OPTIMIZATION MODELS AND ALGORITHMS FOR AN INTEGRATED DEMAND RESPONSIVE FEEDER BUS SYSTEM

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1. GENERAL CONTEXT OF THE SERVICE

In this paper we analyze a new transportation system where several transit lines of different kinds are operated in an integrated environment; here, traditional fixed lines, with regular timetable and fixed itinerary, interact with flexible lines providing demand responsive service. The feature of this system is to provide, by way of integration, an almost personalized transportation service at the cost of a traditional transportation system; such a system is particularly fitting for medium/low demand transportation settings, and it represents a solution toward a sustainable mobility in urban and sub-urban areas. Moreover, this system is perfectly suited for feeding high swift, high capacity main lines, such as light tramways or underground, thus providing a suitable connection to the regular transit system to users located in low demand areas. According to the recent review by Gendreau and Potvin (1998), there is a lack of both literature and models in this field.

The system here proposed is designed as follows. Let us consider a set bearer swift lines, such as underground or light surface rail, and a set of flexible lines, possibly acting as feeders of the previous ones. The traversal of any line can be described, as usually, in terms of a list of time-tabled trips. Moreover, flexible lines can be described in terms of compulsory stops, where the vehicle must transit within a specified time window and where passengers may transfer from and to connecting lines (both fixed and flexible). Beside compulsory stops, a set of on-demand stops (optional stops hereafter) is available to the users; each optional stop is univocally located within a pair of compulsory stops which are consecutive along the line. Within this framework, the user selects among the whole set of stops, optional and compulsory, those which are closest to its origin and to its destination, and issues a request to the system concerning the optional stops involved, if any. Note that, depending on the actual design of the service network, the optimal itinerary of each user is not univocally given but it depends on the current schedule and on the availability of the transportation system. In general, the itinerary of a user is described by a sequence of portion of trips on flexible and fixed lines.
On flexible lines, optional stops are visited on-demand, while service is guaranteed at compulsory stops, thus providing a basic transportation service to users not issuing any call (passive users). When no requests involve optional stops, the vehicle travels along the shortest path on the network between each pair of consecutive compulsory stops. Indeed, serving a request involving an optional stop requires rerouting the vehicle for that part of route within the associated compulsory stops, eventually delaying transit time at the later stops.

Traveling times of the arcs of the physical network are known a priori. Time windows at compulsory stops provide a certain degree of flexibility along the line, while acting as a warranty on the service quality for passive users.

An additional issue to be considered is synchronization at compulsory stops in order to guarantee requested line transfers, as discussed in the following. Indeed, synchronization might be dealt on an on-demand base, as optional stops are, even though at the detriment of system's robustness.

Let us identify which are the main aspects to be considered in the proposed transport system, given that capacity constraints are not explicitly handled; this is due to passive users and to the assumption that capacity is not a tight constraint on a low demand setting:

Synchronization:
Given two or more lines sharing the same compulsory stop (or with stops in walking distance) and intersecting time windows, two possible ways to deal with synchronization can be considered:

1.a) synchronization is solved at design level, that is, it is enforced regardless of existing demand for transfer. In this case the involved time windows are accordingly modified so that all vehicles must be present at the same time at the same stop (or proximity).

1.b) synchronization is determined dynamically, depending on the actual set of request. This approach implies that a decision must be taken each time requests are processed. In fact allowing a synchronization may enforce a delay in a vehicle schedule, making later detours unfeasible. In this case a suitable evaluation function must be provided in such a way that the time lost to enforce synchronization can be compared with the detours that could have been made instead.

Request management
From the management point of view, we consider two possible ways to collect requests; the first one goes towards a real-time service, the second allows better service levels and a more profitable management:

2.a) Requests are collected and managed on-line and continuously during the service operation. This means that the optimization of the itineraries (both from the user and from the service management viewpoint) is done by considering one request at a time. Once a request have been accepted, the flexible line itineraries are possibly modified from their next compulsory stop on. Note that, although the position of the vehicle must be kept updated, a sophisticated vehicle positioning system is not necessary, since the knowledge of the last served compulsory stop already defines the position of the vehicle. In this context, the initial itinerary might already include some optional stops on the basis of "historical" stochastic information, anticipating the requests to come. In any case, the optimization problem to be solved for each new incoming
request is rather trivial, though the attained result could be far from the global optimality that could have been reached if knowing in advance all the requests.

2.b) Requests are collected off-line, and some time before the requested departure time (say 30') are reconfirmed providing the users with their complete itineraries (boarding and alighting stops, departure and arrival times, connections). Obviously, in this case the optimization of the vehicle itineraries can be made on a broader basis, thus better results can be obtained with respect to the on-line approach. This means that more difficult optimization problems must be solved each time a set of requests is processed.

Request classification
Each user specifies either the earliest pick up time at the origin point or the latest arrival time at destination. The user travel time is computed as the difference either between the actual arrival time and the desired pick up time (if given), or between the desired arrival time and the actual departure time (if given). Moreover, for each pair of origin/destination stops of the network (or more broadly for pairs of origin/destination areas) an indicative ideal travel time is known. This information is used to define the minimum service level that the transit company assures to its users: for example the company guarantees that the maximum travel time is no more than a given multiple of the ideal travel time. This maximum allowed travel time, along with the desired pick up or drop off time, defines the so called customer time window. Requests are distinguished on the basis of the minimum service level guaranteed. We have two cases to consider:

3.a) Single class of users: the same service level is guaranteed to all users.

3.b) Multiple classes of users: several classes of users are considered each having its own level of service and its own service fare; for example we may think of an express service with a small customer time window, and a regular service with larger time window.

Additional flexibility
In order to satisfy the maximum number of requests, it may be useful to introduce additional flexibility in the service. For example suppose that some requests cannot be satisfied because the vehicle can not feasibly visit the requested stops at the requested time. In this case the operator may propose to the user a displacement in space (proposing him/her to board or alight in stops located nearby the desired ones) or a displacement in time, (proposing alternative pick up or drop off times). These displacement may involve a discount in the service fare.

Finally, we assume that, in case a request can not be served by the regular service, a collective taxi or other forms of personalized transportation services are provided to the user without extra fares so that the minimum guaranteed service level is maintained.

2. MATHEMATICAL MODELS

We summarize two mathematical models used to represent the optimization problem of managing a specific flexible transportation system which can be described in terms of the framework defined above. More details can be found in (Crainic, Malucelli and Nonato 2001a). In particular, let us
make some assumptions with respect of the different implementation choices that have been discussed above. Let us focus on a transportation system where synchronization is defined at design level (1.a), requests are managed off-line (2.b), a single class of users is considered (3.a), no additional flexibility is allowed neither in time nor in space, and overlapping optimization periods are considered. However, the following mathematical models can be generalized in order to consider all the other possible cases. The following data are given:

- a set of lines \( L \) and for each line \( l \in L \) a set of occurrences of the trip \( 1, \ldots, K_l \);
- each line \( l \) and trip occurrence \( k \) is defined by a sequence of compulsory stops \( f^l_k \) with \( h = 1, \ldots, n^l \) and \( f^l_1 = f^l_n \) is the terminal of the line;
- for each compulsory stop \( f^l_k \) of line \( h \) and trip occurrence \( l \), a time window \( [a^l_{hk}, b^l_{hk}] \) defines when the vehicle operating that trip occurrence is allowed to leave from the stop; of course, for fixed lines, all stops are compulsory and each time window reduces to a single value (i.e. the scheduled departure time);
- between any two consecutive compulsory stops \( f^l_k \) and \( f^l_{k+1} \) a set of optional stops \( N^l_{hk} \) is defined; the union of all \( N^l_{hk} \) and of all compulsory stops \( f^l_k \) defines the set of stops served by the system; let us call \( N \) this set. Moreover let us denote by \( N^l_k \) the set of stops involved by the \( k \)th occurrence of line \( l \).
- \( R \) is the set of requests currently available; each request \( r \) is specified by the desired departure stop \( s(r) \in N \), the desired arrival node \( d(r) \in N \), the customer time window \( [a(r), b(r)] \), that is the interval of time in which customer of request \( r \) is allowed to travel, the benefit of the request \( u(r) \) (for example the ticket fare);
- the travel time and the travel cost between any two nodes \( i \) and \( j \) of the physical network are known and are given by \( \tau_{ij} \) and \( c_{ij} \) respectively;
- the planning horizon is \( T \), where a suitable value for \( T \) can be twice the minimum common multiple of the line headways.

A straightforward representation of the transportation system is by means of a suitable space-time network. For practical reasons the time horizon is discretized (for example into one minute intervals), so that time is an integer in \([0, \ldots, T]\). The nodes of the network are pairs of physical locations and instants of time: in practice they correspond to the stops in \( N \) associated with the time when a vehicle may feasibly visit that particular stop. Therefore, there may exist several nodes for each stop since a flexible line stop can be feasibly visited anytime within a time interval, the stop can be served by more vehicles operating the same line, and finally it may belong to more than one line. In particular for each compulsory stop \( f^l_k \) the graph contains all the nodes \( \langle f^l_k, t \rangle \) for each integer \( t \in [a^l_{hk}, b^l_{hk}] \). In addition, the node set must include a pair of nodes \( r' \) and \( r'' \) for each request \( r \), corresponding to the event of departing and arriving, respectively; these nodes are not associated with a time. The arcs represent boardings, alightings, waitings and travels. Special arcs from \( r' \) to \( r'' \) represent a taxi ride from the origin to the destination of each request; they are introduced in order to guarantee the existence of a feasible solution.
The problem of optimally routing the vehicles and assigning requests to vehicles can be formulated as a multicommodity network flow on the space-time network defined above. The size of the multicommodity flow problem depends on the time horizon $T$, on the number of requests, on the number of vehicles and on the range of the time windows. It is very unlikely that this kind of formulation, though straightforward, may lead to efficient solution approaches.

The fact that vehicle itineraries and passenger itineraries (called routes) can be decomposed into sequences of paths between consecutive compulsory stops suggests an alternative formulation of the problem, which has been succesfully used for the single line case (see Crainic, Malucelli and Nonato 2001b,c). Paths between consecutive compulsory stops can be taken as basic elements of the formal description of the problem. We call segment $h$ of vehicle $lk$ the graph whose node set is $N_{lk}^h \cup \{j_{h-1}^l, j_h^l\}$ and whose arcs correspond to possible vehicle movements between pairs of sites. $P_{lk}^h$ be the set of feasible paths for vehicle $lk$ in segment $h$. Assuming that the feasible paths are given for each segment, we can construct a suitable path graph in order to formulate the problem. The node set is given by the union of all $P_{lk}^h$, for each $l, k$ and $h$. Moreover the node set includes the pairs $r'$ and $r''$ for each request $r$, representing the starting and the ending point of the request route. The arcs of the path graph are of four kinds: compatibility arcs, connecting paths belonging to consecutive segments that can be operated by the same vehicle, connection arcs connecting paths belonging to segments of different lines such that passenger can transfer from one line to the other, boarding and lighting arcs, taxi ride arcs. The problem consists of selecting exactly one path per segment so that paths of consecutive segments are compatible, and to determine the optimal routes of all request by using the selected paths.

REFERENCES


