ANALYTICAL MULTICLASS DYNAMIC TRAFFIC ASSIGNMENT USING A DYNAMIC NETWORK LOADING PROCEDURE

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1 INTRODUCTION

In the literature, several dynamic traffic assignment (DTA) models have been proposed. Some are based on simulation, some are analytical. Due to their complexity, analytical models were mostly only able to handle small theoretical transportation networks. An overview of analytical DTA models is given by Bliemer (2001). Faster heuristic models have been proposed, but a consistent analytical approach able to deal with larger practical networks was not described until recently by Chabini and He (1999), in which they use a very efficient analytical dynamic network loading (DNL) procedure. The first true multiclass analytical DTA model with different vehicle types – having different driving characteristics, infrastructure usage and route choice behavior – was proposed in Bliemer and Bovy (2001). The problem, formulated as a quasi-variational inequality problem, was solved using a nested modified projection scheme. This scheme turned out to be very time consuming and not useful for practical problems. In this paper we propose an analytical multiclass DTA model combining the multiclass principles from Bliemer and Bovy (2001) and the DNL-based scheme from Chabini and He (1999). This model has been implemented and preliminary results show that the model is feasible for multiclass traffic, even on larger networks.
2 FORMULATION

The multiclass DTA model can be split into two parts: (a) a multiclass route choice model, and (b) a multiclass dynamic network loading model. Figure 1 illustrates the framework of the multiclass DTA model. The extension towards multiple vehicle types essentially necessitates, among others, the following modifications:

- Defining multiclass specific input, such as OD demand for each vehicle type and dedicated road infrastructure for a certain vehicle type (e.g. truck lanes);
- Adapt the route choice model such that each vehicle type can have its own route cost function and their own route choice sets;
- Adapt the dynamic network loading model such that the traffic flows are consistent with the driving behavior of the vehicle types. This means that each vehicle type has its own link travel time function such that not all vehicle types move at the same speed through a link. Furthermore, vehicle types on the same link influence each other (i.e. a vehicle type can have a certain impact on the link travel times of other vehicle types), possibly in an asymmetric fashion.

Details on these modifications can be found in Bliemer (2001). First, we will briefly describe the multiclass route choice model, and then the multiclass dynamic network loading model will be formulated.

The route choice model considers $M$ driver types and $N$ vehicle types. The vehicle types have different route cost functions, whereas the driver types are only distinguished based on the information they have. Each vehicle type $n$ has route cost functions $c_{np}^r(s)$, denoting the actual cost of path $p$ from origin $r$ to destination $s$ departing during time interval $k$. The (vehicle type specific) route travel time is usually one of the (main) components in the cost function. For each vehicle type different driver types exist, choosing either a fixed route (not depending on the route cost functions), a perceived cheapest route, or an actual cheapest route (for details, see Chabini and He (1999)). The multiclass route choice problem can be written as a variational inequality problem and can be solved using an iterative procedure in which for each iteration the travel demand for each vehicle type is distributed between the alternative available routes for that vehicle type, dependent on the route costs. If the route flows do not fluctuate much anymore, the model converges.
In order to determine the network conditions in each iteration of the multiclass route choice model, a multiclass dynamic network loading (DNL) model is used. The following system of equations describes the multiclass DNL model, adapted from the single class model of Chabini and He (1999):

**Flow conservation**

\[
 u_{anp}^r(t) = \begin{cases} 
 f_{np}^r(t), & \text{if link } a \text{ is the first link on path } p \in \mathcal{P}_{nrs}, \\
 v_{anp}^r(t), & \text{if link } a \text{ is after } a',
\end{cases}
\]

where \( u_{anp}^r(t) \) and \( v_{anp}^r(t) \) respectively are the link \( a \) inflow rate and outflow rate at time \( t \) of vehicle type \( n \) following path \( p \) from \( r \) to \( s \), \( f_{np}^r(t) \) is the OD route flow rate of vehicle type \( n \) over path \( p \) from \( r \) to \( s \) departing at time \( t \), and \( \mathcal{P}_{nrs} \) is the set of routes from \( r \) to \( s \) available to vehicles of type \( n \).

**Flow propagation**
\[ V_{\text{amp}}^r(t) = \int_{W} u_{\text{amp}}^r(w) dw, \quad \text{with} \quad W = \{ w \mid w + \tau_{an}(w) \leq t \}, \]

where \( V_{\text{amp}}^r(t) \) is the cumulative outflow rate and \( \tau_{an}(t) \) is the link \( a \) travel time for vehicle type \( n \) when entering the link at time \( t \).

**Definition**

The outflow rate: \( v_{\text{amp}}^r(t) = \frac{dV_{\text{amp}}^r(t)}{dt} \).

The cumulative inflow rate: \( U_{\text{amp}}^r(t) = \int_{w=0}^{t} u_{\text{amp}}^r(w) dw \).

The number of vehicles of type \( n \) on link \( a \) at time \( t \):

\[ X_{an}(t) = \sum_{(r,s) \in \mathcal{P}_{\text{amp}}} X_{\text{amp}}^r(t), \quad \text{where} \quad X_{\text{amp}}^r(t) = U_{\text{amp}}^r(t) - V_{\text{amp}}^r(t). \]

**Multiclass Link Travel Time Functions**

\[ \tau_{an}(t) = D_{an}[X_{a1}(t), \ldots, X_{an}(t), \ldots, X_{aN}(t)], \]

where \( D_{an}(\cdot) \) is a nondecreasing function from \( \mathbb{R}^N \) to \( \mathbb{R} \).

For solving the multiclass DNL model, we have adapted the C-Load algorithm proposed by Chabini and He (1999) such that it can take the interactions and their impact on the traffic operations into account. This algorithm has a close correspondence to a simulation process and in contrast to other DNL procedures proposed in the literature (e.g. Astarita (1996), Wu et al. (1998), and Xu et al. (1999)) it is not an iterative procedure but an exact procedure. Even the first-in-first-out (FIFO) condition is not required.

### 3 Preliminary Results

The original implementation of Chabini and He (1999) had a high computational performance, and was able to perform a dynamic network loading on the beltway of Amsterdam (196 nodes, 310 links, 1000 OD pairs and 1500 paths) in less than 2 minutes, using a 2-hour travel demand and time steps of 3.5 seconds. Our multiclass implementation shows that the computation time increases only linear with the number of vehicle types. Typical case studies with two vehicle types (passenger cars and trucks) and dedicated road infrastructure (a truck lane) will be presented in the full paper.

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4 REFERENCES


Bliemer and Bovy (2001) Quasi-Variational Inequality Formulation of the Multiclass Dynamic Traffic Assignment Problem. To be published in Transportation Research B.

