1 INTRODUCTION

Bus transportation companies utilise vehicles (buses) to transport passengers on given routes according to a given timetable. The general problem in such a situation is: how to assign particular buses to given routes?

In the last decades there were many attempts to solve the assignment of vehicles to transportation jobs (routes). In the simplest form the assignment problem can be formulated in terms of linear programming and solved with a help of simplex method, network algorithms (Cook 1985) or assignment method (Lotfi et al. 1989). In real life situations, however, the vehicle assignment problem is more complicated and requires more advanced methods to be solved. Some authors (Löbel 1998, Rushmeier et al. 1997) formulate the vehicle assignment problem in terms of the linear, integer programming. Some others (Beaujon et al. 1991) transform the linear, discrete model into a non-linear, continuous form. Many models are based on the queuing theory (Green et al. 1995, Whitt 1992). The proposed models consider either the homogeneous (Beaujon et al. 1991) or a non-homogeneous fleet (Ziarati et al. 1999). Some of the models combine the vehicle assignment problem with other fleet management problems, such as: fleet sizing (Beaujon et al. 1991) or fleet scheduling (Löbel 1998).

The models usually refer to specific transportation environments, such as: urban transportation (Löbel 1998), rail transportation (Ziarati et al. 1999) or air transportation (Rushmeier et al. 1997). In the majority of cases the proposed vehicle assignment models have a single objective character. M. Zeleny (Zeleny 1982) proposes an extended multicriteria model for the vehicle assignment problem and a solution procedure based on an assignment algorithm – Hungarian method (Bradley et al. 1977). Based on the authors’ research and experience the vehicle assignment problem has a multiobjective character. In the problem formulation the interests of both passengers (customers) and company’s management should be taken into account. The most popular solution procedures are decomposition techniques, such as: Frank-Wolfe’s, Benders’ or Dantzig-Wolfe’s decomposition algorithms (Bradley et al. 1977). Heuristics and branch-and-bound algorithm are also utilised (Rushmeier et al. 1997).
In this paper the authors propose an original multicriteria model for an optimal assignment of non-homogeneous fleet of buses to a given set of routes in an international passenger transportation company. The solution procedure is based on a metaheuristic (Pareto Memetic Algorithm) and on interactive multicriteria method called Light Beam Search.

2 MATHEMATICAL MODEL OF THE PROBLEM

The analysed problem is expressed in terms of multicriteria, non-linear, integer, combinatorial mathematical programming (Bradley et al. 1977, Vincke 1992). A one week time horizon is assumed for the problem analysis.

2.1 Data
- $S_i$ – length of route $i$ [kilometres],
- $P_i$ – average number of passengers travelling weekly on route $i$ [persons],
- $p_{\text{pas}i}$ – average income per one passenger travelling on route $i$ (ticket price) [monetary units],
- $w_i$ – average load index of a bus on route $i$ [-], expressed as a quotient of an average number of tickets sold for a particular ride on route $i$ and an average number of passengers in a bus during this ride, $w_i \in \{0,1\}$,
- $k_{ij}$ – fixed cost per route $i$ and bus $j$, including drivers’ salaries, highway fares, tolls, insurance and licence fees etc. [monetary units / ride],
- $k_{wkm\ ij}$ – variable (vehicle-kilometre) cost per bus $j$ and route $i$, including fuel and maintenance cost [monetary unit / kilometre],
- $c_j$ – capacity of bus $j$ – number of seats [-],
- $f_j$ – comfort level of travelling by bus $j$ [-], expressed in points according to the following characteristics of bus $j$: production year, seats’ comfort (size, softness), air conditioning, toilet, video etc., $f_j \in \{1, 2, 3, 4, \ldots, f_{\text{max}} = 10\}$.

2.2 Decision variables
The integer decision variable $\omega_{ij} \in \{0, 1, 2, 3, \ldots\}$, denominates a number of rides carried out weekly by a vehicle $j$ on route $i$.

2.3 Criteria

1. Total weekly profit – $Z$.

$$\text{Max } Z = \sum_{i=1}^{I} \left[ W_i - \left( S_i \sum_{j=1}^{J} \omega_{ij} k_{wkm\ ij} \right) - \sum_{j=1}^{J} \omega_{ij} k_{ij} \right],$$

where: $W_i = (P_i - SK) \cdot p_{\text{pas}i}$ is a weekly income per route $i$. (2)
2 Capacity utilisation – WL.

\[
\text{Min } WL = \left| WL_{opt} - \left[ \sum_{i=1}^{I} \sum_{j=1}^{J} \left( \frac{\min\{P_{oi}, c_j\}}{c_j} \cdot \omega_{ij} \right) \right] \right|, \quad (3)
\]

where: \( P_{oi} = \left( \frac{P}{\sum_{j=1}^{J} \omega_{ij}} \right) \cdot w_i \) is an average number of passengers travelling on route \( i \). \quad (4)

This criterion measures the distance between the optimal (\( WL_{opt} = 0.8 \)) and the real utilisation of bus capacity.

3 Total number of weekly lost (rejected) customers (passengers) – SK.

\[
\text{Min } SK = \sum_{i=1}^{I} SK_i, \quad (5)
\]

where: \( \forall i \ ST = \max \left\{ 0, P_i - \sum_{j=1}^{J} \left[ \min\{P_{oi}, c_j\} \cdot \omega_{ij} \right] \right\} \) is a number of weekly lost passengers on route \( i \). \quad (6)

4 Comfort of travel for passengers – WK.

\[
\text{Max } WK = \sum_{i=1}^{I} \sum_{j=1}^{J} \min\{P_{oi}, c_j\} \cdot \omega_{ij} \cdot f_j \left/ \sum_{i=1}^{I} \sum_{j=1}^{J} \min\{P_{oi}, c_j\} \cdot \omega_{ij} \cdot f_{max} \right. \quad (7)
\]

2.4 Constraints
- Real riding time by bus \( j \) on route \( i \) should be consistent with the timetable.
- Weekly working time of bus \( j \) should not be greater than its maximal weekly working time, including maintenance (repair and service) times.

3 SOLUTION PROCEDURE

A two-stage approach has been proposed to solve the analysed problem:
- stage one: generating a sample (set) of solutions with an application of a multicriteria metaheuristic method called the Pareto Memetic Algorithm (PMA) (Jaszkiewicz 2002),
- stage two: selection of the most satisfactory solution with an application of the Light Beam Search (LBS) method (Jaszkiewicz et al. 1995).

3.1 PMA

Pareto memetic algorithm (PMA) is a multiple-objective hybrid evolutionary algorithm utilising simultaneously recombination operators and local heuristics. The algorithm starts by generation of an initial set of solutions. Then, in each iteration, a weight vector defining the current scalarising function is drawn at random. After that, two solutions (parents) being good on the current scalarising function are drawn for recombination with the use
of tournament selection. A local heuristic is applied to the solution obtained in result of recombination. If the new solution is better on the current scalarising function than at least one of the parents, it is added to the temporary set of solutions. The outcome of the algorithm is a set of potentially Pareto-optimal solutions - $PP$. $PP$ is updated with each newly generated solution, i.e. the new solution is added to $PP$, if it is not dominated by any solution from $PP$, and solutions dominated by the new solution are rejected.

### 3.2 LBS method

LBS method falls into a category of interactive procedures for multiobjective mathematical programming and combinatorial problems. The procedure is characterised by phases of decision alternating with phases of computation. At each computational phase a sample of solutions is generated. This sample is examined and evaluated by a Decision-Maker (DM) in the decision phase, according to his/her preferences. These preferences can be defined using the outranking binary relation and the thresholds of indifference, preference and veto. As a result of the examination, the DM inputs some preferential information, which leads to the improvement of the solutions to be generated in the next computational phase.

The process finishes when the most satisfactory solution is found.

### 4 COMPUTATIONAL EXPERIMENT

A Polish, passenger transportation company is analysed. This company operates on the 17 routes between 34 Polish and 47 European cities. All the routes are characterised by the following parameters:
- length $S_i$ between 1818 and 4048 kilometres,
- average number of passengers travelling weekly on particular routes $P_i$ between 2 and 796,
- average income per one passenger (ticket price) $p_{\text{pas}i}$ between 188 and 721 $PLN$,
- average load index $w_i$ between 0.25 and 0.46,
- fixed cost $k_{ij}$ per route $i$ and bus $j$ between 3 530 and 14 809 $PLN/ride$.

Analysed company utilises a fleet of 30 buses (Hyundai, Neoplan, Scania, Volvo) characterised by:
- vehicle-kilometre cost $k_{\text{wkm}ij}$ between 1.49 and 2.01 $PLN/kilometre$,
- number of seats (capacity) $c_j$ between 31 and 57,
- comfort level $f_j$ between 3 and 9 points (comfort level ranges from 2 to 10 points).

In the first stage of the solution procedure, after 60 000 iterations (recombination and local improvements), the set, being a good approximation of all Pareto - optimal solutions,

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1 $PLN$ – Polish New – Polish currency. 1 $PLN = 0.24 USD$ in December 2001
has been generated, with a help of PMA. This set, composed of 2 985 solutions, gives the following ranges on particular criteria (the best values are underlined):

- criterion 1 (profit – $Z$): -2 669 000 $\div$ 1 802 570 PLN (for solution A2959: 1 752 140 PLN),
- criterion 2 (capacity utilisation – $WL$): 0.11 $\div$ 0.74 (for solution A2959: 0.15),
- criterion 3 (number of lost passengers – $SK$): 0 $\div$ 181 (for solution A2959: 22),
- criterion 4 (comfort of travel – $WK$): 0.85 $\div$ 0.90 (for solution A2959: 0.87).

In the second stage of the solution procedure DM, with a help of LBS method, selects the most satisfactory solution according to his/her preference model. At first DM rejects all non-profitable solutions. This reduces the generated set of Pareto – optimal solutions to 1 435, which essentially does not influence on the value ranges of the remaining criteria (criteria 2 $\div$ 4). After that DM expresses his/her preferences by the definition of:

- the reference point – expected values of criteria (criterion 1: 1 800 000 PLN, criterion 2: 0.15, criterion 3: 0, criterion 4: 0.88),
- indifference, preference and veto thresholds; in the analysed case DM uses an indifference threshold only (criterion 1: 100 000 PLN, criterion 2: 0.03, criterion 3: 30, criterion 4: 0.005).

Using this preferential information the LBS method generates a middle point (see the first row in Fig. 1) and its neighbourhood composed of 357 solutions. At this point DM is not able to evaluate the whole sample of solutions. That is why he/she uses the LBS method to filter the 10 most representative solutions (Fig. 1).

![Figure 1. The most representative solutions from a neighbourhood of the middle point](image)

DM is interested in solution A2960 which outranks the present middle point on criterion 1 (by 300 000 PLN) and is indifferent on the other criteria. Solution A2960 becomes a new middle point. Its neighbourhood consists of 38 solutions including solution A2959 which has been selected by DM as the most satisfactory, compromise solution. This point,
presented in the decision variables space is shown in Table 1. It guaranties the above mentioned (bold printed) values of criteria.

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**5 CONCLUSIONS**

The outcome of this research is an original, multiobjective model of the vehicle assignment problem and the computational procedure, that leads to an optimal solution of the problem. The model refers to a non-homogeneous fleet of buses utilised in a long-distance passenger transportation company.

The proposed approach lets us take into account the interests of both passengers and the company’s management. Consequently, the generated compromise solution satisfies the stakeholders’ interests to some degree. It reduces the operational costs and increases the quality of a transportation service at the same time.

The methodology leads to the profitability analysis of particular routes. Based on the analysis of criterion 1 certain, non-profitable routes can be eliminated from the existing portfolio of the transportation services. It also allows possible to define the minimal ticket price for each route to assure its acceptable profitability and maintain this service in the portfolio.

**REFERENCES**


