ORIGIN/DESTINATION TRIP MATRIX
ESTIMATION WITH GENERATION CONSTRAINTS

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1 INTRODUCTION

The estimation of the Origin/Destination (O/D) trip matrix is a fundamental problem for the planning and evaluation of transportation systems. Different approaches have been proposed in literature in order to resolve the problem of estimating the O/D matrix, and they can be grouped in three classes: direct estimation, model estimation, and estimation from traffic counts.

Over the last years increasing attention has been devoted to methods for the estimation of O/D matrices from traffic counts, named in the following O/D count based estimation, ODCBE. These methods efficiently uses traffic flow measures combining them with other available information, in order to correct and improve an initial estimation of the O/D trip matrix possibly obtained by the two previous methodologies.

All the methods proposed to resolve the ODCBE need an initial estimate of the O/D matrix (called target matrix) and a set of link traffic flows measured on the transportation network considered. Virtually all of them are formulated as optimisation or mathematical programming problems with an objective function and a set of constraints. ODCBE models also need an estimation of the assignment matrix, that is of the O/D flow percentages that use each link of the network for which the traffic counts are available. These percentages depend on the generalised costs for all the links of the network, but for congested networks generally these costs are not known by the analyst and they are computed on the basis of the link flows forecasted through the assignment model, that again depend on the O/D matrix to be assigned. In the literature the problem of the circular dependence among the O/D matrix estimation and the traffic flow assignment has been studied by different authors as a problem of bi-level mathematical programming for deterministic user equilibrium assignment models (Fisk, 1988, 1989; Bell, 1991; Chen and Florian, 1994, Yang, 1995; Cascetta and Postorino, 2001).

In this paper a generalised least squares model and a stochastic assignment model are used to resolve the ODCBE problem. Particularly, the objective function is specified in order to take into account some generation constraints that prevent the emission from each origin zone to be greater than the actual one, as could be the case if no constraints in that sense are introduced.
2 FORMULATION OF THE ESTIMATION PROBLEM

It has been shown (Cascetta and Nguyen, 1985) that most ODCBE models can be cast in a general statistical framework. Among the different kind of estimators, particularly the GLS model is robust with respect to distribution assumptions, as it has been proved also numerically in Cascetta and Nguyen, 1988, Di Gangi, 1991.

In all cases the ODCBE problem can be cast in the general form:

\[ \mathbf{t}^* = \arg\min_{\mathbf{t} \geq 0} f_1 (\hat{\mathbf{t}}, \mathbf{t}) + f_2 (\mathbf{Ht}, \mathbf{v}) \]  

(1)

where \( \mathbf{t} \) is the O/D matrix to be estimated, \( \hat{\mathbf{t}} \) is the target matrix, \( \mathbf{v} \) are the traffic count values, and \( \mathbf{H} \) is the assignment matrix. The expression \( \mathbf{Ht} \) provides the traffic flow values forecasted by the assignment model using the current O/D trip matrix estimate.

The GLS solution of the ODCBE problem (1) requires the use of an assignment model in order to obtain the link flows from the current O/D matrix and to suitably compare them with those measured. When congested networks are considered, the ODCBE problem becomes more complex because for a congested network the link cost \( c \) depends on the traffic flows on it, and when the O/D demand values vary at each iteration (during the estimation procedure) the link choice probabilities also vary (\( \mathbf{H(t)=AP(c(t))} \), \( \mathbf{A} \) being the link-path incidence matrix, \( \mathbf{P} \) the path use percentage matrix).

The proposed formulation is an extension of the previous one (1) because a trip generation constraint is explicitly considered in the model. In fact, the model explicitly take into account the fact that the estimated trips generated by each zone cannot be greater than the maximum actual generation of the zone itself. It is well known that, given a set of counted traffic flows, there are more O/D matrices that, assigned to the network, reproduce the same observed flow values. The introduction of the trip generation constraint allows to select, among the possible solution of the problem, the matrix that satisfies the generation constraint and then is more reliable.

From an analytical point of view, the problem can be formulated as:

\[ \mathbf{t}^* = \arg\min_{\mathbf{t} \geq 0} f_1 (\hat{\mathbf{t}}, \mathbf{t}) + f_2 (\mathbf{Ht}, \mathbf{v}) + f_3 (\mathbf{t}, \mathbf{t}_{\text{o}}) \]  

(2)

where \( \mathbf{t}_{\text{o}} \) is the current estimation of the trip generation form the zone \( \text{o} \), \( \hat{\mathbf{t}}_{\text{o}} \) is the maximum forecasted emission from the zone \( \text{o} \), \( f_3 \) is a measure of the distance between \( \mathbf{t}_{\text{o}} \) and \( \hat{\mathbf{t}}_{\text{o}} \). The other quantities are the same as in the expression (1).
3 THE RESULTS

In order to evaluate the performances of the proposed model, some numerical experiments were carried out on a small test network, using a GLS objective function and a stochastic assignment model. The results have been evaluated with reference to the statistical index RRMSE (Relative Root Mean Square Errors).

First, the RRMSE is evaluated for the target matrix, \( RRMSE(\tilde{t}, t') \), and the estimated O/D matrix solution of the problem, \( RRMSE(t^*, t') \), as regards a true O/D matrix, known for the experiment. A second set of RRMSE values is computed for the traffic flows estimated with the target matrix, \( RRMSE(v(\tilde{t}), v) \), and the solution O/D matrix, \( RRMSE(v(t^*), v) \), where \( t' \) and \( v \) are the true O/D matrix and link flows.

The capability to reproduce flows can be evaluated not only as regards the counted links, but also the total link flow vector, and particularly the no-counted links (hold-out-sample), that is \( RRMSE(v(t^*)_{no\_counted}, v_{no\_counted}) \). This latter indicator is useful in order to verify the reliability of the O/D matrix for the whole network.

Just as an example, some results are reported in Table 1.

<table>
<thead>
<tr>
<th>Problem (1)</th>
<th>Problem (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without generation constraint</td>
<td>With generation constraint</td>
</tr>
<tr>
<td>( RRMSE(t^*, t') )</td>
<td>0.38</td>
</tr>
<tr>
<td>( RRMSE(v(t^*), v) )</td>
<td>0.06</td>
</tr>
<tr>
<td>( RRMSE(v(t^*)<em>{no_counted}, v</em>{no_counted}) )</td>
<td>0.25</td>
</tr>
</tbody>
</table>

It can be seen that the \( RRMSE(t^*, t') \) value obtained by resolving the problem (2) is significantly better than that obtained by resolving the problem (1), even if the traffic counts are well reproduced in both cases. But, with reference to the no-counted traffic flows, the problem (2) provides very good results if compared with the results provided by problem (1), i.e. the trip O/D matrix is more reliable in the first case, because it allows to better reproduce the traffic situation on the whole transportation network.

Further improvements, both in the O/D estimate and traffic count reproduction, have been obtained by using a fixed point approach, as proposed by Cascetta and Postorino (2001).

REFERENCES


