OPTIMIZATION MODELS FOR THE URBAN PARK PRICING PROBLEM

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1 INTRODUCTION

Park pricing strategies are an important tool for rebalancing the modal split between personal car and transit systems in urban area. In fact, the high levels of congestion are mainly due to the preference of users for the private car system. In order to obtain a more equilibrate modal split it is possible, jointly with the improvement of transit system quality, to impose fares on use of private cars; it can be obtained by road pricing and/or park pricing strategies. Park pricing strategies are the simplest ones, since they can be managed without the adoption of advanced technologies.

In this paper some park pricing strategies are proposed and some optimization models are formalized; these optimization models search for the optimal parking fares optimizing the value of an objective function. In the paper, therefore, some solution algorithms are proposed and numerical results obtained on a test site are reported.

2 PARK PRICING STRATEGIES

The park pricing problem can be framed as a Multimodal Network Design Problem (Montella et al., 2000) in which the decisional variables are the parking fares; the need for a multimodal approach arises by the aims of the park pricing (rebalancing the modal split).

In order to model the park pricing problem there are two main aspects to establish:

– organization of parking zones and respective fares;
– choice of objective function.

There are two commonly adopted methods of choosing the parking zones:

– to decide only if a zone should be fared: all the priced zones have the same fare;
– to decide different fares for different priced zones (zone pricing).

In this paper we propose another strategy based on the possibility to park price in function of the OD couple (OD pair pricing); in this way it is possible to avoid a strong reduction of relative accessibility for the zones than are not efficiently served by transit systems. Using this strategy it is possible to raise the fares for the OD couple that have the transit as a valid alternative, so to limit the reduction of total accessibility.
Different objective functions can be adopted in the park pricing model; in this paper the following are proposed (and tested):
- system cost minimization;
- social cost minimization;
- total accessibility maximization.

3 OPTIMIZATION MODELS AND ALGORITHMS

Park pricing optimization model can be formalized as:

\[
y^* = \arg \min_y w(y, f_M^*)
\]

s.t.

\[
f_M^* = f_M[c_M(y, f_M^*)]
\]

\[0 \leq y_j \leq y_{MAX} \quad \forall j\]

where:
- \(y\) is the decisional variable vector (park fare vector); each component of vector, \(y_j\), is the value of parking fares for a zone (or for a couple OD) \(j\);
- \(f_M^*\) is the multimodal equilibrium flow vector; each component of vector, \(f_M l\), is the equilibrium flow on the link \(l\) of the multimodal network;
- \(c_M\) is the multimodal link cost vector; each component of vector, \(c_M l\), is the cost on the link \(l\) of the multimodal network;
- \(y_{MAX}\) is the maximum value for the parking fare.

The relationship (2) represents the multimodal equilibrium assignment model, approached as a fixed-point problem, that can be solved using different methods (Cascetta, 2001); in Montella et al. (2002) there are proposed and compared some efficient algorithms to solve the multimodal assignment problem. All the constraints on the feasibility of flows are implicit in multimodal equilibrium assignment model (2).

The multimodal network should be one network in which there are represented all modes (hypernetwork) or the formal union of separated networks, one for each mode; this choice depends on the multimodal assignment model used.

The decisional variables \(y_j\) can be assumed continuous or discrete; if the variables are assumed discrete, the fare of a zone \(j\), \(p_j\), can be expressed as:

\[p_j = p_0 y_j \quad (€/h)\]

where:
- \(p_0\) is a prefixed value, e.g. 0,25 €/h;
- \(y_j\) is an integer number, constrained in the interval \([0, y_{MAX}]\) (e.g. \(y_{MAX} = 8\)).
Under this assumption, if the park pricing strategy uses a unique fare for all zones, it is possible to enumerate all the feasible solutions \((y_{MAX} + 1)\) and calculate the objective function for each of them.

For the other two strategies the total number of solution is:

- \((y_{MAX} + 1)^{nz}\), for the zone pricing strategy;
- \((y_{MAX} + 1)^{[nz \times (nz - 1)]}\), for the OD pair pricing strategy.

In these cases, the exhaustive approach is intractable also for only 5 zones (and \(y_{MAX} = 8\)); in fact, the feasible solutions are 59,049 for the zone pricing strategy and \(1.22 \times 10^{19}\) for the OD pair pricing strategy. The combinatorial problem is NP-hard and should be tackled with heuristic or meta-heuristic algorithms.

If the decisional variables are assumed continuous, a (numerical) gradient algorithm should be adopted, but with calculation times unacceptable for real dimension cases (at each iteration, in order to calculate the descent direction, it is necessary to run a multimodal assignment for each variables).

In this paper a meta-heuristic algorithm for the solution of the discrete formulation of the problem is proposed.

### 3.1 Objective function specifications

Objective function \(w(.)\) can specified in different ways. In this paper there are proposed three different objective functions: System Optimum; Accessibility Optimum and Social Optimum. The first is a classical specification of the System Optimum Problem (Cascetta, 2001); the second is a maximization accessibility problem and the latter is a minimization social costs problem. The three optimisation problems can be formulated as:

- **System Optimum**:

\[
y^* = \arg \min_y \left[ w(y, f_M^*) \right] = \arg \min_y \left[ UC(y, f_M^*) \right]
\]

s.t.

\[
f_M^* = f_M[c_M(y, f_M^*)]
\]

\[
0 \leq y_j \leq y_{MAX} \quad \forall j
\]

where:

\(UC\) is the sum of the multimodal user costs.

- **Accessibility Optimum**:

\[
y^* = \arg \max_y \left[ w(y, f_M^*) \right] = \arg \max_y \left[ \beta_{Acc}^T Acc(y, f_M^*) \right]
\]

s.t.
\[ f_M^* = f_M[c_M(y, f_M^*)] \]
\[ 0 \leq y_j \leq y_{MAX} \quad \forall j \]

where:

- \( \beta_{Acc} \) is the accessibility relative weight vector; each component of vector, \( \beta_{Acc}^i \), is the relative weight value for the zone \( i \) with \( \sum_i \beta_{Acc}^i = 1 \);
- \( Acc \) is the accessibility vector; each component of vector, \( Acc_i \), is the multimodal passive accessibility value for the zone \( i \).

- **Social Optimum:**

\[ y^* = \arg \min_y \left[ w(y, f_M^*) \right] = \arg \min_y \left[ TRC(y, f_M^*) + (\beta_{VOT})^T UC(y, f_M^*) - (\beta_{TSU})^T D_{TSU}(y, f_M^*) \right] \]

s.t.
\[ f_M^* = f_M[c_M(y, f_M^*)] \]
\[ 0 \leq y_j \leq y_{MAX} \quad \forall j \]

where:

- \( TRC \) is the Transit Regional Contribution;
- \( \beta_{VOT} \) is the Value of Time (VOT) vector; each component of vector, \( \beta_{VOT}^i \), is the VOT value for user category \( i \);
- \( UC \) is the user costs vector; each component of vector, \( UC_i \), is the sum of the multimodal user costs for user category \( i \);
- \( \beta_{TSU} \) is the social value of Transit System User (TSU) vector; each component of vector, \( \beta_{TSU}^i \), is the social value for each person, belonging to user category \( i \), that uses transit system;
- \( D_{TSU} \) is the Transit System Users vector; each component of vector, \( D_{TSU}^i \), is the demand flow, belonging to user category \( i \), that uses transit system.

### 3.2 Solution algorithm

The solution algorithm proposed in this paper can be adopted for any specification of objective function. This algorithm, usable for discrete variables, is meta-heuristic; the phases of the algorithm are:

**Phase 1 – Exhaustive monodimensional optimization**

Each variable \( y_j \) is optimised in exhaustive way independently by other variables (assumed equal to 0); the resulting value is indicated as \( y_j^{MO} \). This phase requires \([y_{MAX} + 1] \times n_z\) objective function evaluations, for the zone pricing strategy, and
[(y_{MAX} + 1) \times (n_z \times (n_z - 1))], for the OD pair pricing strategy. At the end of this phase is obtained a starting solution for the next phase; this solution is the vector \( y^{MO} = [..., y_j^{MO}, ...] \). The corresponding value of objective function is \( w(y^{MO}) \).

**Phase 2 – Neighbourhood Search local optimization**

Starting from the solution obtained in the previous phase, a Neighbourhood Search algorithm is performed; this algorithm evaluates the solutions (Neighbours) that can be obtained changing the current solution by an elementary move (e.g. \( y_j \rightarrow y_j + 1 \) or \( y_j \rightarrow y_j - 1 \), operating on only one variable). The algorithm ends when no elementary move is able to improve the objective function value.

The obtained solution can be assumed as a “good” local optimum, considering the starting solution, \( y^{MO} \), of local optimisation algorithm.

### 4 NUMERICAL RESULTS

The proposed model and the algorithm were tested on trial and real networks; in the paper there are described the results for different objective functions, pricing policies and demand levels.

### REFERENCES

