PROBABILISTIC PROPERTIES OF PATH TRAVEL TIMES

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ABSTRACT
Observed travel times exhibit many variations: given the path, the traffic conditions, the type of vehicle/mobile, and the user behavior, there still remains an inherent variability. In roadway transport an important source of inherent variability consists in the microscopic interactions among individual mobiles.

The objective of the paper is to provide a physical and probabilistic model of the travel time along a path, which makes explicit the following sources of variability: (i) macroscopic state (e.g. traffic conditions); (ii) user type; (iii) microscopic fluctuations; (iv) stops on behalf of the user.

The paper is comprised of five parts: Part 1 introduces the factors of path travel times, together with a set of behavioral assumptions. These yield physical properties, indicated in Part 2, and probabilistic properties, in Part 3. Part 4 is devoted to advanced probabilistic properties based on further assumptions (stochastic process, markovian model). Lastly, Part 5 provides empirical results, based on various observations recently carried out in France.

Keywords
Travel time; Probabilistic model; Stochastic process; Markov model

1 FACTORS OF TRAVEL TIME

Let us consider a path P on a transportation network: P may be either an arc (oriented link), or a path comprised of several arcs and nodes.

The path is used by a variety of trips, under diverse temporal conditions, traffic conditions, weather conditions (e.g. rain, snow) etc, which influence the local speed hence the path travel time. As any modality of theses conditions is common to all users that travel in a given temporal period, we define the set of relevant common conditions as the macroscopic state in the trip period, and we denote it by M. This enables us to take account of period conditions in a concise, synthetic way.

We also include in the macroscopic state the volume and structure of the trips made during the period: trip rate for each user type, hence also the relative proportions of user classes.

In roadway transport, relevant user types are traffic classes: light vehicles (mostly cars) as opposed to heavy vehicles (mostly trucks). Within a traffic class, there may remain a variety of individual behavior: let us focus on the preference toward speed, denoted by a velocity index $\alpha$: between two users of respective indices $\alpha_1$ and $\alpha_2$, if $\alpha_1 \leq \alpha_2$ means that user 2 is more rapid than user 1.
As the velocity index is an ordinal variable, we consider its cumulative distribution function and denote it by \( A(x) = \Pr\{ \alpha \leq x \} \).

For simplicity we do not introduce any index for traffic class: it is implicitly denoted by notation \( \alpha \) or \( A \).

As the structure of user behaviors may depend on the macroscopic state \( M \), we may specify \( A_M \). However, it may often be assumed that velocity is a relative index among all users, and that the structure of the user population in any period does not differ much from a standard structure in that respect (apart from traffic class).

Lastly, each individual trip is faced to local, specific conditions: in roadway transport a given car may be impeded by other cars and trucks. The microscopic conditions are random fluctuations resulting from the macroscopic state: we denote them by an index \( \omega \).

To sum up, under macroscopic state \( M \), we model the travel time of a user with velocity index \( \alpha \) as a random variable \( t_{P\alpha M}(\omega) \).

If a population of users is considered with no respect to velocity indices, then the travel time is a random variable \( t_{PM}(\alpha, \omega) \), in which randomness applies to both \( \omega \) and \( \alpha \). Paying no respect to macroscopic state implies in turn a random variable \( t_P(M, \alpha, \omega) \).

## 2 Physical properties

Let us denote by \( \mathbb{E}_\omega[X] \) the expectation (mean) of a random variable \( X \) with respect to factor \( \omega \). For any two velocities \( \alpha \leq \beta \), the core assumption of velocity index implies that

\[
\mathbb{E}_\omega[t_{P\alpha M}] \geq \mathbb{E}_\omega[t_{P\beta M}], \forall M
\]  

(1)

Assuming further that \( M \) measures the level of congestion, then for any two macroscopic states \( M \leq N \) it holds that

\[
\mathbb{E}_\alpha[t_{P\alpha M}] \leq \mathbb{E}_\alpha[t_{P\alpha N}], \forall \alpha
\]  

(2)

hence by aggregation over velocity indices

\[
\mathbb{E}_\omega[t_{PM}] \leq \mathbb{E}_\omega[t_{PN}],
\]  

(3)

## 3 Probabilistic properties

We have already used the theorem of total expectation, viz.

\[
\mathbb{E}_\omega[t_{PM}] = \mathbb{E}_\omega[\mathbb{E}_\omega[t_{P\alpha M}]]
\]  

(4)

We can also establish properties about the variances: denoting by \( \mathbb{V}_\omega[X] \) the variance of random variable \( X \) with respect to \( \omega \), by the theorem of total variance

\[
\mathbb{V}_\omega[t_{PM}] = \left( \int \mathbb{V}_\omega[t_{P\alpha M}] dA(\alpha) \right) + \left( \int \left[ \mathbb{E}_\omega[t_{P\alpha M}] - \mathbb{E}_\omega[t_{PM}] \right]^2 dA(\alpha) \right).
\]  

(5)

The assumption of ergodicity is that each individual trip, on a long path \( P \), encounters so many microscopic events that these "compensate" one another and make the individual path
time close to the path time averaged over all microscopic conditions. Stated formally, denoting by $L_P$ the length of path $P$, the ergodicity property is that

$$V_{\omega}[t_{P,M}/L_P] \xrightarrow{L_P \to \infty} 0.$$  

(6)

### 4 ADVANCED PROPERTIES

A more detailed analysis, notably on ergodicity, temporal and spatial homogeneity, is available in Leurent (2001).

This reference also includes two advanced models, which are introduced hereafter in a very abridged way.

First, a stochastic process framework for the residual fluctuation $\omega$, which is decomposed into local fluctuations $\omega = (\omega_s)_{s \in P}$ associated with points $s$ along path $P$. The local time (pace) $t_{s,\alpha}$ is a stochastic process indexed by $s$, with mean $\tau_{s,\alpha}$ and spatial covariance function $\tilde{\chi}_{s,\alpha}(u - s) = \text{cov}_{\omega}[t_{s,\alpha}, t_{u,\alpha}]$.

The mean function induces the mean path travel time

$$E_{\omega}[t_{P,M}] = \int_{s \in P} \tau_{s,\alpha} ds$$  

(7)

whereas the covariance function induces the variance of path travel time,

$$V_{\omega}[t_{P,M}] = \int_{s \in P} \tilde{\chi}_{s,\alpha}(u - s) ds du$$  

(8)

The assumption of a negative exponential function $\tilde{\chi}$ implies ergodicity.

This framework enables us to relate local statistical properties to path properties: at the local level, (5) yields that

$$V_{\omega}[t_{s,\alpha}] = E_{\alpha}[V_{\omega}[t_{s,\alpha}]] + V_{\alpha}[E_{\omega}[t_{s,\alpha}]]$$

$$= E_{\alpha}[\tilde{\chi}_{s,\alpha}(0)] + V_{\alpha}[\tau_{s,\alpha}]$$  

(9)

At the path level, assuming spatial homogeneity, ergodicity and a long length $L_P$, then

$$V_{\omega}[t_{P,M} / L_P] = V_{\alpha}[\tau_{s,\alpha}]$$  

(10)

Second, a markovian model for microscopic interaction among mobiles within a trip. Let us assume two traffic classes $A$ and $B$ in the trip direction, with free speed of $v_A$ and $v_B$ respectively: class $A$ represents slow mobiles (e.g. trucks) and class $B$ fast mobiles (e.g. cars). Class $A$ mobiles are unimpeded, whereas a class $B$ mobile may be obliged to slow down behind an $A$-mobile and to wait until possible to overtake. Here the key variables are the (spatial) transition rates $\gamma_M$ from $v_B$ to $v_A$, and $\beta_M$ from $v_A$ to $v_B$, for any $B$-mobile.

These depend on the macroscopic state $M$ and may be derived from both physical and probabilistic assumptions (Leurent, 2001, 2002).

The results of interest here are that, in the stationary case,

$$E_{\omega}[t_{P,B,M}] = L_P \frac{\beta_M}{\gamma_M + \beta_M v_B} + \frac{1}{\gamma_M + \beta_M v_A}$$  

(11)
\[ V_\omega[t_{PBM}] = 2 \frac{\gamma_M \beta_M}{(\gamma_M + \beta_M)^3} \left( \frac{1}{v_B} - \frac{1}{v_A} \right)^2 \left( (\gamma_M + \beta_M) L_P + e^{-(\gamma_M + \beta_M) L_T} - 1 \right) \] 

which again is consistent with ergodicity, and provides an analytical formula for the covariance function \( \tilde{\chi} \) in this simple framework.

These advanced probabilistic models have considerable effects on the specification and observation of travel times and local speeds; notably Wardrop's formula and the floating vehicle method are reconsidered in Leurent (2001).

5 **Empirical Results**

The observation of traffic on French interurban motorways (Leurent and Simonet, 2001) has revealed the following facts pertaining to light vehicles:

1. a reference order of magnitude for \( \tilde{\chi}_{sBM}(0) = V_\omega[t_{aBM}] \) is \((0.05 \text{ mn/km})^2\).
2. this is less than the observed variance of local pace \( V_\omega[t_{PM}] = (0.07 \text{ mn/km})^2 \) due to the diversity of velocity indices \( \alpha \).
3. over a long section, \( V_\omega[t_{PM} / L_P] \approx (0.045 \text{ mn/km})^2 \). Interpreted as \( V_\alpha[t_{aBM}] \) on the basis of (10), this is consistent with the two previous points and (9).
4. on the most congested interurban arc, over all macroscopic conditions in a year, the variation in car travel times due to traffic conditions amounts to about 30% of all variations, i.e. \( V_M[E_\omega[t_{PM}]]/V_M[t_{PM}] = 30\% \).
5. all of the variations considered here pertain to pure travel time \( t_P \), as opposed to the total travel time \( t_T \) which also includes user's stops. For trips more than 50 km long, \( V[t_P]/V[t_T] = 15\% \) only.

6 **References**


