1 THE NETWORK DESIGN PROBLEM

The concentration of human activities in urban areas has induced an increase in travel demand. This has brought urban transport systems to considerable levels of congestion, with negative environmental effects and external diseconomies leading to a poor quality of life. Therefore quantitative methods for urban network design have been developed in order to assure more effective results.

The network design problem (reviews in Magnanti and Wong 1984, Cascetta 2001) has recently been treated in the literature using different models and algorithms. The proposed algorithms in the literature can first of all be classified according to type of decision variables: integer (Branch and Bound, Billheimer and Gray, Poorzahedy and Turnquist), continuous (Marcotte, Davis, Friesz and Shah, Clegg et al.) and mixed integer (Cantarella and Vitetta). In relation to solution algorithms, network design can be divided into heuristic (Billheimer and Gray, Cantarella and Vitetta, Friesz and Shah, etc.), exact (brute force, Branch and Bound, etc.) and local convergent (Davis, Clegg et al.).

This article deals with the problem of urban road network design, with the hypothesis of a congested network and rigid travel demand. The decision variables are the network layout and the signal settings at the junctions; the objective is the minimization of the user total travel time. The users’ behaviour is simulated with a deterministic user equilibrium model. Solution algorithms proposed in this paper are based on heuristic methods with mixed integer variables.

The network design problem has been variously formulated in the literature. Its more consolidated formulation is (Cascetta, 1999):

$$y^* = \arg \min_y \left( \max_w \{ f^*(y) \} \right)$$

subject to
\[ f^* = \Delta(y)P[y, g(f^*, y)]d[g(f^*, y)] \quad \text{SUE} \]
\[ \text{or} \]
\[ c(f^*, y)^T (f - f^*) \geq 0 \quad \forall f \in S_y(y, d) \quad \text{DUE} \]
\[ y, f^* \in E \quad \text{(external constraints)} \]
\[ y, f^* \in T \quad \text{(technical constraints)} \]

with

- \( w = \gamma_{usr} \cdot w_{usr} + \gamma_{man} \cdot w_{man} + \gamma_{com} \cdot w_{com} \) objective function\(^1\), with:
  - \( w_{usr} \) objective of transport system users;
  - \( w_{man} \) objective of transport system manager;
  - \( w_{com} \) objective of community;
  - \( \gamma_{usr}, \gamma_{man}, \gamma_{com} \) weights of the different categories of objectives;
- \( y^T = [y_{top}^T, y_{cap}^T, y_{pri}^T] \) decision variables vector, with
  - \( y_{top} \) vector of variables related to network topology (for example: directions of lanes of road links or routes of transit lines);
  - \( y_{cap} \) vector of variables related to links capacity (frequency of transit lines, signal settings, etc.);
  - \( y_{pri} \) vector of variables related to prices and rates (road and park pricing, bus and railway rates, etc.);
- \( \Delta \) link-path incidence matrix, which has as many rows as the number of links of the network and as many columns as the number of paths joining all the origin-destination pairs; the generic element \( \delta_{ij} \) assumes the value 1 if the link \( i \) belongs to the path \( j \), otherwise it assumes the value 0;
- \( f \) link flow vector;
- \( c \) link cost vector, which depends on an appropriate cost function\(^2\) from the link flow vector \( f \) in equilibrium condition, and so it depends on users’ behaviour and on network configuration \( y \);
- \( g \) path cost vector;
- \( d \) origin-destination demand flow vector, which has as many elements as the number of origin-destination pairs;
- \( P \) path choice probability matrix;

The formulation used in this study is obtained from the general formulation expounded above applying the following specifications:

- \( d \) constant (rigid travel demand);
- \( g = \Delta^T c \) (absence of non-additive path costs);

\(^1\) Instead of this formulation in terms of linear combination, a multicriteria analysis can be effected.
\(^2\) The hypotheses of continuous cost function and of strictly increasing monotonic cost function are assumed, so the existence and the singleness of the solution of the equilibrium model are guaranteed.
- \( w = c^T f \) to be minimized (objective function that considers only the objectives of transport system users, through the generalized transport cost supported by transport system users);
- \( y^T = [y_{top}^T, y_{cap}^T] \) decision variables vector, with
  - \( y_{top} \) vector of links' layout;
  - \( y_{cap} \) vector of signal settings in the junctions;
- \( c(f^*, y)^T (f - f^*) \geq 0 \quad \forall f \in S_f (y, d) \) (deterministic user equilibrium);
- \( E = T = \emptyset \) (absence of external and technical constraints).

The used formulation is justifiable by the aim of this paper, which is testing efficiency of different algorithms proposed. The use of more complex objective functions (at worst the use of a multicriteria analysis) could make the comparison of obtained results more difficult.

### 2 PROPOSED ALGORITHMS

In this paper several heuristics were proposed, specified and applied to solve the network design problem in an urban area. Genetic Algorithms (Holland 1975), Tabu Search (Glover 1989), Simulated Annealing (Kirkpatrick \textit{et al.} 1984) and a local convergence algorithm were used.

All proposed algorithms proceed initially to define integer decision variables (topology) through procedures specific to each method, and subsequently optimise values of the continuous variables (green times at junctions) through an iterative procedure (Cantarella \textit{et al.} 1991, Webster 1958). None of the methods (except for local convergence) have any stopping criteria, but to guarantee accurate comparison, in terms of the same calculation time, they perform an identical number of calculations of the objective function. The methods proposed to solve the formulation in question are specified below.

**Genetic Algorithms (GA)**

Application of GA to the network design problem requires specification of the string that represents the solution (chromosome), of the fitness function and of crossover and mutation operators. The chromosome was constructed as a sequence of genes (decision variables), each of which determines the configuration of a sequence of links. The fitness function is a negative exponential transformation of the objective function. The crossover operator, given two solutions randomly chosen among the current population, replaces them with as many solutions obtained by swapping two corresponding pieces of chromosomes. The mutation operator, given a randomly chosen solution, modifies the value of a randomly chosen gene of this solution.

This algorithm is controlled by four parameters: cardinality of population, coefficient of reproduction selectivity, frequency of crossover and frequency of mutation. In this study
we use parameters which are already calibrated and well-known in the literature (Cantarella and Vitetta 1994).

**Tabu Search (TS)**

A Short Term Tabu Search was applied, and thus it was necessary to specify the way in which each solution is codified and to define the generic move, the structure of the tabu list and the aspiration criterion. The TS was applied by codifying a generic solution in the same way as GA. A generic move is defined as a transformation from one solution to another by varying the value of one decision variable (the configuration of one and only one sequence is different). Each item of the tabu list forbids all the moves that restore the configuration of a modified sequence. The aspiration criterion assumes as a value to be overridden the best value of the objective function of the explored solutions at the moment. As a control parameter, tabu list size is considered.

**Simulated Annealing (SA)**

In the SA algorithm the generic solution and the generic move are defined in the same way as the TS algorithm. The annealing law is a geometric progression. Control parameters are the annealing rate (ratio of the geometric progression) and the starting temperature. The length of the plateau is fixed at the value of 100.

**Local Convergence (LC)**

In the LC algorithm the generic solution and the generic move are defined in the same way as the TS algorithm. This algorithm in each iteration proceeds to make the best available move (it selects the best solution in the neighbourhood), and stops when the value of the objective function cannot be further improved (in other words it stops when a local minimum is reached).

**Combined methods**

The following combined methods are proposed: GA applied by starting with a population given from a set of the best solutions obtained with TS; TS applied by starting with the best solution obtained with GA.

**3 APPLICATIONS AND RESULTS**

Application to a real network allows the efficiency and convergence of the proposed solution algorithms to be tested and compared. Two networks were used, both of which are relative to towns of the south of Italy:

- Barcellona Pozzo di Gotto, with 30,000 inhabitants, 510 links, 185 nodes, 26 origin-destination pairs;
- Villa San Giovanni, with 10,000 inhabitants, 310 links, 120 nodes, 20 origin-destination pairs.

In the first case the designed area corresponded to the central business district (approximately equivalent to 8% of the whole length of the urban network); instead in the second case the
designed area corresponds to the whole urban area. In both cases there are 34 sequences of links in the designed area; the junctions to be designed are those concerned with the sequences in the designed area (in other words the nodes with at least one link of the backward star or of the forward star belonging to a sequence that has to be designed).

All the elaborations perform 20000 calculations of the objective function before stopping (except for local convergence).

The following symbols are used:
- \( w_0 \) is the value of the objective function calculated using the starting configuration (that corresponds to the present situation);
- \( w^* \) the best value found of the objective function;
- \( \Delta w\% \) the proportional decrease: \( \Delta w\% = \left(\frac{w^* - w_0}{w_0}\right) \cdot 100 \).

The results of the application of TS and SA with different parameters are summarized in Tab. 1. The average best results obtained with the GA, TS, SA, LC algorithms and the combined methods are compared in Tab. 2. The comparison reported in Tab. 2 is to be considered preliminary because the results are obtained in different ways for each algorithm: best values obtained by using many different parameters for GA, TS and SA are reported; the value calculated without parameters for LC is reported; the value calculated with best value for single algorithms for local convergence is reported.

### Tab. 1 - Tabu Search and Simulating Annealing for the cities of Villa San Giovanni and Barcellona P. di G.

<table>
<thead>
<tr>
<th>List size</th>
<th>Villa San Giovanni</th>
<th>Barcellona P. di G</th>
<th>Starting temperature</th>
<th>Annealing ratio</th>
<th>Villa San Giovanni</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( w^*(h) )</td>
<td>( \Delta w% )</td>
<td>( w^*(h) )</td>
<td>( \Delta w% )</td>
<td>( w^*(h) )</td>
</tr>
<tr>
<td>0</td>
<td>681.1</td>
<td>13.07%</td>
<td>1765.6</td>
<td>1.82%</td>
<td>776.1</td>
</tr>
<tr>
<td>10</td>
<td>635.5</td>
<td>18.89%</td>
<td>1752.2</td>
<td>2.57%</td>
<td>719.8</td>
</tr>
<tr>
<td>15</td>
<td>626.9</td>
<td>19.99%</td>
<td>1747.1</td>
<td>2.85%</td>
<td>692.0</td>
</tr>
<tr>
<td>20</td>
<td>657.3</td>
<td>16.11%</td>
<td>1747.5</td>
<td>2.83%</td>
<td>692.5</td>
</tr>
<tr>
<td>30</td>
<td>631.4</td>
<td>19.42%</td>
<td>1735.7</td>
<td>3.48%</td>
<td>721.9</td>
</tr>
<tr>
<td>40</td>
<td>675.9</td>
<td>13.73%</td>
<td>1726.5</td>
<td>4.00%</td>
<td>690.9</td>
</tr>
<tr>
<td>50</td>
<td>675.9</td>
<td>13.73%</td>
<td>1751.4</td>
<td>2.61%</td>
<td>691.8</td>
</tr>
<tr>
<td>60</td>
<td>675.9</td>
<td>13.73%</td>
<td>1747.4</td>
<td>2.84%</td>
<td>708.4</td>
</tr>
</tbody>
</table>

\( w_0 = 783.5 \text{ h} \) \quad w_0 = 1798.3 \text{ h} \quad 2.61\% \quad w_0 = 783.5 \text{ h}

### Tab. 2 - Comparison between algorithms in the cities of Villa San Giovanni and Barcellona P. di G.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Villa San Giovanni</th>
<th>Barcellona P. di G</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( w^*(h) )</td>
<td>( \Delta w% )</td>
</tr>
<tr>
<td>Genetic Algorithms (GA)</td>
<td>719.4</td>
<td>8.18%</td>
</tr>
<tr>
<td>Tabu Search (TS)</td>
<td>626.9</td>
<td>19.99%</td>
</tr>
<tr>
<td>Simulating Annealing (SA)</td>
<td>690.9</td>
<td>11.82%</td>
</tr>
<tr>
<td>Local Convergence (LC)</td>
<td>681.1</td>
<td>13.07%</td>
</tr>
<tr>
<td>Genetic and Tabu Search</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( w_0 = 783.5 \text{ h} \) \quad w_0 = 1798.3 \text{ h}
Initial results show that TS is often more efficient than other algorithms. In particular there is a range of the tabu list size that allows better results. The efficiency of GA, SA and the local convergent algorithm is difficult to rank; more numerical results are probably required. However, the local convergence algorithm is sometimes more efficient than GA and SA in spite of its simplicity. There seems to be quite a strong influence of the network (and of the selected area to be designed) in the efficiency of algorithms. A possible development of this study is the conception of new combined methods.

REFERENCES


