1 INTRODUCTION

Calibration of mode choice models has been developed in the literature, particularly with reference to trips in urban areas with fixed choice sets. In this paper the calibration of a mode choice model and an alternative perception model is proposed. Research developed from other branches of learning, like economy and psychology, suggest that human behaviour can be divided into two processes: at the beginning the user tends to eliminate alternatives considered unacceptable and to accept the other alternatives with a non-compensatory process, including them in his choice set; then he chooses the best alternative belonging to the choice set on the basis of a compensatory analysis (Fukuda et al. 2001).

For the first time, in 1977 Manski methodically discussed this subject, introducing a model with explicit simulation of the choice set formation process. In 1995 Morikawa re-proposed the Manski model, but he made use of a different formulation: through a two by two comparison of alternatives he obtained a form computationally treatable even with a large number of alternatives. Nonetheless, this model can be utilized only if the random components of utility of choice models are identically and independently distributed.

Later other models for specification and calibration of choice set were proposed like captivity models (Gaudry, Dagenais, 1979), random constraint models (Swait, Ben Akiva, 1987), latent choice set models (Ben Akiva, Boccaria, 1995; Ben Akiva et al., 1997), implicit generation, (Cascetta, Papola, 1997); and dominance among alternatives (Cascetta, Papola, 2000). Also in recently studied route choice models, (Ben Akiva et al., (1984); De La Barra et al., (1983); Russo and Vitetta (1996); Cascetta et al., (1996); Bakhor et al., (2001); Cascetta et al., (2001)) the need to specify the model of route alternative generation is evident. In these models the construction of route potential “analytical” sets and then selection of “attractive” choice sets whose alternatives are perceived by appropriate models are being developed. In this paper we propose the specification, calibration and validation of some mode choice models to explicitly simulate the perception of alternatives. In short, the proposed model, taking advantage of conditional probability, assigns to every possible choice set a probability
2 METHODOLOGY

The models most widely treated in the literature for the choice set formation process can be classified according to two different approaches: implicit approach, which consists in the simulation of the perception/availability of an alternative in the choice model of such alternative; explicit approach, which consists in the simulation of the choice set with a suitable model which is used before choosing among the alternatives.

In the following $n$ is the generic user who chooses an alternative, $i$ or $j$, belonging to the universal set of alternatives $M$, which has cardinality $m$ ($i, j \in M$); $C$ is the generic choice set belonging to $G$, $(C \in G)$, with $G$ the set of all the non-empty subsets that may be extracted from $M$ ($\emptyset(M) - \emptyset$); $P_n(i)$ is the probability that the user $n$ chooses the alternative $i$ ($i \in M$); $P_n(i|C)$ is the probability that the user $n$ chooses the alternative $i$ among those contained in the choice set $C$; $Q_n(C|G)$ is the probability that the choice of the users $n$ takes place inside the choice set $C$; $X^n_S$ is the matrix of the attributes associated to the alternative choice for user $n$ and $X^n_P$ it is the matrix of the attributes associated to the perception of the alternatives for the user $n$.

So the general formulation of the model with implicit approach is:

$$P_n(i | X^n_S, X^n_P) = P_n(i | C^n \equiv M)\left[X^n_S, X^n_P\right]$$

in which $C^n$ is the choice set of the user $n$ which coincides with the universal set of alternatives and in which the functional dependence on $X^n_S$ and $X^n_P$ means that the choice probability of the generic alternative and the perception probability are simulated in the same level. The “utility” attributes are confused with the “availability” attributes.

The general formulation of the model with explicit approach is:

$$P_n(i | X^n_S, X^n_P) = \sum_{C \in G} P_n(i | C)\left[X^n_S, X^n_P\right] \cdot Q_n(C|G)\left[X^n_P\right]$$

in which the probability that user $n$ chooses the alternative $i$ among those contained in a choice set $C$ exclusively depends on the choice attributes $X^n_S$, while the probability that the choice of user $n$ takes place inside the choice set $C$, given $G$, exclusively depends on the perception attributes $X^n_P$. From the set of all the existing alternatives we identify a subset that
represents the choice set for the generic user, which can be defined with some deterministic rules or probabilistic rules. The deterministic rules are appropriate only in a few contexts, so in the following the explicit stochastic approach is exclusively examined in detail.

The explicit stochastic choice models are made up by two choice behaviour levels: the process of formation of the choice set, which associates a probability to every possible set; the choice process given the choice set, which is usually made by using compensatory utility models, i.e. in which the various attributes considered by the user can compensate each other. The first formulation proposed by Manski (1977) considers all the combinations of potential choice sets. Therefore the number of combinations exponentially grows with the growth of the number of alternatives. The general form of the model presented by Manski is expressed as:

\[ P_n(i) = \sum_{C \in G} P_n(i \mid C) \cdot Q_n(C \mid G) \quad (1) \]

in which \( Q_n(C \mid G) \) is the probability that the choice of user \( n \) takes place inside the choice set \( C \) given \( G \), and may be obtained as follows:

\[
Q_n(C \mid G) = \frac{1}{1 - Q_n(\emptyset)} \cdot \prod_{i=1 \ldots m} q_n(i)^{d_i} \cdot \{1 - q_n(i)\}^{1-d_i}
\]

where

- \( Q_n(\emptyset) \) is the choice probability of the empty set in the random constraints model for the user \( n \);
- \( d_i \) is 1 if the alternative \( i \) is an element of \( C \), 0 otherwise;
- \( q_n(i) \) is the probability that the alternative \( i \) is included in the choice set of user \( n \).

Assuming that the generic choice set is composed by alternatives that satisfy some independent constraints, the probability that the alternative \( i \) is included in the choice set of user \( n \) can be expressed as:

\[
q_n(i) = \prod_{k=1}^{K} q_{kn}(i)
\]

in which

- \( k \) is the generic independent constraint;
- \( K \) is the total number of independent constraints;
- \( q_{kn}(i) \) is the probability that the \( k \) constraint is satisfied for the user \( n \).

A generic alternative can be included in the set if, and only if, it satisfies a series of imposed constraints. More specifically, when one of the constraints is not satisfied for an alternative, this cannot be included in the choice set, even if the other constraints are satisfied; the constraints inside the model do not compensate each other. The second level, \( P_n(i \mid C) \) consists in making a choice given the choice set and can be formulated by a discrete choice model (like Logit or Probit) with a compensatory utility function. Hence, for example, the Logit model:
\[ P_n(i | C) = \exp(\beta_{i,S}^T X_{i,S}) / \sum_i \exp(\beta_{i',S}^T X_{i',S}) \]

in which \( \beta_{i,S} \) is a vector of parameters, associated to the choice, to be estimated, for the alternative \( i \); and \( X_{i,S} \) is a vector of variables, associated to choice, regarding the alternative \( i \).

### 3 PROPOSED APPROACH

In this paper a mode choice model, able to interpret the essence of the mechanism entailed in perceiving alternatives is proposed. The correct identification of user’s choice set is important in mode choice models; beforehand some alternatives can be rejected by the analyst. However, in general, it is not possible to know beforehand the real user’s choice set.

In mode choice the mechanism of alternative perception essentially depends on the following factors: accessibility of transport services, availability of the transport services, user’s economic availability, and user’s particular requirements.

Therefore it seems more correct to proceed as follows:
- use a deterministic model to identify a set of considerable alternatives, \( M \), excluding those clearly inadmissible in the context considered;
- use a stochastic model of alternative perception/availability simulation \( Q_n(C | G) \) to identify the choice set \( C \) of the user \( n \).

After choice set definition we may proceed to use a model which simulates the user’s choice inside this set, where every alternative has a probability of being perceived. More specifically, for the simulation of the choice set, the explicit stochastic model (1) is used.

In the choice context of the generic user there are \( K \) independent constraints, where “independent” means that the \( K \) constraints have a non-compensatory nature and are statistically independent. The generic constraint can be applied to a subset of the universe of the alternatives, \( I_k^n \), and therefore various alternatives can be subject to various constraints (by contrast the Morikawa model (1995) assumes that all alternatives are subject to the same constraints). The probability \( Q_n(C | G) \) is expressed as:

\[
Q_n(C | G) = \frac{1}{1 - Q_n(\emptyset)} \prod_{i=1,m} \left( \prod_{k=1}^{K} \left( \frac{1}{1 + e^{-[\beta_{k,i,P}^n X_{k,i,P}^n - \mu_k]}} \right)^{d_{i,k}^n} \right) \left[ \prod_{k=1}^{K} \left( \frac{1}{1 + e^{-[\beta_{k,i,P}^n X_{k,i,P}^n - \mu_k]}} \right) \right]^{1-d_{i,k}^n}
\]

in which \( \beta_{k,i,P} \) is a vector of parameters, associated to perception, to be estimated, for the constraint \( k \); \( X_{k,i,P}^n \) is a vector of variables, associated to perception, regarding the constraint \( k \) and \( \mu_k \) is the threshold for the constraint \( k \).

The choice probability for the user \( n \) given the choice set \( C \) is calculated with a discrete choice model like the Multinomial Logit.
4 RESULTS AND CONCLUSIONS

Some models were calibrated and validated in urban areas; in particular we focused on homework trips during the morning rush hour. The choice and perception models were tested on an available data-base (comprising 500 interviews), concerning trips made in Parma (Italy), obtained by the Special Transport Project (PFT) of the Italian National Research Council and compared with the choice models (Multinomial Logit).

More complex models are gradually specified: considering only a choice level (Multinomial Logit) with fixed choice set or choice and perception levels.

The coefficients are estimated with the Maximum Likelihood method. The models with combined simulation of alternative perception and choice behaviour were calibrated using suitably implemented software, there being no suitable software available on the market. To carry out formal validation several statistical indicators were calculated.

The comparisons among some of the various specifications proposed, with and without perception of the alternatives, are summarized in the Tab. 1. In this table we report the values assumed by the calibrated coefficients and Student’s t test (shown between brackets). At the bottom of the table the values of the calculated indicators (Log likelihood at convergence, Log likelihood at zero, $\rho^2$, correct $\rho^2$, %right) are reported.

Table 1. Comparison among choice models and perception and choice models

<table>
<thead>
<tr>
<th>Mode</th>
<th>Attribute</th>
<th>Choice with fixed choice set</th>
<th>Perception and Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedestrian</td>
<td>Time [1/h]</td>
<td>-9.863 (-10.0)</td>
<td>-10.652 (-11.668)</td>
</tr>
<tr>
<td>All modes accept</td>
<td></td>
<td>(-10.652 (-11.668))</td>
<td>(-12.565 (-17.417))</td>
</tr>
<tr>
<td>pedestrian</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Car driver</td>
<td>Cars available</td>
<td>-3.033 (-9.1)</td>
<td>-2.498 (-2.768)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2.768 (-2.303))</td>
<td></td>
</tr>
<tr>
<td>Car passenger</td>
<td>Constant</td>
<td>-4.718 (-12.7)</td>
<td>-5.006 (-4.564)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-4.564 (-4.707))</td>
<td></td>
</tr>
<tr>
<td>Motorcycle</td>
<td>Constant</td>
<td>-4.041 (-9.5)</td>
<td>-4.172 (-4.335)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-4.335 (-4.169))</td>
<td></td>
</tr>
<tr>
<td>Bus</td>
<td>Constant</td>
<td>-3.992 (-9.6)</td>
<td>-4.385 (-4.821)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-4.821 (-5.0))</td>
<td></td>
</tr>
<tr>
<td>Business centre</td>
<td></td>
<td>-2.285 (-4.7)</td>
<td>-1.646 (-2.6)</td>
</tr>
<tr>
<td>Car</td>
<td>Cars available</td>
<td>2.238 (6.7)</td>
<td>6.203 (3.9)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.9)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\mu$</td>
<td>2.216 (4.5)</td>
<td>3.787 (4.4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Acc/egr [1/h]</td>
<td>-3.305 (-2.9)</td>
<td>-2.858 (-2.3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2.3)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>µ</td>
<td>-15.97 (-2.3)</td>
<td>-16.701 (-2.7)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2.7)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Waiting+on board</td>
<td>-4.333 (-2.0)</td>
<td>-4.582 (-2.5)</td>
</tr>
<tr>
<td></td>
<td>µ</td>
<td>(-2.5)</td>
<td></td>
</tr>
</tbody>
</table>

Log likelihood at zero | -804.72 | -804.72 | -804.72 | -804.72 | -804.72 |
Log likelihood at convergence | -542.07 | -500.85 | -465.22 | -492.77 | -448.53 |
$\rho^2$ | 0.326 | 0.378 | 0.422 | 0.388 | 0.443 |
$\rho^2_{e}$ | 0.318 | 0.364 | 0.409 | 0.366 | 0.426 |
%right | 0.604 | 0.648 | 0.662 | 0.618 | 0.660 |
The results of the calibration are highly significant and there is a growth of reproductive capacity of the models in which perception attributes were present, independent of the specifications used.

Analysis of the results shows that the perception and choice models increase the goodness of fit of the used statistics with respect to the choice model with fixed choice set. Indeed, in the models which explicitly simulate perception, the signs of the parameters are all consistent with the behavioural hypothesis. Besides, all the statistical indicators present some net improvements; in particular the %right improves by about 3+10%, while $\rho^2$ improves by about 20%.

There is a significant dependence between the mechanisms of decision of the individual and the parameters of the transport systems that are both quantitatively and qualitatively conditioned by the transportation system characteristics. Probably due to the perception models and attributes it is possible to identify the choice set most probably perceived by every user. Thus the problem of fixed choice set for each user is avoided, albeit with a greater computational price.

REFERENCES


