1 INTRODUCTION

Traffic flow is the result of different phenomena that may have different scales. For example, a queue at a traffic signal is very dynamic and has a hundred meters of geographical spread, whereas a traffic jam on a freeway can spread over several kilometres and during several hours. For this reason, but also because it is important to model a whole network in a comprehensive way, we are interested in a hybrid model mixing two types of representations known in the literature as microscopic and macroscopic models.

This paper intends to be a first step towards such a model and we will address here some theoretical issues of hybridation. In a first part, we will browse the different kinds of traffic flow models regarding their scale of representation and we will propose a new classification that is more illustrative than the classical micro versus macro one. In a second part, we will study the theoretical aspects of hybridation and in particular different limitations introduced by the mixing of different representation scales. Finally, we will present a sort of “ideal” hybrid model based on the LWR traffic flow theory (Lighthill and Whitham 1955; Richards 1956) and we will propose some extensions to this model.

2 DIFFERENT REPRESENTATION SCALES FOR A MODEL

2.1 Classification of traffic flow models

Two types of traffic models are generally distinguished: “microscopic”, dealing with vehicles, and “macroscopic” dealing with flows. However, no precise and theoretical definition exists for those terms, and if this classification suits well for some models, others cannot be classified correctly. For example, models like INTEGRATION (van Aerde 1995) giving the trajectories of vehicles (or packets of vehicles) and using flow rules are microscopic in their representation but macroscopic in their behavioural rules.

Thus it is our opinion that models have to be differentiated regarding their representation scale (flow or vehicles) and their behavioural rules. For example, the classical car-following models (based on General Motors work; see for example (Gazis, Herman et al. 1961)) use a representation where information is driven by vehicles and are based on individual rules, including reaction time, etc. On the opposite, traffic stream models like LWR use a continuous flow representation and collective rules, based on density, capacity, etc.

So, we propose to classify models depending on:

- Their representation scale: flow representation (FR), where density or flows are calculated, or vehicle representation (VR), where trajectories are calculated.
• Their type of behavioural rules: individual or collective, depending on what they consider, the preceding vehicle trajectory or traffic density, for example.

2.2 Consequences of this classification

A first consequence of this classification is that comparing two models with different representation scale (VR and FR) is not straightforward. One gives trajectories and the other gives flows and density. How can those results be compared?

Even with two VR models, the comparison is difficult. Indeed, a distance measure between models can be constructed, based on distances between vehicles trajectories, but they are not satisfactory and it is more effective to gather those individual trajectories into a flow representation. Regarding the comparison between VR and FR models, several tools will be presented in the full paper, based on cumulative flows or density calculation (on a grid or between trajectories). None of them is fully satisfactory and comparing two models remains a difficult task. But it is important to notice that this difficulty is not linked to behavioural rules, which may be individual or collective.

Another consequence is that none of the classical microscopic or macroscopic approaches is the better. Indeed, several authors claim that one is faster than the other or “cheaper” (in terms of amount of necessary data to design the model). It is our opinion that both types of representation (VR or FR) can be more or less precise depending on the needs and this precision has a cost in terms of calculation speed. On the other hand, individual or collective rules can be more or less sophisticated depending on the needs of the user of the model; this leads to a necessary cost in term of calibration data.

3 THEORETICAL ASPECTS OF HYBRIDATION

We are more specifically interested in a special case of hybridation that deals with the introduction of a VR segment into a FR model of a road network (Fig.1). In this part, we will develop some theoretical elements necessary to the construction and the validation of such type of model.

3.1 Constraints to be satisfied

In order to avoid counterintuitive results, several constraints are to be satisfied by the hybrid model. They can be gathered into two groups:

• There must be flow or vehicle conservation at the interfaces (at the entrance as well as the exit of the VR segment)
• In stationary conditions, the VR segment has to behave in the same way as if it was represented with the FR model.

Those constraints may seem quite trivial but they are not straightforward to define precisely. Thus, for the first problem, conservation has necessarily to be defined through a time horizon to compare the continuous flow and the discrete vehicle departure or arrival. As for the second problem, coherence is difficult to measure because model comparison is difficult to draw, as we already mentioned.

Three typical scenarios can be isolated to verify that those constraints are satisfied: stationary state; congestion going upstream; and platoon crossing downstream the segment.

3.2 Limitations inherent to the hybridation

Even if those constraints are satisfied, it can be shown that hybridation has consequences that cannot be avoided, whatever is the “quality” of the model. In particular, the difference of representation scales of the two zones to be considered introduces some errors that cannot be recovered.

Changing flow into vehicles comes down to converting continuous information into discrete one and this might be a source of loss of information. For example, when considering a stationary continuous flow, different solutions can be drawn and the only constraint is that interval between vehicles is equal to the inverse of the flow. Nothing is said about the first vehicle creation time. Are those solutions all equivalent? Are they all correct? Which is the best one? We think that there is no definitive answer to those questions.

Another limitation is that information has to be carried by vehicles in the VR segment. This trivially means that no information can pass through this segment during the time interval between two vehicles.

More generally, it implies that information can propagate at a maximum speed inside the VR segment depending on the length of this segment and on the vehicle density.

4 An example of hybrid model based on the LWR model

We can apply those theoretical considerations to a sort of “ideal” hybrid model. This model is based on two different scales of representation (VR and FR) but with the same collective behaviour: the LWR theory of traffic flow. The objective of such a model is to give a basis, a first step, for the development of a “real” hybrid model, mixing different representation scales and different behaviours.

4.1 The LWR model, its entropic solution and its two representation scale

The LWR model is based on three equations describing the state of traffic in an arterial, with flow $Q$, density $K$ and speed $V$:

- $\frac{\partial Q}{\partial x} + \frac{\partial K}{\partial t} = 0$ (conservation of vehicles)
- $Q = KV$ (flow-speed definition)
- $Q = Q_e(K)$ (equilibrium relation between flow and density)
The well known entropic solution of those equations is made of two kinds of elements: shock waves, when density increases, and acceleration fans, when it decreases (Godlewski and Raviart 1990). Such a solution has interesting aspects (in terms of stability for example) but it is quite harsh to calculate in the general case due to fans calculation.

To solve this difficulty, several authors advocate the discretization of space and time (see for example (Daganzo 1994; Lebacque 1996)) so that differential equations are then easy to solve incrementally. Others propose to use a triangular equilibrium relationship, so that acceleration fans are reduced to simple shock waves calculation (Newell 1993; Coifman 2002).

We propose in this paper a new solution for the LWR model based on a discretization of the acceleration fans in a finite number of accelerating shock waves. Those waves behave in the same way as the classical decelerating shock waves (when density increases) and this model is proved to be as close to the entropic solution as desired, depending on the number of considered waves in the fan. Figure 2 depicts the entropic solution (2.a) as well as the fan discretization model (2.b) for a traffic signal in an arterial.

Figure 2: Different solutions of the LWR model at a traffic signal: entropic solution (a); fan discretization FR model (b); VR model (c)
Another new solution of LWR model will also be presented; it is based on a vehicle representation. In this model, vehicles have piecewise constant speed and their speed-change positions correspond to the shock waves of the fan discretization model (see Fig.2.c). However, no shock wave is calculated and each vehicle only reacts to the speed changes of the preceding vehicle. The model calculates the time lag between a speed change and the reaction of the following vehicle. Thus, information is fully driven by vehicles and not by any flow and the proposed model really is a VR model.

4.2 The hybrid model

In our case, the two traffic models are based on the same LWR behavioural rules, and they use common information: shock waves (although not calculated explicitly in the VR segment). So, the interfaces between the two models receive this information from both sides and transmit it: if a shock wave comes from the FR zone, it is transmitted to the first vehicle present in the VR zone and conversely. This ensures a correct transmission of information through the interfaces without deformation. To enable information to pass through the interfaces, we propose to create two “fictitious vehicles” in the FR zone: one at each interface. Those vehicles react to shock waves present in the FR zone in order to transmit shock waves to vehicles in the VR model. They also react to speed changes of vehicles, which allows to transform a speed change into a shock wave. At the upstream interface, this vehicle corresponds to the next vehicle to be created. At the downstream interface, it is the last vehicle which has left the VR zone.

We have calculated this model in two simple cases: stationary situation (see Fig. 3.a) and propagation of a shock wave (see Fig. 3.b). It can be seen that shock waves propagate correctly through the VR zone and reappears in the FR zone without distortion.

More complicated situations could be calculated in the same way, the only difference being the number of shock waves to be considered.

Figure 3: The hybrid model in stationary conditions (a) and with propagation of a shock wave (b)
4.3 Extensions of the hybridation

The model we have just presented is very theoretical and has no practical application. In order to solve this issue, we have to introduce some different behavioural rules in the VR segment than in the FR ones.

Two possible extensions to this “ideal” model will be presented here. One is based on a stochastic creation of vehicles in the VR segment and the other is based on a classical car-following model.

4.3.1 Stochastic generation of vehicles

In this paragraph, we keep the previous “ideal” model where we introduce a stochastic behaviour at the generation of vehicles using random headways (Erlang distribution has been used, but other distributions would have lead to the same results). So, in this new model, vehicles speeds are still deterministic and are the same as in the initial model.

We can observe that shock waves are correctly represented in the VR segment as well as at the interfaces, and propagate in the same way as in the “ideal” model (see fig. 4).

As for the flow conservation, it seems to be globally respected, on the considered time period. Nevertheless, this raises the question of its local definition. Perhaps an average conservation constraint could be defined.

4.3.2 Car-following model

Another extension of the “ideal” hybrid model is to substitute the VR model with another one, based on different behavioural rules. A simple model to be used is a car-following one where speed is calculated with the density equal to the inverse of the space headway (this has been used for example in INTEGRATION, (van Aerde 1995)). Such a model is easily shown to be equivalent to the FR model in stationary phases if the speed-density relation which is used is equivalent to the flow-density used in the FR segment.

Nevertheless, flow conservation at interfaces is very difficult (and even hardly possible) to verify, whatever is the way interfaces are built. Indeed, transitory phases are handled in very different ways by the two models and there is no common information between them.

Figure 4: The hybrid model with stochastic headways
It seems that the only individual model corresponding to the conservation constraints and able
to transmit information from a FR segment to the other, is the “ideal” model we have
presented above.

5 CONCLUSIONS

An “ideal” hybrid model has been proposed, based on the LWR traffic flow theory. This
model seems to work correctly and respect the constraints we have defined for hybridation.
But, in fact, it comes down to mixing too similar behaviours and it is not directly useable. As
soon as we try to extend it, in order to introduce different behaviours, difficulties (and even
impossibilities) appear.

This questions us on the relevancy (and feasibility) of a model mixing different type of
behavioural rules.

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