EXTENSIONS OF THE UNCAPACITATED HUB LOCATION PROBLEM FOR APPLICATIONS IN INTERMODAL PUBLIC TRANSPORTATION

Alfredo Marín* Stefan Nickel† Anita Schöbel‡ Tim Sonneborn†§

Abstract

In this talk we present extensions of the uncapacitated hub location problem (UHL) with multiple allocation (see e.g. [Campbell, 1994], [Hamacher et al., 2000]), which can be applied to network design problems in intermodal public transportation. These extended models can be considered as a mixture of location and network design problems. We show how the extensions are related to the basic UHL and to each other. Moreover, we introduce some strengthened mixed integer formulations and propose efficient solution algorithms for all models.

1 THREE DIFFERENT UHL MODELS

A typical hub location problem in a network consists of two parts: the location part, in which certain nodes have to be chosen to be established as transshipment points (or hubs), and the allocation part, in which the request of certain origin-destination pairs has to be routed through the hubs.

*Departamento de Estadística e Investigación Operativa, Facultad de Matemáticas, Universidad de Murcia, Spain. Author supported by Plan Nacional de Investigación Científica, Desarrollo e Innovación Tecnológica (I+D+I), project TIC2000-1750-C06-06.
†Fraunhofer Institut Techno- und Wirtschaftsmathematik (ITWM), Kaiserslautern, Germany
‡Fachbereich Mathematik, Universität Kaiserslautern, Germany
§corresponding author, email: sonneborn@itwm.fhg.de
In all types of UHLs considered in this paper and in the literature, we have at least two kinds of costs occurring in the network: transportation costs for routing each unit (e.g. passenger), and fixed costs for establishing hub nodes. Transportation costs usually satisfy the triangle inequality. In addition, the transportation cost of each unit between two hub nodes is smaller than between a hub and a non-hub node. As we allow multiple allocation, units sharing the same origin (or destination, respectively) may be routed through different hubs.

**Basic UHL**

In the basic UHL model, introduced in [Campbell, 1994] and often referred to in the literature (see e.g. [Hamacher et al., 2000]), we have three additional characteristics:

(a) No fixed costs for establishing hub edges have to be paid.

(b) No fixed costs for establishing spoke edges have to be paid.

(c) The flow of every origin–destination pair has to be routed via hub nodes only.

As a consequence, every unit of flow is routed via one or two hub nodes (see Figure 1).

![Diagram](attachment:image.png)

**Figure 1**: Example of a transportation path for an origin–destination pair in a basic UHL network, routed via two hubs. Black circles denote hub nodes, white ones non-hub nodes. The transportation path is described by arrows, additional hub edges by dashed lines.

This model is suitable e.g. for air passenger and cargo transportation, where small planes connect small airports to hub airports, and big planes operate directly between hub airports.
**Model PT-UHL**

In the first UHL model for public transportation, called PT-UHL and introduced in [Nickel et al., 2001], the assumption \((a)\) of the basic UHL model is not satisfied. For every hub edge to be established, a certain fixed cost has to be paid. Furthermore, we require an additional property:

\((d)\) Spoke edges are allowed as first and last edge of every path only.

It follows that every unit of flow is routed via one, two, three, or more hubs (see Figure 2).

![Diagram](image)

**Figure 2:** Example of a transportation path for an origin-destination pair in a PT-UHL network, routed via three hubs.

This extended model can be applied to an urban public transportation network, in which rapid transit lines (metro or light rail) run in the inner part of the city, and shuttle buses or taxis connect the bus stations in the outskirts directly to a nearby transit station. Recently, a similar model has been developed for an application in the telecommunication sector [Choi et al., 2001].

**Model GPT-UHL**

The second UHL model for public transportation is a generalization of the first one, therefore called GPT-UHL and introduced in [Nickel et al., 2001]. Here, all three assumptions of the basic UHL are not satisfied. Fixed costs have to be paid for every hub or spoke edge to be built, and the routing of units is not restricted through hub nodes only. However, to guarantee a certain comfort for each trip, we require the following assumption to be hold:

\((e)\) At most two changes are allowed for each trip. More specifically, the path of each
origin–destination pair may change first from a spoke to a hub edge, and then from a hub to a spoke edge.

Hence, the routing of each unit can use any number of hubs, including zero (see Figure 3).

![Diagram of transportation path](image)

**Figure 3:** Example of a transportation path for an origin–destination pair in a GPT-UHL network, routed via three hubs and two non-hubs.

This generalized model is suitable in an urban public transportation network with rapid transit lines in the hub level (as in PT-UHL), but normal buses in the spoke level. These buses collect all passengers from different bus stations, and bring them to a nearby transit station.

## 2 Relations between the Models

We observe the following interactions between the three models.

**Basic UHL ⇒ PT-UHL:** Once the location part of the basic UHL is solved, there remains a network design problem (see e.g. [Manganti and Wong, 1984]) for the hub level, i.e. the hub edges have to be located, and the origin–destination pairs have to be routed from their first to their last hub node.

**PT-UHL ⇒ GPT-UHL:** If the location part of PT-UHL is solved, i.e. the location of hub nodes and hub edges are known, another network design problem remains for the spoke level: the spoke edges have to be located, and the origin–destination pairs have to be routed from their origin to their first hub, and from their last hub to their destination.

**Basic-UHL ⇒ GPT-UHL:** If the locations of the hub nodes are fixed, we have to solve a two-level network design problem, i.e. locating hub and spoke edges, and determining the complete transportation paths for all origin–destination pairs.
Clearly, the location and network design parts have to be solved simultaneously because the location of hub or spoke edges can also influence the location of hub nodes.

3 MILP Formulations and Solution Algorithms

We present tighter mixed integer linear programs than the ones proposed in literature for all three models. We adapt some of the polyhedral ideas developed in [Hamacher et al., 2000] for the basic UHL to the two new models. Moreover, we present heuristic algorithms, including a Langrangian approach, to solve models PT-UHL and GPT-UHL. We discuss the performance of different solution approaches on a set of numerical examples.

References


