$.

Approach #2: Generalized Benders Decomposition

Theorem

Assume that the social welfare $\psi_1(\hat{x})$ is non-negative for all $\hat{x}$, that the second-level problem's solutions $q(\hat{x}), d(\hat{x})$, and $\bar{q}^{new}(\hat{x})$ are unique for given $\hat{x}$, and that the network is connected. Then, the algorithm terminates within a finite number of iterations and returns a globally optimal solution for the trilevel problem.

Enhanced Solution Techniques

Symmetry Breaking Constraints

- Lexicographical ordering
- Méndez-Díaz, Zabala (2001, 2006)
- (Shifted) column inequalities by Kaibel and Pfetsch (2008)

Primal Heuristics

- Min k-cut approximation heuristic based on Gomory–Hu trees
- Relaxation-based rounding heuristic
- 1-opt improvement heuristic

Enhanced Solution Techniques

Genuine First-Level Costs

Since no genuine first-level costs are present up to now, the optimality cuts only act as no-good cuts!

- New objective function

$$\psi_1 \leftarrow \psi_1 - \psi_{\text{acc}}, \quad \psi_{\text{acc}} := \sum_{i \in [k]} x_i^{\text{acc}}$$

- Zonal acceptance costs

$$x_i^{\text{acc}} \geq c_{n,m}^{\text{acc}} x_{n,i} x_{m,i}, \quad i \in [k],\ n, m \in N,\ n < m$$

- Additional cuts

$$\psi_{\text{IGTC}}^* - \psi_{\text{acc}}(x) \geq \psi_1^{\text{inc}}$$

Outline

Motivation

A Mixed-Integer Multilevel Model

Solution Approaches

Computational Results

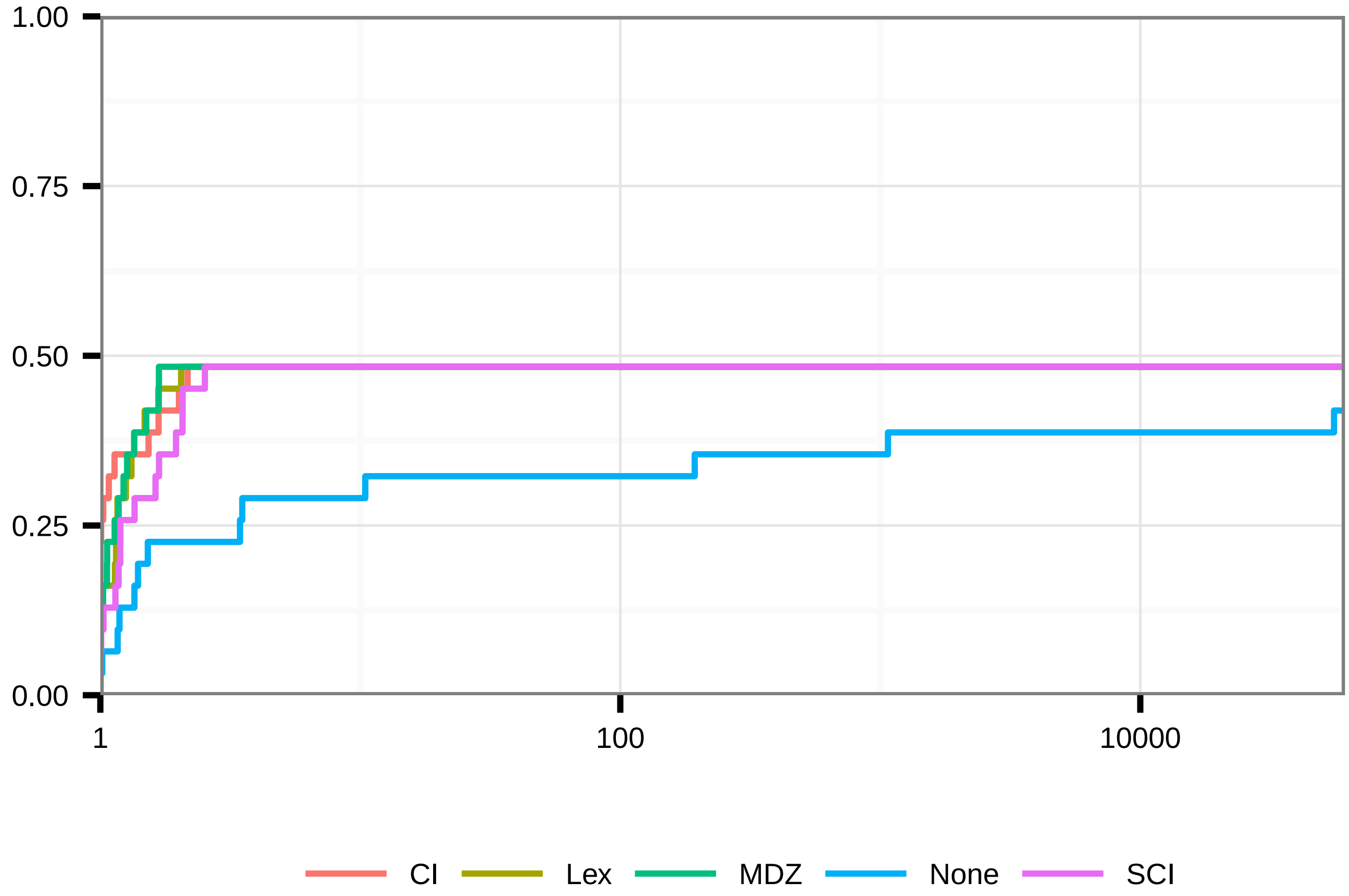
Instances & Setup

Network name	$\lvert N \rvert$	$\lvert L \rvert$	$\lvert T \rvert$	Zones (MIQP)	Zones (Benders)
Grimm-et-al-2015-3	3	3	4	all	all
Chao-Peck-1998	6	6	4	all	all
Grimm-et-al-2016-6	6	6	52	all	all
DE-09	9	19	52	$\{1, 2, 3, 7, 8, 9\}$	all
DE-12	12	23	52	$\{1, 2, 10, 11, 12\}$	all
DE-16	16	27	52	$\{1, 2, 14, 15, 16\}$	all
DE-23	23	39	52	$\emptyset$	$\{1, 2, 3, 4, 20, 21, 22, 23\}$
DE-28	28	39	52	$\emptyset$	$\{1, 2, 3, 4, 25, 26, 27, 28\}$

- Python implementation
- Gurobi 6.5.2 as MIQP, MIP, and QP solver

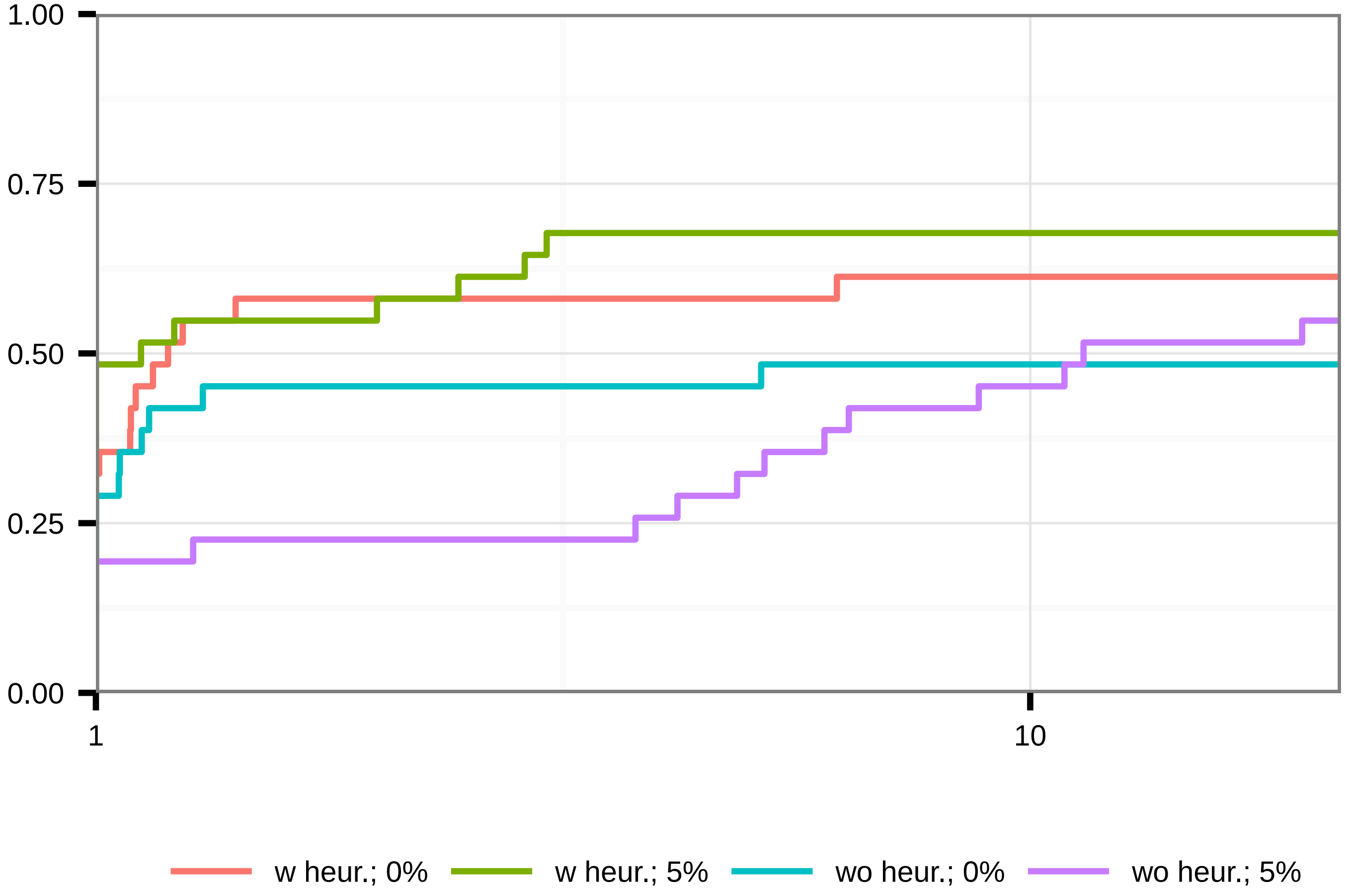
MIQP Results

Symmetry Breaking Constraints



MIQP Results

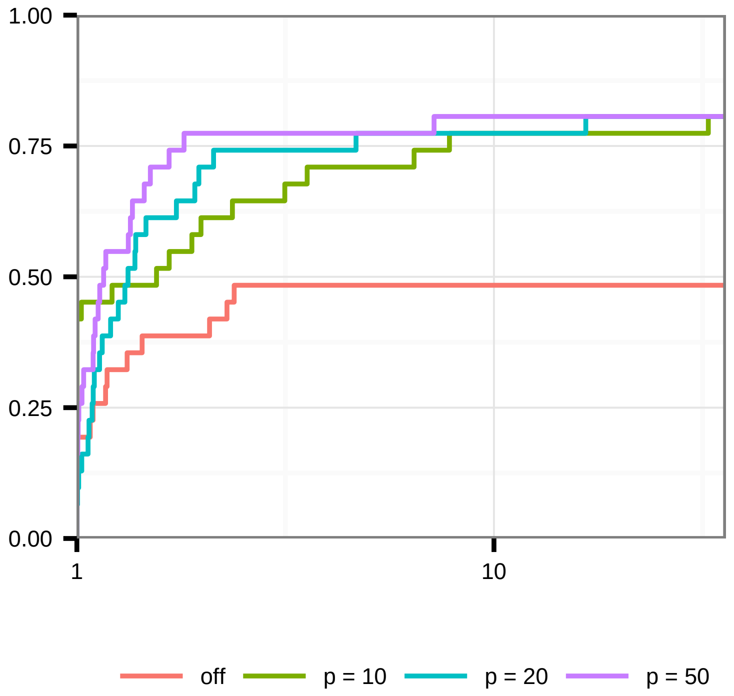
Minimum *k*-Cut Approximation Heuristic



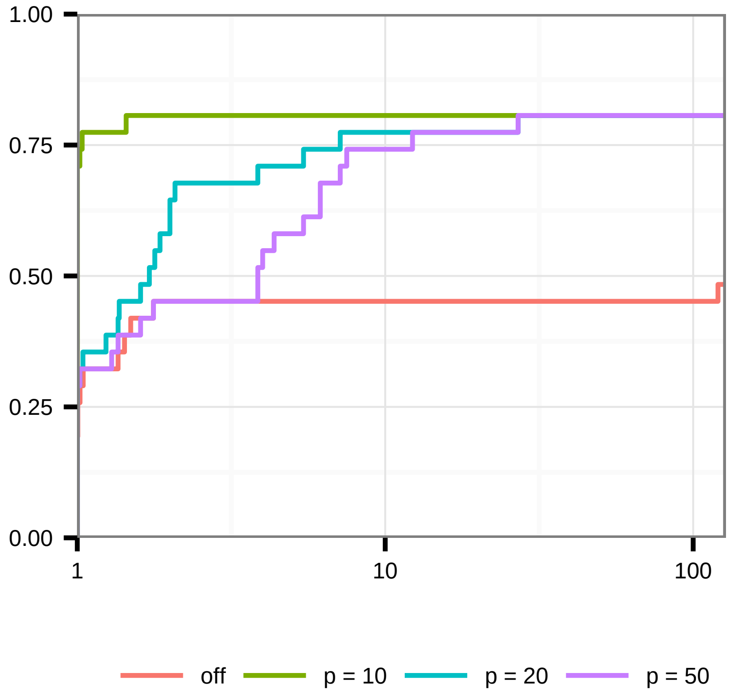
MIQP Results

Rounding + 1-Opt Improvement Heuristic

Running times

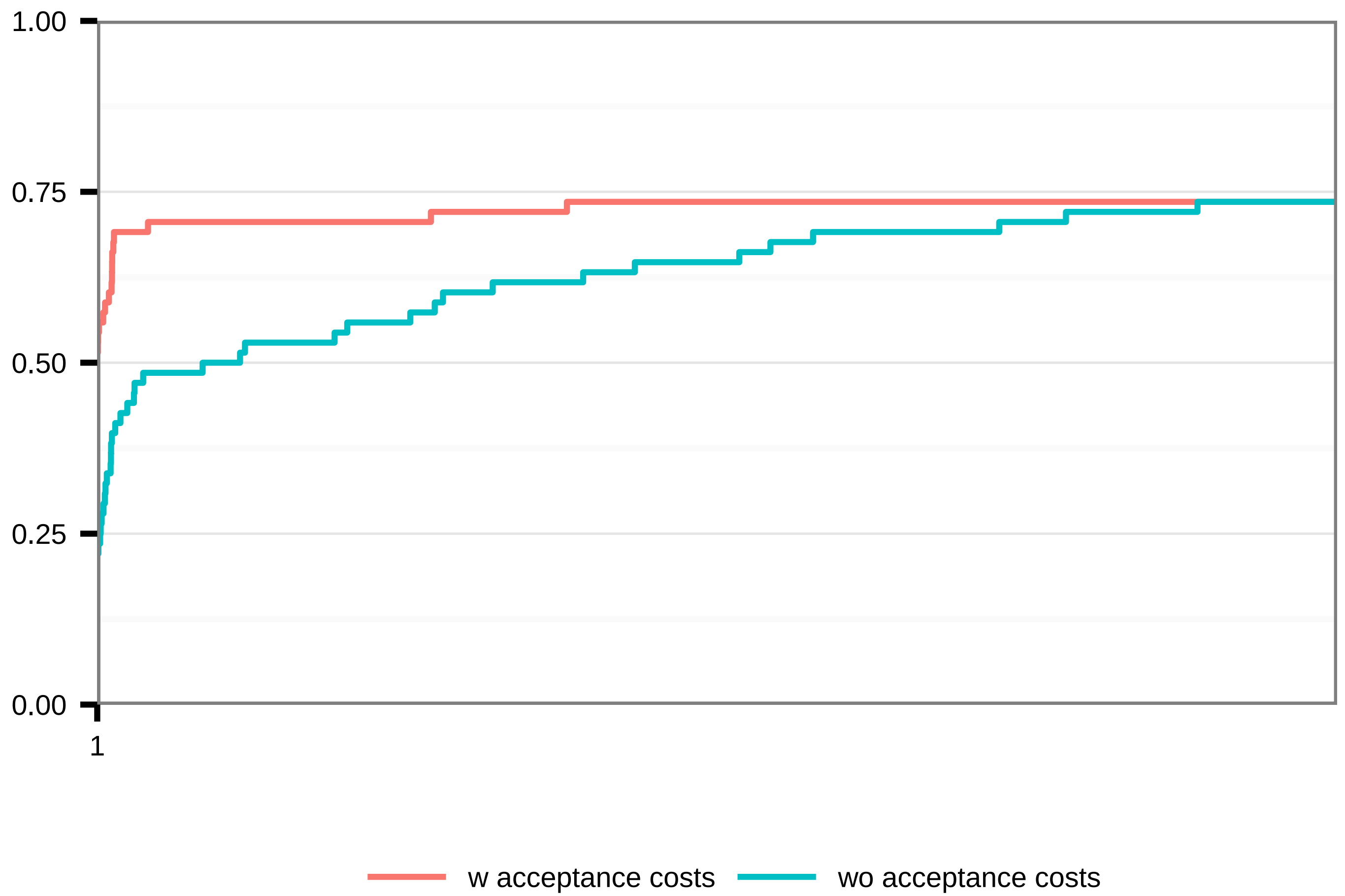


Node counts



Generalized Benders Results

Impact of Acceptance Costs



Comparison: MIQP vs. Benders

Without acceptance costs

Network	Zones	MIQP	Benders
Chao-Peck-1998	1	0.04	0.04
Chao-Peck-1998	2	10.39	0.16
Chao-Peck-1998	3	26.22	0.30
Chao-Peck-1998	4	17.93	0.30
Chao-Peck-1998	5	9.54	0.13
Chao-Peck-1998	6	0.11	0.06
Grimm-et-al-2015-3	1	0.02	0.02
Grimm-et-al-2015-3	2	0.05	0.04
Grimm-et-al-2015-3	3	0.02	0.03
Grimm-et-al-2015-6	1	0.06	0.23
Grimm-et-al-2015-6	2	222.58	0.57
Grimm-et-al-2015-6	3	—	1.06
Grimm-et-al-2015-6	4	—	0.97
Grimm-et-al-2015-6	5	6834.90	0.56
Grimm-et-al-2015-6	6	54.65	0.37

Comparison: MIQP vs. Benders

Without acceptance costs

Network	Zones	MIQP	Benders
DE-09	1	13.20	0.78
DE-09	2	1656.49	5.20
DE-09	3	4081.41	33.97
DE-09	4	—	61.13
DE-09	5	—	65.13
DE-09	6	—	36.36
DE-09	7	3645.49	12.54
DE-09	8	75.11	3.12
DE-09	9	8.52	1.33
DE-12	1	4.85	0.82
DE-12	2	3470.03	19.60
DE-12	3	—	434.34
DE-12	4	—	6097.09
DE-12	8	—	3684.91
DE-12	9	—	334.37
DE-12	10	—	38.60
DE-12	11	303.54	4.99
DE-12	12	10.75	1.66

Comparison: MIQP vs. Benders

Without acceptance costs

Network	Zones	MIQP	Benders
DE-16	1	64.31	1.22
DE-16	2	—	48.64
DE-16	3	—	6672.61
DE-16	13	—	5286.82
DE-16	14	—	129.30
DE-16	15	350.91	9.73
DE-16	16	12.48	3.26
DE-23	1	—	1.01
DE-23	2	—	241.18
DE-23	21	—	705.10
DE-23	22	—	13.83
DE-23	23	—	3.35
DE-28	1	—	1.02
DE-28	2	—	83.62
DE-28	26	—	1042.95
DE-28	27	—	15.28
DE-28	28	—	3.74

Thanks for your attention!

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