MODE CHOICE IN THE MORNING COMMUTE

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1 Introduction

The Jordanstown campus, hosting more than eleven and a half thousand students, is the largest one of the four campuses of University of Ulster. Jordanstown’s proximity to Belfast, 7 miles away, enables students and staff to live in the Belfast city and enjoy the nightlife there. On another hand, the easy access to various travel modes further ease students and staff’ commuting, including trains, university buses, taxis and private cars. Currently, more than 6,000 students and staff live in the Belfast city. This paper addresses the problem of mode choice arising from the students’ and staff’ morning commuting from Belfast to Jordanstown.

There are two ways available between Belfast and Jordanstown, one is the rail track the other the motorway. The motorway crosses the rail rack by flyovers and hence they have no interactions. However, the interactions among buses, taxis and private cars on the motorway are not ignorable. Also, in the recent years ever-growing traffic loads cause more and more serious, recurrent congestion on this motorway. In the Jordantown campus, there are about 800 parking spaces and all of them are free to access. Now every weekday morning so many students and staff commute by their private cars that many cars have to park on the grassland or roadside. Contrary to this, many university buses often operate at a not-full capacity. In addition, there is only one bus stop on the campus and taxis (and even private cars) dropping students and staff off at the bus stop further effect the operational efficiency of buses. Furthermore, there are a large number of seats available on trains every morning.

From the practical viewpoint, we need investigate the demand distribution over various modes (i.e., mode choice) and further observe those factors which have great impacts on individuals’ mode choice. As the final purpose, it is hoped this work can provide some implications for the management of the university parking facilities and the public transit services oriented to universities. To accomplish these, we first should have an effective tool. In the immediately following section we shall formulate a convex optimization model equivalent to the logit formula which is employed to carry mode split out. This model presumes that the disutility functions of all travel modes are separable and that hence there are no interactions among these travel modes. To get the disutilities, each mode is associated with a disutility generator. Subsequently, a solution algorithm is developed based on the convex combinations method. Extensions aim at treating the scenarios with symmetric and asymmetric mode interactions in Section 4. The last section offers a brief conclusion.

2 Model formulation

2.1. A convex optimization model for mode split

The mode split problem deals with mode choice, which results from a complex decision process. This process is influenced by a large number of factors, many of which are difficult to quantify and measure. To account for behavior of commuters in the morning period, we often employ the theory of discrete choice models, which describes individuals’ choices between competing alternatives. The underlying hypothesis of discrete
choice theory is that when facing with a choice situation, an individual prefers to an alternative with the least disutility, which, reflecting the individuals' preferences, is a function of the attributes of the alternatives as well as the decision makers’ characteristics. Since these factors are deterministically unmeasurable and unobservable and hence uncertain, the disutility measure is often treated as random.

Assume there are $M$ travel modes available, indexed by $m = 1, 2, \ldots, M$, each of which is associated with a disutility, $\Phi_m(a)$, where $a$ denotes the vector of variables composed of the commuters’ characteristics and the attributes of mode $m$. As such, an individual chooses mode $m$ only when $\Phi_m(a) \leq \Phi_{m'}(a)$ holds for all $m' = 1, 2, \ldots, M$. This implies the probability an individual with a given vector $a$ chooses mode $m$ is

$$p_m(a) = \text{Prob}\{\Phi_m(a) \leq \Phi_{m'}(a), \forall m' = 1, 2, \ldots, M\} \quad \forall m = 1, 2, \ldots, M$$

(1)

If the disutility random variables $\Phi_m(a)$ are independently and identically distributed Gumbel variates, the choice probability is then given by

$$p_m(a) = \frac{\exp[-\theta \phi_m(a)]}{\sum_{m'=1}^{M} \exp[-\theta \phi_{m'}(a)]} \quad \forall m = 1, 2, \ldots, M$$

(2)

where $\phi_m = E(\Phi_m)$, a deterministic disutility component, and $\theta$ is a scale parameter. This is the widely-used logit formula. In this context, we assume that all commuters have the identical characteristics and that the disutility of each mode only depends on the number of commuters choosing this mode and the others. This does not rule out the possibility the disutility is dependent on other factors, e.g., weather conditions.

Now replace $a$ with $d = (d_1, d_2, \ldots, d_M)$, where $d_m$ is the number of the commuters choosing mode $m$, $m = 1, 2, \ldots, M$. Then $p_m(a) = p_m(d)$.

An equivalent interpretation of $p_m$ is that it represents the fraction of commuters who choose mode $m$ to commute. This interpretation gives the number of the commuters who choose model $m$ within the study horizon, as follows

$$d_m = N \cdot \frac{\exp[-\theta \phi_m(d)]}{\sum_{m'=1}^{M} \exp[-\theta \phi_{m'}(d)]}$$

(3)

where $N$ denotes the number of students and staff from Belfast to the Jordanstown campus every morning. Obviously, $\sum_{m=1}^{M} d_m = N$ holds.

If only 2 modes are available, it is easy to solve the mode split pattern using Eq. (3). However, if the number of modes available is greater than 2, it is better to transform Eq. (3) into an equivalent convex optimization model to solve. The following theorem shows this is possible.

**Theorem 1** The mode split pattern can be obtained by solving the following mathematical program

$$\min \ z(d) = \sum_{m=1}^{M} \int_{0}^{d_m} [\phi_m(w) + \theta \ln w]dw$$

(4)

subject to

$$\sum_{m=1}^{M} d_m = N$$

(5)

$$d_m \geq 0 \quad \forall m = 1, 2, \ldots, M$$

(6)

which is strictly convex if each $\phi_m$ is nondecreasing.
Remark  This theorem assumes the disutility functions are separable, that is, the disutility of a mode is only dependent on the number of commuters choosing this mode. In practice, this condition seldom holds. As discussed in Section 1, the buses, taxis and privates cars move on the same motorway and, when the traffic flow increases up to the capacity of this motorway their interactions are not negligible. We will extend this program to treat this scenario with interactions among these modes later.

The strict convexity of program (4)–(6) ensures there is one and only one optimal solution. To solve this program, we need the determination of the disutility of each mode. The next section is set up for this purpose.

2.2. Disutility generators

Theorem 1 proposes an equivalent convex mathematical program to the logit choice model. To apply it to the morning commuting problem involving the multi-mode and departure choices, we further devise a technique which helps obtain the averaged or expected disutility of each travel mode. For this purpose, first review what bottleneck models are available.

The existing bottleneck models with endogenous individuals’ departure times can be grouped into two categories: deterministic and stochastic. The deterministic models require that congestion evolve over the period of departure in such a way that no commuter can strictly better off by changing his/her departure time. Within each class $m$ of commuters who choose the common mode $m$ this equilibrium condition reduces the condition that over the study horizon $T$ travel cost be the same for every commuter, i.e.,

$$\psi_m(t) = \begin{cases} \mu_m & h_m(t) \neq 0 \\ \geq \mu_m & h_m(t) = 0 \end{cases} \quad \forall t \in T$$

(7)

where $\psi_m(t)$ denotes the travel cost experienced by an individual who chooses mode $m$ to travel and depart at instant $t$, $\mu_m = \min \{ \psi_m(t) | t \in T \}$ and $h_m(t)$ the departure rate of class $m$ at instant $t$. This definition of equilibrium is widely employed in the practice of modelling the bottleneck congestion, including Hendrickson and Kocur (1981), Mahmassani and Herman (1984), Newell (1987, 1988), Arnott et al. (1993), Chu (1995), Yang and Huang (1997) and Ge and Zhang (2001) and others. If models of this kind are employed, the disutility is determined by

$$\phi_m(d_m) = \mu_m \quad \forall m = 1, 2, \ldots, M.$$  

(8)

The deterministic equilibrium model presumes that commuters have the perfect information about prevailing traffic conditions. This presumption faces the challenges of stochasticity in travel time perception arising from weather conditions, road work, incidents, vehicle disabilities, day-to-day fluctuations in demand and capacity, etc. On the contrary, the stochastic equilibrium model provides an alternative to capture one or multiple aspects of the uncertainty in traffic operation. De Palma et al. (1983) and Ben-Akiva et al. (1984) consider the uncertainty of commuters’ travel time perception and assume the commuters make their decision on departure times in terms of the logit choice model. As such, the probability (density) an individual in class $m$ chooses to depart at instant $t$ is

$$f_m(t) = \frac{\exp[-\eta \psi_m(t)]}{\int_T \exp[-\eta \psi_m(w)]dw}$$

(9)

where $\eta$ is a scale parameter. Therefore, the expected travel price of this individual is

$$\phi_m(d_m) = \int_T \psi_m(w)f_m(w)dw \quad \forall m = 1, 2, \ldots, M$$

(10)

Eqs (8) and (10) define two disutility generators, respectively corresponding to the deterministic and the stochastic equilibrium. Once the disutility of each mode is determined, we can solve program (4)–(6) for a pattern of demand split into each mode.
3 Solution algorithm

Since program (4)–(6) is convex the convex combinations method is applicable. Applying this method to program (4)–(6) requires, at every iteration $n$, a solution of the linear program below

$$\min z^n(e) = \nabla z^n(d^n)e^T = \sum_{m=1}^{M} [\phi_m(d^n_m) + \theta \ln d^n_m]e_m$$ (11)

over all feasible values of $e = (\cdots, e_m, \cdots)$, which satisfies the conservation and nonnegativity constraints. In this formulation, $d^n$ is known at this iteration and $e$ the auxiliary demand split pattern. In this program, since $\phi_m(d^n_m) + \theta \ln d^n_m$ are constant for all $m (= 1, 2, \cdots, M)$ the objective value is minimized when all commuters choose the mode with the least value of $\phi_m(d^n_m) + \theta \ln d^n_m$. If $l = \arg \min \{\phi_m(d^n_m) + \theta \ln d^n_m | 1 \leq m \leq M\}$ then the resultant auxiliary demand split pattern is $e^n_l = N$ and $e^n_m = 0 (m \neq l)$. This solution defines the descent direction $x^n = e^n - d^n$. This is similar to the all-or-nothing assignment in solving the stationary traffic assignment models.

The optimal move size can be determined by solving the equation

$$\frac{\partial z[n + (d^n + e^n - d^n)]}{\partial \alpha} = \sum_{m=1}^{M} (e^n_m - d^n_m)\{\phi_m[d_m + \alpha(e^n_m - d^n_m)] + \theta \ln[d_m + \alpha(e^n_m - d^n_m)]\} = 0$$ (12)

Since $\phi_m(\cdot)(m = 1, 2, \cdots, M)$ and $\ln(\cdot)$ are monotonic increasing it is easy to solve for $\alpha$ by the existing linear search techniques, e.g., the bisection method. The resultant move size $\alpha_n$ gives a better pattern $d^{n+1} = d^n + \alpha_n(e^n - d^n)$.

The algorithm can then be outlined as follows

Step 0: Initialization. Find an initial feasible demand split pattern $d^1$. Set $n = 1$.

Step 1: Disutility updating. Set $\psi^n_m = \psi_m(d^n_m), \forall m$.

Step 2: Descent direction finding. Search for a mode with the least value of $\phi_m + \theta \ln d_m$ ($m = 1, 2, \cdots, M$) and assign all commuters $N$ to the mode. This assignment yields an auxiliary demand split pattern $e^n$ and hence the descent direction $x^n = e^n - d^n$.

Step 3: Move size determining. Find $\alpha_n$ that solves Eq. (12).

Step 4: Demand split updating. Find $d^{n+1}$ by using $d^{n+1} = d^n + \alpha_n(e^n - d^n)$.

Step 5: Convergence testing. If the error measure $\sqrt{\sum_{m=1}^{M}(d^{n+1}_m - d^n_m)^2/N}$ satisfies the given tolerance, terminate. Otherwise, set $n = n + 1$ and go to step 1.

4 Extension to the scenario with mode interactions

This section addresses the problem of mode choice with mode interactions. As a special case the scenario with symmetric interactions can be formulated, by means of the line integral technique, as

$$\min z(d) = \int_L \Phi(w) \cdot dw + \sum_{m=1}^{M} \int_0^{d_m} \theta \ln wd\theta$$ (13)

constrained by (5)–(6), where $L$ denotes a curve from the origin to the point $(d_1, d_2, \cdots, d_M)$, and $\Phi = (\phi_1, \phi_2, \cdots, \phi_M)$. The first term in the objective function is a line integral. It can be shown that, under the assumption of symmetric interactions, i.e.,

$$\frac{\partial \phi_m(\cdots, d_m, \cdots, d_l, \cdots)}{\partial d_l} = \frac{\partial \phi_l(\cdots, d_m, \cdots, d_l, \cdots)}{\partial d_m}$$ (14)
the above formulation is equivalent to the logit formula. This proof is left to readers. Condition (14) ensures that the line integral is independent of the shape of curve $L$ and hence unique.

If the interactions are asymmetric (i.e., condition (14) does not hold) the mode split pattern can be generated by means of the following diagonalization procedure

Step 0: Initialization. Find an initial feasible mode split pattern $d^1$. Set $i = 1$.

Step 1: Diagonalization. Apply the algorithm developed in Section 3 to solve the diagonalized problem

$$
\min z(d) = \sum_{m=1}^{M} \int_0^{d_m} \left[ \phi(d_i^1, \ldots, d_i^{m-1}, w, d_i^{m+1}, \ldots, d_i^M) + \theta \ln w \right] dw
$$

(15)

constrained by (5)–(6). This produces $d^i$.

Step 2: Convergence test. If there is no significant improvement in the solution, terminate. Otherwise, set $i = i + 1$ and go to Step 1.

5 Concluding remarks

This paper has investigated the mode choice problem in the morning commute. If the operations of all modes are independent of one another, the problem can be formulated as a convex optimization model. The line integral and diagonalization techniques are employed to model those scenarios with symmetric and asymmetric mode interactions, respectively. In these formulations, the averaged or expected disutilities of each mode is produced by means of a disutility generator. If the deterministic equilibrium is realized for a mode, those commuters choosing the mode would experience the same disutility. If only the stochastic equilibrium is possible, the disutility associated with the mode is the expectation of the travel cost of the commuters travelling by the mode.

This paper has reported the theoretical results and the ongoing work is to calibrate/validate the proposed model.

References