# PATH RELINKING FOR MULTIPLE OBJECTIVE COMBINATORIAL OPTIMIZATION. TSP CASE STUDY

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**Abstract.** The paper presents a new metaheuristic algorithm for multiple objective combinatorial optimization based on the idea of path relinking. The algorithm is applied to the traveling salesperson problem with multiple link (arc) costs, corresponding to multiple objectives. The multiple costs may for example correspond to financial cost of travel along a link, time of travel, or risk in case of hazardous materials. The algorithm searches for new good solutions along paths in the decision space connecting two other good solutions. It is compared experimentally to state of the art algorithms for multiple objective TSP.

## 1. Introduction

Many real-life problems require taking into account multiple points of view corresponding to multiple objectives. At the same time the problems are computationally hard and of large scale, thus difficult to solve with exact methods. Such problems may be, however, efficiently solved by multiple objective metaheuristics (MOMHs) generating a set of approximately Pareto-optimal solutions [2]. Of course, MOMHs should be based on the best methods for single objective optimization.

Path relinking is a relatively new and very promising metaheuristic [3]. It shares with evolutionary algorithms (EAs) the idea of constructing new solutions by recombination of features of other good solutions (parents). However, while recombination in EAs is rather a random process, path relinking constructs new solutions by analyzing whole paths linking the parents in the decision space and selecting high quality solution(s) from the paths. The offspring solution generated in EA also lays on a path linking the parent. However, this path needs not to be particularly good (needs not to go through good solutions), and, even if it contains some very good solutions, the offspring solution needs not to be among the best on the path.

According to our knowledge, the only multiple objective version of path relinking has been proposed by Basseur et al. [1]. Their method is based on Adaptive Genetic Algorithm

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(AGA). In this paper we propose a new multiple objective version of path relinking using the general scheme of Pareto Memetic Algorithm (PMA) [4][5]. Unlike AGA that is based on Pareto ranking PMA uses scalarizing functions with weight vectors drawn at random for each iteration. This mechanism assures both convergence towards the Pareto set and good dispersion of the search over the whole Pareto set, which is an often weak point of the methods based on Pareto ranking.

## 2. Multiple objective symmetric traveling salesperson problem

Traveling salesperson problem (TSP) is a basis for manifold optimization problems appearing in transportation. Its single objective version is a typical benchmark for metaheuristics. It is defined by a set of cities and a cost (distance) of travel between each pair of cities. In symmetric TSP the cost does not depend on the direction of travel between two cities. The goal is to find the lowest cost hamiltonian cycle.

We use the same problem formulation and instances that have been used in [4] and [6]. In $J$-objective TSP, $J$ different cost factors are defined between each pair of cities. In practical applications the cost factors may for example correspond to cost, length, travel time, risk or tourist attractiveness. In our case, $J$-objective symmetric TSP instances are constructed from $J$ different single objective TSP instances having the same number of cities. Thus, $j$-th cost factor, $j$=1,...,$J$, between a pair of cities comes from $j$-th single objective instance. Individual optima of particular objectives are equal to optima of corresponding single objective instances. In our case, the single objective instances are completely independent, so, also objectives are independent and therefore non-correlated.

## 3. Multiple objective path relinking

Our version of path relinking algorithm shares the general scheme with PMA (see figure 1). In each iteration it draws at random a weight vector defining a scalarizing function. We use *achievement scalarizing functions* defined in the following way:

$$s_a\left(\mathbf{z},\mathbf{z}^0,\boldsymbol{\lambda},\rho\right)=\max_j\left\{\lambda_j\left(z_j^0-z_j\right)\right\}+\rho\sum_{j=1}^{J}\lambda_j\left(z_j^0-z_j\right)$$

where $z^0$ is a reference point, $\boldsymbol{\lambda}=\left[\lambda_1,...,\lambda_J\right]$ is a weight vector such that $\lambda_j \geq 0\ \forall j$ and $\exists j \mid \lambda_j > 0$, $\rho$ is a small positive number such that $0<\rho<<1$.

The algorithm maintains a set of current solutions $CS$ playing a role of a large population from which parents may be selected. Initial solutions are generated by local search starting from randomly generated solutions. In the main phase, the algorithm selects two parents from $CS$ being good on the current scalarizing function. The parent are drawn with tournament selection in which a significant portion of solutions from $CS$ takes part. The parents are linked in the decision space by a path of feasible solutions. Each solution in the path is used to update the set potentially Pareto-optimal solutions $PPO$. Then the best solution from the point of view of the current scalarizing function is selected from the path. This solution is further improved by a local heuristic, e.g. local search.

**Parameters:** *Er* – expected rank of linked solutions, *S* – stopping criterion for the phase of generation of initial solutions, stopping criterion of the algorithm

*Initialization:*

The set of potentially Pareto-optimal solutions $PPO:=\varnothing$

The current set of solutions $CS:=\varnothing$

*Generation of the first approximation of the ideal point:*

**for each** objective $f_j$

Construct a new feasible solution **x** by a randomized algorithm

Apply a local heuristic optimizing objective $f_j$ to solution **x** obtaining **x**'

Add **x**' to the current set of solutions *CS*

Update set *PPO* with **x**'

*Generation of the initial set of solutions:*

**repeat**

Draw at random a weight vector $\lambda$

Construct a new feasible solution **x** by a randomized algorithm

Apply a local heuristic optimizing $s(\mathbf{z},...,\boldsymbol{\lambda})$ to solution **x** obtaining **x**'

Add **x**' to the current set of solutions *CS*

Update set *PPO* with **x**'

**until** the stopping criterion for the phase of generation of initial solutions is met

*Main loop:*

**repeat**

Draw at random a weight vector $\lambda$

From *CS* draw at random with uniform probability a sample *T* of $3|CS|/2Er$ solutions

Find path *PT* between the second best and the best solution on $s(\mathbf{z},...,\boldsymbol{\lambda})$

Update set *PPO* with each solution in *PT*

Select the best solution in *PT* $\mathbf{x}_1$

Apply a local heuristic optimizing $s(\mathbf{z},...,\boldsymbol{\lambda})$ to solution $\mathbf{x}_1$ obtaining $\mathbf{x}_1$'

**if** $\mathbf{x}_1$' is better on $s(\mathbf{z},...,\boldsymbol{\lambda})$ than the second best solution in *T* **then**

Add $\mathbf{x}_1$' to the current set of solutions *CS*

Update set *PPO* with $\mathbf{x}_1$'

**until** the stopping criterion is met

**Figure 1. Path relinking for multiple objective combinatorial optimization. $s(\mathbf{z},...,\boldsymbol{\lambda})$ stands for a scalarizing functions controlled by weight vector $\lambda$. $s_a\left(\mathbf{z},\mathbf{z}^{**}(PP),\boldsymbol{\lambda},\rho\right)$ is used in this experiment**

## 4. Adaptation of the multiple objective path relinking to TSP

As a local heuristic we use local search based on the standard 2-arcs exchange neighborhood. The new neighbor solution is obtained by removing two nonadjacent arcs from the current solution and substituting them with the only two arcs that restore the Hamiltonian cycle.

The path building process is based on the same kind of local search. The distance in objective space is measured by the number of uncommon arcs in two solutions. Path building process starts from one of the parents and considers only moves that reduce the distance to the second parent. Among such moves the algorithm selects the one leading to a neighbor solution with the best evaluation on the current scalarizing function.

## 5. Computational experiment

The algorithm is evaluated on bi- and three-objective instances of TSP with 50, 100 and 150 nodes. The results of the computational experiment will be presented in the full version of the paper. The preliminary experiments indicate that the algorithm is competitive to the state of the arc metaheuristics for the multiple objective TSP [4], [6].

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