Informatica Teorica. 16 September 2008.

1. Prove that the set of the continuous functions from a cpo A to a cpo B is itself a cpo. How functions are ordered in a cpo of functions?

2. State and illustrate the relative completeness theorem of the Hoare's calculus of triples. Give a proof of why this calculus is undecidable.

3. Give the rules for the construction of the least Herbrand Model of a definite logic program.

4. Compute the minimal fixpoint and the maximal fixpoint of the equation $X = aX+b$, where the unknown X is a language subset of $\{a, b, c\}$.

5. A set $A \subseteq N$ is r.e. iff A is empty or A is the range of a total p.r.f. from N to N. Show that a set $A \subseteq N$ is r.e. iff it is the domain of a partial recursive function.

6. Show the total correctness of the following program by showing both partial correctness and termination (by `div` we mean integer division by 2, that is, for instance, `7 div 2` = `3`):

```
      {n >= 0}
  begin
    k := n; y := 1; z := x;
    while k \not = 0 do if odd(k) then begin k := k-1; y := y * z end;
                            k := k div 2; z := z * z;
                        od
  end
      {y = x ^ n }
```

7. Show that the eager semantics of $let\ x \Leftarrow e\ in\ t$ is equal to the eager semantics of $(\lambda x.t)e$.

Assume that:

$[\![(t_1 t_2)]\!]\rho = let\ \varphi \Leftarrow [\![t_1]\!]\rho,\ v \Leftarrow [\![t_2]\!]\rho.\ \varphi(v)$

$[\![\lambda x.t]\!]\rho = \lfloor \lambda v.[\![t]\!]\rho[v/x] \rfloor$

$[\![let\ x \Leftarrow e\ in\ t]\!]\rho = let\ v \Leftarrow [\![e]\!]\rho.[\![t]\!]\rho[v/x]$

8. Prove that if a formula can be proved by Park induction it can also be proved by Scott induction.

Consider a cpo $D_\perp$, its least element $\perp$, a continuous function f from $D_\perp$ to $D_\perp$, and an inclusive unary predicate $P \subseteq D_\perp$.

Recall that a predicate $P \subseteq D_\perp$ is said to be *inclusive* iff for all ω-chains $d_0 \sqsubseteq d_1 \sqsubseteq \ldots$ in $D_\perp$ we have that *if* for all $i \geq 0$, $P(d_i)$ holds *then* $P(\bigsqcup_{i \geq 0} d_i)$ holds.

Recall also that *fix* is a function in $[[D_\perp \to D_\perp] \to D_\perp]$ such that $f(\mathit{fix}(f)) = \mathit{fix}(f)$.

The *Scott induction* (or *fixpoint induction*) rule is as follows:

$$\frac{P(\perp) \qquad \forall d \in D_\perp.\ P(d) \to P(f(d))}{P(\mathit{fix}(f))}$$

The *Park induction* rule is as follows. For all $d \in D_\perp$:

$$\frac{f(d) \sqsubseteq d}{\mathit{fix}(f) \sqsubseteq d}$$