CONTINUOUS DYNAMIC NETWORK LOADING MODELS

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1 INTRODUCTION

The dynamic network equilibrium models have a renewed interest in Europe, Japan and North America. This is due to the design and implementation of Advanced Transport Telematic (ATT) systems require of network models which may evaluate dynamically such actions.

The main modeling approaches for the dynamic network equilibrium are: Aggregated macroscopic models in which traffic flows are regarded in an aggregated way as a fluid and microscopic approaches in which it is tried to understand the behavior of the system by modeling the individual vehicles. See Barceló (1998) for a overview of some currently used microscopic traffic simulators.

Daganzo (1995) considers that a basic theory of macroscopic traffic models for networks in the context of dynamic assignment would have to include a the least realistic models of:

1. Traffic behavior when the vehicle paths are known. This sub model is known as the dynamic network loading problem (DNLP) and consists of finding temporal arc volumes, arc travel times and path travel times given time-dependent path flow rates for a given time period.
2. Path choice when the time-varying arc times are known.
3. Equilibrium to reconcile the predictions items 1 and 2.

The macroscopic models for DNLP can be classified according to several criterion (see Wu (1998)). In this paper we consider two categories defined by whether arc exit functions are used or not:

1. Travel time function models (Astarita (1996), Ran and Boyce (1996), Wu (1998), Xu (1999)). These approaches assume that the travel time from the beginning to the end of an arc of the network can be expressed as an increasing function of the flow on

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the arc at the time. These models can be viewed as an extension of the static assignment model.

2. **Kinematic wave theory** (Daganzo (1995), Lebacque and Khoshyaran (1998)). These models represent the traffic phenomena by partial differential equations, where the behavior of the traffic at a given point in time-space is solely affected by the state of the system in a neighborhood of that point. These approaches rely on the discretization of their basic equations.

Any CNDLP must address three basic questions: (a) a performance link model, (b) intersection modeling and (c) partial flows modeling. The objective of this paper is to compare computationally and theoretically the item (a) on the cell transmission model of Daganzo (1994) and an extension of the travel time function of Xu (1999).

### 2 MODEL A: A CONTINUOUS TIME LINK MODEL BASED ON TRAVEL TIME FUNCTION

A first model for the CDNLP is obtained from the static assignment models where the traffic volume on a link is time dependent and the travel link traversal time of a user arriving at link \( a \) at instant \( t \) is solely determined by the (total) traffic volume still on link \( a \) at the instant \( t \). The exit time at the head of arc \( a \) of a user who enters the tail of the arc at time \( t \), is equal to

\[
\tau_a(t) = t + s_a(v_a(t)) \quad \text{for all } t \in [0, T_a]
\]

where \( T_a \) denotes the latest arrival instant at the tail node of arc \( a \), \( s_a(v_a(t)) \) is a traversal time of link \( a \) at instant \( t \) and \( v_a(t) \) is the traffic volume on link \( a \) at instant \( t \). This model considers that the arc dynamics on link \( a \) obey the equation:

\[
v_a(t) = \int_0^t b_a(y) dy - \int_0^t e_a(y) dy \quad \text{for all } t \in [0, T]
\]

where \([0, T]\) is the system period; \( e_a(t) \) is the exit flow rate into link \( a \) at instant \( t \in [0, T] \), and \( b_a(t) \) is the arrival flow rate into link \( a \) at instant \( t \in [0, T] \).

Our formulation assumes that the model is well posed defining solely the travel cost function and the entry flow rate into the arc \( a \). The resulting behavior does not necessarily satisfy the FIFO condition. Consider the following auxiliary function to give a full definition of this model which indicates if a user, who entered at the instant \( s \), has arrived at the link head at the instant \( t \).

\[
\delta_a(t) = \begin{cases} 
1 & \text{if } t \geq \tau_a(s) \\
0 & \text{otherwise}
\end{cases}
\]

The total amount of users that have gone out of arc \( a \) at the instant \( t \) satisfies:

\[
\int_0^t e_a(t) = \int_0^t b_a(y) \cdot \delta(t, y) dy
\]
In this work the model travel time function includes explicitly the capacity constraint on the link’s flows and the traffic light control system. The model takes into account the constraint

\[ v_a(t) \leq u_a \text{ for all } t \]

where \( u_a \) is the upper bound on the flow on the link \( a \). This constraint imposes when it is active an over saturation delay at the begin of the arc.

A function \( \Gamma_a \) is introduced in order to tackle the traffic signals. The total time of traversing an arc \( a \) at the instant \( t \) is given by formula:

\[ \Gamma_a(v_a(t)) \]

3 MODEL B: THE LWR MODEL FOR ONE LINK

The first order fluid approximation of traffic flow dynamics proposed by Lighthill and Whitham (1955) and Richards (1956) (the LWR model), provides a description of traffic behavior for a link using three variables that vary in time and space: flow, \( q_a \), density, \( k_a \) and speed \( v_a \).

All road link \( a \) without entrances or exist, satisfies the following conservation principle

\[
\frac{\partial}{\partial t} \int_{x_1}^{x_2} k_a(x,t) \, dx + q_a(x_2,t) - q_a(x_1,t) = 0
\]

for any locations \( x_1 \) and \( x_2 \) of the link and for any time \( t \). Under the assumption of \( k_a \) is differentiable, the previous equation can be expressed as the partial differential equation

\[
\frac{\partial k_a(x,t)}{\partial t} + \frac{\partial q_a(x,t)}{\partial x} = 0
\]

The basic LWR model assumes the relation between flow and density observed under steady state condition for a homogenous road, where \( Q \) is a differentiable nonnegative, function that is zero for \( k_a=0 \) and \( k_a=kJ \).

\[ q_a(x,t) = Q_a(k_a(x,t)) \]

The third variable, \( v_a \) is, by definition, \( v_a(x,t) = q_a(x,t)/k_a(x,t) \), then the relationship between speed and density as \( v_a(k_a) = Q_a(k_a)/k_a \).

Model A do not able to guarantee the respect to FIFO rule. Codina and Barceló (1996) show that the LWR model and a first order implicit regressive finite difference approximation hold FIFO properties.

Daganzo (1994) introduces a numerical procedure, the cell-transmission model, in order to approximate the LWR equations. This method converges to the proper solution under the general conditions that arise in traffic problems and it has been used in this work.
4 NUMERICAL RESULTS

From a theoretical point of view, it would be interesting to find weaker conditions on the traversal time functions that ensure the FIFO condition (Xu (1999)). We have illustrated that it is impossible for a simple case.

4.1 Comparison of models

The first computationally test that it is developed raises the comparison of the models described before, subject to general conditions that allow to make them comparable.

This numerical experiment has been carried out using $b_a(t) = 40000 \cdot t \cdot e^{-10\cdot t}$ and the following models:

A. It establishes the time of trip through the arc is given by a traversal time function. The considered function is of the type $s_a(v_a) = L/\nu_0 + \beta \cdot (v_a)^\alpha$, where $L$ is the length of the road link, $\nu_0$ is the free-flow speed, $v_a$ is the number of vehicles in the arc, and $\alpha$ and $\beta$ they are parameters of the function.

A’. It constitutes the previous model but taking into accounts the capacity constraint.

B. It is the LWR model and it is solved using the cells transmission algorithm raised by Daganzo (1995), considering a trapezoidal relation between the intensity of flow ($q$) and the density ($k$), with which the parameters to consider are the free flow speed ($\nu_0$), the maximum flow intensity ($q_{\text{max}}$), the speed of congestion disturbance propagation ($w$) and the possible maximum density or (jam density) in the arc ($k_{\text{max}}$). The value of $w$ is settled like 25 kilometers per hour.

B’. Based on model B, but instead of using a trapezoidal $q-k$ relationship, considers a functional form derived from the model A.

The obtained results are shown in Figure 1. It is observed the loss of FIFO property in the model A once stops the arrival of vehicles to the arc and the drained one begins, situation that does not take place in the other models.

![Figure 1. Test 1](image-url)
It is worth the trouble to make notice that the models A’ and B’ have a very similar performance throughout the simulation, since in both occasions the capacity is restricted and the travel speed is given by an equivalent function.

Finally it is possible to write down that the difference between models B and B’ takes place at the moment at which model B reaches the situation reproduced by the model A’ situation in which the simulation of model B’ takes care of the variations of the travel speed whereas model B stays simulating the same initial speed.

As conclusion, model A requires the capacity constraint for congested networks in order to palliate the loss of FIFO condition and for obtaining accurate predictions of travel time.

4.2 Empiric test

This test looks for to compare the real behavior of a way section with the estimations made by each one of the studied models. It was developed on the data of a traffic study made by the Transit and Transport Research Program (PIT) of the Universidad Nacional de Colombia in Bogotá (Colombia), study where tow groups distanced 3.96 kilometers, registered the matriculations of the vehicles were observed that circulated around the Luis Carlos Galán Avenue. It is important to mention that the Luis Carlos Galán Avenue is a 10 meters wide channel of a road, with two-way senses of circulation, that is located to the West of Bogotá and is characterized by non-congested flow conditions.

The obtained information corresponds to two days (Thursday, February 11 of 1999 and Monday, February 15 of 1999) in the East - South circulation sense (leaving the city) and is consolidated in 15 minutes intervals during a period of two hours (10:00 to 12:00), in where it was related the interval of analysis, the number of coincident vehicles (vehicles observed in both gauging points) and the average time used by these to cross the 3.96 kilometers. Additionally the percentage that represented the volume observed in points respect to the total of observed vehicles.

Figure 2. Test 2
The function that appears in the Figure 2 represent the main simulation result, including the estimations done by the models A and B with the real performance of the arc. Because the Luis Carlos Galán Avenue, object of the simulation, is not congested, the models A’ and B’ such produces the same results that the models A and B, thus are excluded from the representation. In agreement with the results observed in the graph, in non-congested conditions, as much the model A as the B suitably represents the performance of the traffic in a real arc, with which the goodness of both models acquires knowledge.

REFERENCES


