A Tight MIP Formulation of the Unit Commitment Problem with Start-up and Shut-down Constraints

C. Gentile[a,1], G. Morales-España[b,2], A. Ramos[c]

[a] *Istituto di Analisi dei Sistemi ed Informatica "A. Ruberti", C.N.R., Via dei Taurini 19, 00185 Roma, Italy*

[b] *Department of Electrical Sustainable Energy, Delft University of Technology, 2628 CD Delft, The Netherlands*

[c] *Institute for Research in Technology (IIT) of the School of Engineering (ICAI), Universidad Pontificia Comillas, Madrid, Spain*

Abstract

This paper provides the convex hull description of the single thermal Unit Commitment problem with the following basic operating constraints: 1) generation limits, 2) start-up and shut-down capabilities, and 3) minimum up and down times. The proposed constraints can be used as the core of any Unit Commitment (UC) formulation to strengthen the lower bound in enumerative approaches. We provide evidence that dramatic improvements in computational time are obtained by solving the self-UC problem and the network-constrained UC problem for different case studies.

Keywords: Unit Commitment (UC), Mixed-Integer Programming (MIP), Facet/Convex hull description.

1. Introduction

The Unit Commitment (UC) problem requires to optimally operate a set of power generation units over a time horizon ranging from a day to a week. Despite the breakthrough in Mixed-Integer Programming (MIP) solvers, Unit Commitment (UC) problems remain restricted in size and scope due to the time requested to solve these problems. UC problems can be solved significantly faster by improving their MIP formulation by providing the convex hull description of some set of constraints. Even though other constraints in the problem might add some fractional vertices, this solution should be nearer to the optimal solution than the solution of the original model would be [23, 22]. Some efforts in tightening specific set of constraints have been done, such as: the convex hull of the minimum up and down times [9, 10, 19], cuts to tighten ramping limits

Email addresses: `gentile@iasi.cnr.it` (C. Gentile), `g.a.moralesespana@tudelft.nl` (G. Morales-España), `andres.ramos@upcomillas.es` (A. Ramos)

[1] The work of C. Gentile was partially supported by the project MINO grant no. 316647 Initial Training Network of the "Marie Curie" program funded by the European Union and by the project PRIN 2012JXB3YF_003 funded by the Italian Minister of Education, University and Research.

[2] The work of G. Morales-España was supported by the Cátedra Iberdrola de Energía e Innovación (Spain).

[18, 2], tighter approximation for quadratic generation costs [6], new formulations for the time-depending start-up costs [20], and simultaneously tight and compact description of thermal units operation [15, 14, 13, 12].

The main contribution of this paper is a slight modification of the constraints presented in Morales-Espana et al. [14] plus the proof that the new model provides the convex hull description of the solutions satisfying the following set of constraints: 1) generation limits, 2) start-up and shut-down capabilities, and 3) minimum up and down times. This result is a basic step towards the definition of a formulation describing the convex hull of the set of solutions satisfying also general ramp constraints with a linear number of variables. Recently a formulation with $O(T^3)$ variables (where T is the length of the time horizon) describing the convex hull of the feasible solutions have been obtained independently in Frangioni and Gentile [4], Knueven et al. [8], but using formulations based on the Dynamic Programming algorithm in Frangioni and Gentile [3]. Moreover, the techniques used in this paper could be possibly used also to achieve this more general result. These results are in some sense orthogonal to those in Damci et al. [2]. In this paper, we consider both start-up and shut-down capabilities together but we do not consider ramp constraints, in Damci et al. [2] two separate polytopes are defined: the ramp-up polytope considering solutions satisfying ramp-up and start-up limits and the ramp-down polytope considering solutions satisfying ramp-down and shut-down limits. In Damci et al. [2] the convex hull descriptions for ramp-up and ramp-down polytopes are provided for the case of only two periods and some facet defining inequalities are presented for the same polytopes with arbitrary time horizon.

On the application side tighter formulations are usually solved in less time by MIP solvers; however, this must be tested by computational experiments. We compare the new formulation with two other MIP formulations obtaining results significantly faster for three different case studies. The first one consists in solving a self-UC problem only taking into account the constraints proposed in this paper. Self-UC optimizes the net profit of a price-taker generation company, that is a relatively small company that is not able to influence the market price. If we restrict to the above mentioned constraints, we have a convex hull description also for the self-UC problem. The second and third case studies solve the network-constrained UC problem for two IEEE power systems, where other common constraints are taken into account, such as demand balance, reserves, ramping and transmission limits.

The remainder of this paper is organized as follows. Section 2 introduces the main notation used to describe the proposed formulation. Section 3 details the basic operating constraints of a single generating unit. Section 4 contains the facet inducing and convex hull proofs for the proposed linear description of the self-UC subproblem. Section 5 provides and discusses results from several case studies, where a comparison with other three UC formulations is made. Finally, some relevant conclusions are drawn in Section 6.

2. Notation

Here, we introduce the main notation used in this paper. The length of the time horizon is denoted by T and the time is indexed by t. The set of generating units is denoted by $\mathcal{G}$ and indexed with g running from 1 to G.

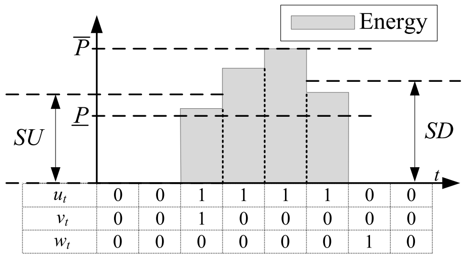

Figure 1: Unit's operation including its start-up and shut-down capabilities

2.1. Unit's Technical Parameters

$\overline{P}_g/\underline{P}_g$ Maximum/minimum power output [MW] for unit g.

SD_g/SU_g Shut-down/start-up capability [MW] for unit g.

TD_g/TU_g Minimum down/up time [h] for unit g.

2.2. Decision Variables

u_{gt} Binary variable for the commitment status of unit g for period t, which is equal to 1 if the unit is online and 0 otherwise.

v_{gt} Binary variable for the start-up status of unit g, which is equal to 1 if the unit starts up in period t and 0 otherwise.

w_{gt} Binary variable for the shut-down status of unit g, which is equal to 1 if the unit shuts down in period t and 0 otherwise.

p_{gt} Power production above the unit's minimum output $\underline{P}$ [MW] for unit g in period t. The total generation output is equal to $u_{gt}\underline{P}_g + p_{gt}$.

3. Modeling the Unit's Operation

This section describes the mathematical formulations of the basic operation of a single generating unit in Unit Commitment (UC) problems. To simplify the notation, here we do not report the unit index. In Section 5 we consider two multi-units UC problems where the single generating unit formulations must be replicated for each unit.

Two main formulations can be found in the literature: *1bin* formulation, so called because it uses only one vector of binary variables u_t denoting the status ON/OFF of the unit for each time period t; *3bin* formulation, so called because it uses three vectors of binary variables by adding to the state variables also the start-up v_t and shut-down w_t variables. The basic constraints of the *1bin* and *3bin* formulations are presented in AppendixA.

In this paper, the following set of constraints are modeled: generation limits, minimum up and down times, and start-up and shut-down capabilities. As shown in Figure 1, the start-up capability SU is the maximum power that a generating unit could produce when it starts up. Similarly, the unit should be producing below its shut-down capability SD when it shuts down.

First, we use the following constraints, which were proposed in [19] to describe the convex hull formulation of the minimum-up and -down time constraints:

$$u_t - u_{t-1} = v_t - w_t \qquad t = 2, \ldots, T \tag{1}$$

$$\sum_{j=t-TU+1}^{t} v_j \leq u_t \qquad t = 2, \ldots, T \tag{2}$$

$$\sum_{j=t-TD+1}^{t} w_j \leq 1 - u_t \qquad t = 2, \ldots, T \tag{3}$$

where inequalities in (2) state that in an interval of TU consecutive time periods a unit can be started-up at most once; inequalities (3) works similarly for the shut-down case.

Here, we present the formulation that we now denote as TC obtained by adding to constraints (1)-(3) the following constraints with start-up and shut-down capabilities:

$$p_1 \leq \left(\overline{P} - \underline{P}\right) u_1 - \left(\overline{P} - SD\right) w_2 \tag{4}$$

$$p_t \leq \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SU\right) v_t - \left(\overline{P} - SD\right) w_{t+1} \qquad t = 2, \ldots, T-1 \tag{5}$$

$$p_T \leq \left(\overline{P} - \underline{P}\right) u_T - \left(\overline{P} - SU\right) v_T \tag{6}$$

Constraint (5) states that the maximum power above the minimum output in period t when the unit is started-up (e.g., $u_t = v_t = 1$ and $w_{t+1} = 0$) is equal to $SU - \underline{P}$, when the unit is shut-down at time $t+1$ (e.g., $u_t = w_{t+1} = 1$ and $v_t = 0$) is equal to $SD - \underline{P}$, and when the unit is continuously online (e.g, $u_t = 1$ and $v_t = w_{t+1} = 0$) is equal to $\overline{P} - \underline{P}$. Constraints (4) and (6) describe the first and the last period cases.

Be aware that (5) may be infeasible in the event that the unit is online for just one period. Indeed, when $v_t = w_{t+1} = 1$ the right side of (5) can be negative. Consequently, (5) is only valid for units with uptime $TU \geq 2$. The correct formulation for units with $TU = 1$ is given by substituting (5) with the following pair of constraints:

$$p_t \leq \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SD\right) w_{t+1} - \max\left(SD{-}SU, 0\right) v_t \quad t = 2, \ldots, T-1 \tag{7}$$

$$p_t \leq \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SU\right) v_t - \max\left(SU{-}SD, 0\right) w_{t+1} \quad t = 2, \ldots, T-1. \tag{8}$$

Finally, the variable bounds are given by

$$0 \leq u_t \leq 1 \quad t = 1, \ldots, T \tag{9}$$

$$v_t \geq 0, \quad w_t \geq 0 \quad t = 2, \ldots, T \tag{10}$$

$$p_t \geq 0 \quad t = 1, \ldots, T. \tag{11}$$

In summary, inequalities (4)-(6) together with inequalities (1)-(3) and (9)-(11) describe the operations for units with $TU \geq 2$, and inequalities (4), (6), (7), (8) together with inequalities (1)-(3) and (9)-(11) for units with $TU = 1$. The main contribution of this paper is that the polytopes thus described always have integral vertices with respect to the binary variables.

In Morales-Espana et al. [14] it was presented a slightly different formulation, where instead of constraints (7)-(8) the following ones were used:

$$p_t \leq \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SD\right) w_{t+1} \quad t = 2, \ldots, T-1 \tag{12}$$

$$p_t \leq \left(\overline{P} - \underline{P}\right) u_t - \left(\overline{P} - SU\right) v_t \quad t = 2, \ldots, T-1. \tag{13}$$

Note that if $SU = SD$ then (7)-(8) and (12)-(13) would be equivalent. We denote the old formulation [14] with the latter constraints as *TC0*.

4. Strength of the proposed inequalities

In this section, we prove that inequalities (1)-(11) describe the convex hull of the feasible solutions. Note that constraints (1) uniquely define the value of the variables w as a function of u and v. Unless differently specified, in the following, we will consider only the space defined by the variables u, v, and p. Moreover, we suppose that all constraints (3)-(5), (7)-(8), and (10) are rewritten by substituting the w variables accordingly.

Definition 1. Let $\overline{C}_T\left(TU, TD, \overline{P}, \underline{P}, SU, SD\right)$ be the convex hull of the feasible integer solution for the problem. That is, for $TU \geq 2$, we write

$$\begin{aligned}\overline{C}_T \quad & (TU \geq 2, TD, \overline{P}, \underline{P}, SU, SD) = \\ & conv\{(u, v, p) \in \{0,1\}^{2T-1} \times \mathbb{R}^T_+ |\, (u, v, p) \text{ satisfy (1)-(6) and (9)-(11)}\};\end{aligned}$$

for $TU = 1$, we write

$$\begin{aligned}\overline{C}_T \quad & \left(TU = 1, TD, \overline{P}, \underline{P}, SU, SD\right) = \\ & conv\left\{(u, v, p) \in \{0,1\}^{2T-1} \times \mathbb{R}^T_+ |\, (u, v, p) \text{ satisfy (1)-(4), (6)-(8), and (9)-(11)}\right\}.\end{aligned}$$

For short we write $\overline{C}_T$ for $\overline{C}_T\left(TU, TD, \overline{P}, \underline{P}, SU, SD\right)$, $\overline{C}_T(TU \geq 2)$ for $\overline{C}_T(TU \geq 2, TD, \overline{P}, \underline{P}, SU, SD)$, and $\overline{C}_T(TU = 1)$ for $\overline{C}_T(TU = 1, TD, \overline{P}, \underline{P}, SU, SD)$.

Proposition 2. *$dim(\overline{C}_T) = 3T - 1$ and thus $\overline{C}_T$ is full-dimensional.*

Proposition 3. *The inequalities (4), (6) and (11) describe facets of the polytope $\overline{C}_T$. Moreover, inequalities (5) describe facets of the polytope $\overline{C}_T\left(TU \geq 2\right)$, and inequalities (7) and (8) describe facets of the polytope $\overline{C}_T\left(TU = 1\right)$.*

The proofs of propositions 2 and 3 can be performed by exhibiting the right number of affinely independent points (details of the proofs can be requested to the authors).

For the convex hull proof, we need a preliminary lemma that is very easy to prove from well-known results (we report a proof suggested by a referee for completeness):

Lemma 4. *Suppose that $P = \{x \in \mathbb{R}^n | Ax \leq b\}$ is an integer polyhedron. Suppose that $y \in \mathbb{R}^K$ are new variables and that $Q = \{(x, y) : d^k x \leq y_k \leq c^k x, k = 1, \ldots, K\}$, with at most one lower bound $d^k x$ and one upper bound $c^k x$ for each variable y_k. If $d^k x \leq c^k x$ for each $x \in P$, then $P \cap Q$ has extreme points with x integer.*

Proof. Consider the linear program LP(P,Q): $\min\{qx + \sum_{k=1}^{K} h_k y_k : (x, y) \in P \cap Q\}$. We prove that for each objective function this LP has an integer solution with respect to x. Set $y_k = d^k x$ if $h_k \geq 0$ and $y_k = c^k x$ otherwise. Solve the resulting LP in the x-space. Then x is integer and the corresponding (x, y) is optimal for the linear program LP(P,Q). □

Theorem 5. *Let $\overline{D}_T\left(TU, TD, \overline{P}, \underline{P}, SU, SD\right)$ be a polyhedron defined as follows:*

- *for* $TU \geq 2$

$$\begin{aligned}\overline{D}_T \quad & \big(TU \geq 2, TD, \overline{P}, \underline{P}, SU, SD\big) = \\ & \big\{(u,v,p) \in [0,1]^{2T-1} \times \mathbb{R}^T_+ \,|\, (u,v,p) \ \textit{satisfy}\ (1)\text{-}(6)\ \textit{and}\ (9)\text{-}(11)\ \big\}\,;\end{aligned}$$

- *for* $TU = 1$

$$\begin{aligned}\overline{D}_T \quad & \big(TU = 1, TD, \overline{P}, \underline{P}, SU, SD\big) = \\ & \big\{(u,v,p) \in [0,1]^{2T-1} \times \mathbb{R}^T_+ \,|\, (u,v,p) \ \textit{satisfy}\ (1)\text{-}(4),\ (6)\text{-}(8),\ \textit{and}\ (9)\text{-}(11)\ \big\}\,.\end{aligned}$$

Then $\overline{C}_T\big(TU, TD, \overline{P}, \underline{P}, SU, SD\big) = \overline{D}_T\big(TU, TD, \overline{P}, \underline{P}, SU, SD\big)$.

Proof. As for $\overline{C}_T$, we use short notations $\overline{D}_T$, $\overline{D}_T\big(TU \geq 2\big)$, and $\overline{D}_T\big(TU = 1\big)$. The proof for $TU \geq 2$ easily follows from Lemma 4. Indeed, $\overline{D}_T\big(TU \geq 2\big)$ is described by the inequalities (1)-(3) and (9)-(10), that describe an integral polyhedron in u and v as proved in [19], together with inequalities (4)-(6) and (11) satisfying the hypothesis of Lemma 4.

For $TU = 1$ let us suppose that $SU \geq SD$. We follow Approach 8 in [23] (see Section 9.2.3, Problem 2, Approach 8). We first introduce an extended formulation of the problem, then we prove that the extended formulation is integral, and finally we prove that the projection of the new polyhedron correspond to $\overline{D}_T\big(TU = 1\big)$. We divide the proof into a series of claims. We define the following new binary variables for $t = 2, \ldots, T-1$: $x_t = 1$ if and only if $v_t = 1$ and $w_{t+1} = 1$, if and only if $v_t = 1$ and $w_{t+1} = 0$, if and only if $v_t = 0$ and $w_{t+1} = 1$, if and only if $u_t = 1$, $v_t = 0$, and $w_{t+1} = 0$. Moreover, $\tilde{u}_T = 1$ if and only if $u_T = 1$ and $v_T = 0$.

Claim 1. The polyhedron P defined by the points $(u, v, w, \tilde{u}, \tilde{v}, \tilde{w}, x)$ satisfying the following inequalities is integral:

$$\begin{aligned}
v_t &\leq u_t && t = 2, \ldots, T && (14)\\
\sum_{i=t-TD+1}^{t} w_i &\leq 1 - u_t && t \in [TD+1, T] && (15)\\
u_t - u_{t-1} &= v_t - w_t && t \in [2, T] && (16)\\
w_{t+1} &= \tilde{w}_{t+1} + x_t && t \in [2, T-1] && (17)\\
v_t &= \tilde{v}_t + x_t && t \in [2, T-1] && (18)\\
u_t &= \tilde{v}_t + \tilde{w}_{t+1} + x_t + \tilde{u}_t && t \in [2, T-1] && (19)\\
u_T &= v_T + \tilde{u}_T && && (20)\\
0 \leq u_t &\leq 1 && t \in [1, T] && (21)\\
v_t, w_t, \tilde{u}_t &\geq 0 && t \in [2, T] && (22)\\
\tilde{v}_t, x_t &\geq 0 && t \in [2, T-1] && (23)\\
\tilde{w}_t &\geq 0 && t \in [3, T] && (24)
\end{aligned}$$

Proof of Claim 1. The proof is carried on by showing that the coefficient matrix associated with the above linear system is totally unimodular.

We exploit this well-known property (proved by Ghouila-Houri, see [17], Chapter III.1, Theorem 2.7): let A be a $\{0,1,-1\}$-matrix, if each subset J of columns of A can be partitioned into J_1 and J_2 such that

$$\left| \sum_{j \in J_1} a_{ij} - \sum_{j \in J_2} a_{ij} \right| \leq 1 \tag{25}$$

for each row i, then A is totally unimodular. This part of the proof has been inspired by the proof of Malkin [10] for the polyhedron defined by (1)-(3).

First we assign the variables $w_i \in J$ alternatively to J_1 and to J_2 in lexicographic order. Then the variables $u_t \in J$ are assigned either to J_1 if $w_k \in J_2$, where $k = \max\{i | 1 \leq i \leq t, w_i \in J\}$, or to J_2 if $w_k \in J_1$, or to the same set with respect to u_{t-1} if $\{i | 1 \leq i \leq t, w_i \in J\}$ is empty. Thus condition (25) is satisfied for constraints (15).

Variables $v_t \in J$ are assigned either to J_1 if $u_t \in J_1$, or to J_2 if $u_t \in J_2$, or to the opposite set with respect to u_{t-1} if $u_t \notin J$, or to the same set as w_t if both $u_{t-1}, u_t \notin J$. This ensures that condition (25) is satisfied for constraints (14) and (16).

If $v_t, w_{t+1} \in J$, then assign $\tilde{v}_t \in J$ to the same subset as v_t, $x_t \in J$ to the opposite set with respect to $\tilde{v}_t$, and $\tilde{w}_t \in J$ to the same subset as w_t. These assignments guarantee that condition (25) is satisfied for constraints (17) and (18) both in the case that v_t and w_{t+1} are in the same set or in different sets. Moreover, the assignment for $\tilde{u}_t$ can be chosen to satisfy condition (25) for constraints (19). If one between v_t and w_{t+1} does not belong to J then proceed as follows: suppose w.l.o.g. that $v_t \notin J$, then assign w_{t+1}, $\tilde{w}_{t+1}$, and $\tilde{v}_t$ to the same set and x_t to the other set, then $\tilde{u}_t$ can be chosen to satisfy condition (25) for constraints (19). Similar choices can be done if some of the variables $\tilde{v}_t$, $\tilde{w}_{t+1}$, x_t, $\tilde{u}_t$ do not belong to J and the claim follows. **End of Claim 1.**

Then we define the polyhedron $\tilde{Q}$ by adding to (14)-(24)

$$p_t^v \leq (SU - \underline{P})\tilde{v}_t \quad t \in [2, T-1] \tag{26}$$

$$p_t^x \leq (SD - \underline{P})x_t \quad t \in [2, T-1] \tag{27}$$

$$p_t^w \leq (SD - \underline{P})\tilde{w}_{t+1} \quad t \in [2, T-1] \tag{28}$$

$$p_t^u \leq (\overline{P} - \underline{P})\tilde{u}_t \quad t \in [2, T] \tag{29}$$

$$p_T^v \leq (SU - \underline{P})v_T \tag{30}$$

$$p_1 \leq (\overline{P} - \underline{P})u_1 - (\overline{P} - SD)w_2 \tag{31}$$

where p^v, p^x, p^w, p^u and p_1 are new non-negative variables.

Claim 2. The polyhedron $\tilde{Q}$ is integral with respect to variables u, v, w, x, $\tilde{u}$, $\tilde{v}$, $\tilde{w}$. **End of Claim 2.**

Claim 2 follows by applying Lemma 4 to the polyhedron P of Claim 1. Then we define the polyhedron Q by adding to (14)-(24),(26)-(31)

$$p_t = p_t^v + p_t^x + p_t^w + p_t^u \qquad t \in [2, \ldots, T-1] \tag{32}$$

$$p_T = p_T^v + p_T^u \tag{33}$$

where p_t for $t \in [2 \ldots T]$ are non-negative variables.

Claim 3. The polyhedron Q is integral with respect to variables u, v, w, x, $\tilde{u}$, $\tilde{v}$, $\tilde{w}$. **End of Claim 3.**

Claim 3 follows from Claim 2 and by the straightforward extension of Lemma 4, where the role of P is played by the integral polyhedron $\tilde{Q}$. Finally we prove that

Claim 4. The projection of Q onto the space of variables u, v, p is equivalent to $\overline{D}_T$.

Proof of Claim 4. We start by eliminating the variables p_t^v, p_t^x, p_t^w, and p_t^u by simply substituting constraints (32)-(33) with the following:

$$p_t \leq (SU - \underline{P})\tilde{v}_t + (SD - \underline{P})x_t + (SD - \underline{P})\tilde{w}_{t+1} + (\overline{P} - \underline{P})\tilde{u}_t \qquad t \in [2, T-1] \tag{34}$$

$$p_T \leq (SU - \underline{P})v_T + (\overline{P} - \underline{P})\tilde{u}_T, \tag{35}$$

which are obtained by using constraints (26)-(30).

Now, we replace $\tilde{u}_T$ from (20) in (35) to obtain $p_T \leq \left(\overline{P} - \underline{P}\right) u_T - \left(\overline{P} - SU\right) v_T$ that coincides with (6). Then we eliminate variables in (34) according to the following order: $\tilde{u}_t$ by using the equation (19); $\tilde{w}_{t+1}$ by using the equation (17); $\tilde{v}_t$ by using the equation (18). It is not difficult to see that for $t \in [2, T-1]$ we obtain the following constraints:

$$p_t \leq (\overline{P} - \underline{P})u_t - (\overline{P} - SU)v_t - (\overline{P} - SD)w_{t+1} + (\overline{P} - SU)x_t \tag{36}$$

$$x_t \geq 0 \tag{37}$$

$$x_t \geq v_t + w_{t+1} - u_t \tag{38}$$

$$x_t \leq v_t \tag{39}$$

$$x_t \leq w_{t+1}.. \tag{40}$$

Now we can apply Fourier-Motzkin elimination to variables x_t by considering the following pairs of constraints: (i) from constraints (39) and (36) we obtain we obtain $v_t \geq 0$; and (38) we obtain (iv) from constraints (40) and (36) we obtain we obtain $w_{t+1} \geq 0$; and (38) we obtain $u_t \geq v_t$. Finally, the claim follows by observing that (31) coincides with (4). **End of Claim 4.**

From Claim 4 it follows that $\overline{D}_T$ is integral with respect to the variables u and v. The proof for $SD \geq SU$ can be performed in a symmetric way. □

5. Numerical Results

To illustrate the computational performances of the formulation presented in this paper, three sets of case studies are carried out: one for a self-UC problem and two others for a network-constrained UC problem. This section compares the computational performance of the proposed *TC* formulation with two other formulations, [1] and [18], which have been recognized as computationally efficient in the literature [16, 14, 21].

The following three formulations are then implemented:

- *TC*: This is the complete formulation presented in this paper. For the network-constrained UC, we include other common constraints such as demand-balance, reserves, ramping and transmission limits. The complete network-constrained UC is presented in AppendixB.

- *1bin*: This formulation is presented in [1] and requires a single set of binary variables (per unit and per period), i.e., the start-up and shut-down decisions are expressed as a function of the commitment decision variables.

Table 1: Generator Data

Gen	$\overline{P}$ [MW]	$\underline{P}$ [MW]	TU/TD [h]	SU [MW]	SD [MW]	p_0* [MW/h]	Ste$_0$⋆ [h]	C^{NL} [\$/h]	C^{LV} [\$/MWh]	C^{SU} [\$]
	Technical Information							Cost Coefficients†		
1	455	150	8	252	303	150	8	1000	16.19	9000
2	455	150	8	252	303	150	8	970	17.26	10000
3	130	20	5	57	75	20	5	700	16.60	1100
4	130	20	5	57	75	20	5	680	16.50	1120
5	162	25	6	71	94	25	6	450	19.70	1800
6	80	20	3	40	50	20	3	370	22.26	340
7	85	25	3	45	55	25	3	480	27.74	520
8	55	10	1	25	33	10	1	660	25.92	60
9	55	10	1	25	33	10	1	665	27.74	60
10	55	10	1	25	33	10	1	670	27.79	60

* p_0 is the unit's initial production prior to the first period of the time span.

⋆Ste$_0$ is the number of hours that the unit has been online prior to the first period of the time span.

†C^{NL}, C^{LV} and C^{SU} stand for non-load, linear-variable and startup costs, respectively.

Table 2: Energy Price (\$/MWh)

$t = 1\dots 12 \rightarrow$	13.0	7.2	4.6	3.3	3.9	5.9	9.8	15.0	22.1	31.3	33.2	24.8
$t = 13\dots 24 \rightarrow$	19.5	16.3	14.3	13.7	15.0	17.6	20.2	29.3	49.5	53.4	30.0	20.2

- *3bin*: The convex hull of the minimum up/down time constraints proposed in [19] (see (1)-(3) and (9)-(10)) are implemented with the three-binary formulation. This formulation is presented in [18]

Notice that different set of constraints are used for the self-UC and for the network-constrained UC problems. For the self-UC problems, *1bin* and *3bin* are modeled only considering 1) generation limits, 2) minimum up and down times, and 4) start-up and shut-down capabilities. For the network-constrained UC problems, *1bin* and *3bin* are modeled taking into account the full set of constraints presented in [1] and its *3bin* equivalent [18], respectively; in addition, these formulations are further extended by introducing downwards reserve (which is modeled in the same fashion as the upwards reserve, see AppendixB), transmission limits (see (B.5) in AppendixB), and wind generation (which is taken into account in the demand-balance (B.2) and transmission-limit constraints (B.5)).

All tests were carried out using CPLEX 12.5 on an Intel-i7 3.4-GHz personal computer with 8 GB of RAM memory. The problems are solved until they hit the time limit of 10000 seconds or until they reach optimality (more precisely to 10^{-4}% of relative optimality gap).

5.1. Self-UC

We illustrate the computational performance of the formulation proposed in this paper by solving the self-UC problem for a price-taker producer for different time spans. The goal of a price-taker producer is to maximize his profit (which is the difference between the revenue and the total operating cost [15]) during the planning horizon:

Table 3: Self-UC: Computational Performance Comparison

Case	Optimum	IntGap (%)				LP time (s)				MIP time (s)*				B&C Nodes			
(days)	(M$)	*TC*	*TC0*	*3bin*	*1bin*	*TC*	*TC0*	*3bin*	*1bin*	*TC*	*TC0*	*3bin*	*1bin*	*TC*	*TC0*	*3bin*	*1bin*
64	7.259361	0	0.09	0.88	2.57	0.57	0.47	0.80	0.95	0.57	1.92	12.01	13.79	0	0	496	487
128	14.517096	0	0.09	0.87	2.57	1.17	1.20	2.06	2.60	1.17	4.81	45.54	(0.033)	0	0	528	603915
256	29.032567	0	0.09	0.87	2.57	3.16	3.29	5.38	6.88	3.16	7.75	199.18	(0.052)	0	0	533	229035
512	58.063509	0	0.09	0.87	2.57	8.08	8.39	14.29	18.83	8.08	17.29	734.03	(0.054)	0	0	488	136128

* If the time limit is reached then the final % of optimality tolerance is shown between parentheses

$$\max \sum_{t=1}^{T} \sum_{g=1}^{G} \left[\pi_t \left[u_{gt} \underline{P}_g + p_{gt} \right] - \left(C_g^{\mathrm{NL}} u_{gt} + C_g^{\mathrm{LV}} \left[u_{gt} \underline{P}_g + p_{gt} \right] + C_g^{\mathrm{SU}} v_{gt} + C_g^{\mathrm{SD}} w_{gt} \right) \right] \tag{41}$$

where subscript g stands for generating units and G is the number of units; π_t refers to the energy prices; C_g^{NL}, C_g^{LV}, $C_g^{\mathrm{SU}}$and C_g^{SD} are the non-load, linear-variable, start-up and shut-down costs of unit g, respectively (for this case study $C_g^{\mathrm{SD}} = 0$ for all units). The objective function (41) is optimized over the solution set described by generation limits, start-up and shut-down capabilities, and minimum up and down times constraints. The self-UC also arises when solving UC with decomposition methods such as Lagrangian Relaxation [5, 3] (where the prices are the Lagrangian multipliers).

The 10-unit system data is presented in Table 1 and the energy prices are shown in Table 2. The power system data are based on information presented in [1, 14].

Here, apart from *TC*, *1bin* and *3bin*, the tight and compact formulation presented in [14], labeled as *TC0,* is also implemented. It is important to note that the formulation *TC0* uses constraints (12) and (13) instead of (7) and (8) for units with $TU = 1$. Apart from those constraints, *TC* and *TC0* are identical. Note however that (7) and (8) are needed to describe the convex hull, as proved in Section 4.

Table 3 shows the computational performances for four cases with different time spans. All formulations achieve the same MIP optimum since all of them model the same MIP problem. However, they present different LP optimums, the relative distance between their MIP and LP optimums is measured with the Integrality Gap [22, 14]. Note that the MIP optimums of *TC* were achieved by just solving the LP over (1)-(11), IntGap$=0$, hence solving the problems in LP time. On the other hand, as usual, the branch-and-cut method was needed to solve the MIP for *TC0*, *3bin* and *1bin.* Table 3 also shows the MIP time and B&C nodes explored that were required by the different formulations to reach optimality. It is interesting to note that although *TC0* reached optimality exploring zero B&C nodes, *TC0* needed to make use of the solver's cutting planes strategy because the relaxed LP solution did not achieve the integer one, IntGap$\neq 0$ (the solver used 227 and 1224 cuts for the smallest and largest case, respectively). This tightening process took more time than the time required to solve the initial LP relaxation, that is why the MIP time for *TC0* is more than twice its LP relaxation time.

Table 4 shows the dimensions of all the formulations for four selected instances. Note that *TC* and *TC0* are more compact, in terms of quantity of constraints and nonzero elements, than *3bin* and *1bin.* The formulation *1bin* presents a third of binary variables

Table 4: Self-UC: Problem Size Comparison

Case	# constraints			# nonzero elements				# real var		# binary var	
(days)	*TC**	*3bin*	*1bin*	*TC*	*TC0*	*3bin*	*1bin*	*TC†*	*1bin*	*TC†*	*1bin*
64	65997	107459	138225	338994	334389	417313	469719	15360	46080	46080	15360
128	132045	214979	276465	678450	669237	835105	939735	30720	92160	92160	30720
256	264141	430019	552945	1357362	1338933	1670689	1879767	61440	184320	184320	61440
512	528333	860099	1105905	2715186	2678325	3341857	3759831	12288	368640	368640	122880

* *TC* is equal to *TC0* for these cases

†*TC, TC0* and *3bin* are equal for these cases

in comparison with the other formulations, but 3 times more continuous variables. This is because the work in [1] reformulated the units' operation model to avoid the start-up and shut-down binary variables, claiming that this would reduce the node enumeration in the branch-and-bound process. Note however that this reformulation considerably damaged the strength of *1bin*, hence it presented the worst computational performance, similar results are obtained in [18, 14]. The formulation *1bin* presents more continuous variables than the other formulations because it requires the introduction of new continuous variables to model the start-up and shut-down costs of the generating units.

In conclusion, *TC* presents a dramatic improvement in computation in comparison with *3bin* and *1bin* due to its tightness (speedups above 90x and 8500x, respectively); and it also presents a lower LP burden due to its compactness, see Table 4. Compared with *TC0*, the formulation *TC* is tighter; consequently, *TC* requires less time to solve the MIP problem (speedup above 4.1x).

5.2. Network-Constrained UC

Here, two IEEE systems are used for different time spans, from 24 to 96 hours, the IEEE 118-bus system and the IEEE 73-bus reliability test system. All data for these two systems can be found in [11] and [24, 7], respectively. The IEEE-118 bus system has 118 buses; 186 transmission lines; 54 thermal units; 91 loads, with average and maximum levels of 3991 MW and 5592 MW, respectively; and three wind generation units, with aggregated average and maximum production of 867 MW and 1333 MW, respectively. For this system, the upwards and downwards reserve requirement are set as the 5% of the total expected wind production for each hour.

The IEEE 73-bus reliability test system has 73 buses; 120 transmission lines; 99 thermal units; 51 loads, with average and maximum levels of 7094 MW and 8547 MW, respectively; and no wind generation. For this system, the upwards and downwards reserve requirement are set as the 1% of the total expected demand for each hour.

Bear in mind that the network-constrained UC problem is considerably more complex than the self-UC problem, described in Subsection 5.1, due to the new complicating constraints that are now included (into all the formulations), such as demand-balance, reserves, ramping and transmission limits (see AppendixB).

Table 5 shows the problem size for all formulations for the two IEEE systems. This table shows the problem size for a time span of 24 hours, larger problem sizes are proportional (approximately) to the quantity of hours. On the other hand, there is no direct size relation between the two systems because they have different proportions in thermal

Table 5: IEEE 118-bus & 73-bus Systems: Problem Size Comparison of the UC Formulations for a Time Span of 24 hours

	# constraints			# nonzero elements			# real var			# binary var	
System	*TC*	*3bin*	*1bin*	*TC*	*3bin*	*1bin*	*TC*	*3bin*	*1bin*	*TC**	*1bin*
IEEE 118-bus	15903	37803	38141	536815	473791	472969	8424	9720	11016	3888	1296
IEEE 73-bus	23425	82846	83524	581704	786268	786310	11862	12384	14760	7110	2358

* *TC* is equal to *3bin* for these cases

Table 6: IEEE 118-bus System Results: Computational Performance of The UC Formulations for Different Time Spans

	Optimum	IntGap (%)			LP time (s)			MIP time (s)*			B&C Nodes		
hours	M$	*TC*	*3bin*	*1bin*	*TC*	*3bin*	*1bin*	*TC*	*3bin*	*1bin*	*TC*	*3bin*	*1bin*
24	0.826814	0.53	1.13	1.75	0.33	2.48	2.9	4.13	585.22	(0.094)	77	93285	889610
48	1.649732	0.49	0.70	1.37	1.17	17.88	19.19	26.15	(0.095)	(0.269)	546	260545	40115
72	2.472651	0.46	0.56	1.24	2.57	40.59	57.21	474.85	(0.136)	(0.336)	2411	50593	20657
96	3.295570	0.44	0.48	1.18	4.29	93.85	102.79	1193.92	(0.180)	(0.317)	4295	40601	14605

* If the time limit (10000 s) is reached then the final % of optimality tolerance is shown between parentheses

and wind units as well as transmission lines. For example, the IEEE 73-bus system has 45 (83%) more units than the IEEE 118-bus system, but 66 (35%) less transmission lines. Similarly to the self-UC case study (Subsection 5.1), *TC* is more compact than the others, in terms of quantity of constraints. For the IEEE 118-bus system, having a larger number of transmission lines, *TC* presents more nonzeros than the others because *TC* uses $\underline{P}_g u_{gt} + p_{gt}$, which appear in each of the line constraints, to represent the total unit's production, unlike other formulations that use one variable to represent the total production. Beware, however, that a new variable could be introduced representing the total unit's production, thus decreasing the number of nonzeros but this will increase the number of variables and constraints. Despite this increase in nonzeros, the LP complexity of *TC* for the IEEE 118-bus system is significantly lower than that of both *3bin* and *1bin*, which took in average 15.1 and 17.9 times longer than *TC* to solve the LP problem, respectively (see Table 6). Similarly, for the IEEE 73-bus system, *TC* could solve the LP problem in average 15.6 and 14.2 times faster than *3bin* and *1bin*, respectively (see Table 7). In short, *TC* presents a lower LP burden than the others due to its compactness, as also concluded in the self-UC case in Subsection 5.1.

Table 6 and Table 7 show the computational performance of the network-constrained UC problem for both IEEE test systems and for all formulations and different time spans (up to 96 hours). For these experiments, *TC* is the tightest formulation since its IntGap is always lower than that of *1bin* and *3bin*. On the other hand, although *1bin* has a third of binary variables in comparison with the others, it has the largest quantity of constraints and it is the least tight (see IntGap Table 6); consequently, presenting the worst computational performance, as also discussed in Subsection 5.1.

Interestingly, for the IEEE 118-bus system, all three formulations achieved the same optimum integer solution (all of them model the same integer problem), although *TC* was the only formulation that could prove optimality within the time limit. *3bin* could prove optimality for only one case, the smallest case; and *1bin* could not prove optimality for

Table 7: IEEE 73-bus System Results: Computational Performance of The UC Formulations for Different Time Spans

	Optimum	IntGap (%)			LP time (s)			MIP time (s)*			B&C Nodes		
hours	M$	*TC*	*3bin*	*1bin*	*TC*	*3bin*	*1bin*	*TC*	*3bin*	*1bin*	*TC*	*3bin*	*1bin*
24	1.695434	0,02	0.23	1.18	0.36	1.42	1.31	22.60	(0.107)	(0.134)	24510	1009500	897063
48	3.327422	0,02	0.24	1.13	0.80	15.3	9.66	123.17	(0.151)	(0.180)	22378	245587	264653
72	4.959410	0,02	0.24	1.11	1.33	22.14	21.87	(0.010)	(0.187)	(0.239)	1245643	100358	27756
96	6.591398	0,02	0.24	1.10	1.97	45.07	48.66	(0.012)	(0.175)	(0.374)	655694	12768	2363

* If the time limit (10000 s) is reached then the final % of optimality gap is shown between parentheses

Table 8: IEEE 118-bus and 73-bus System Results: Computational Performance of *TC* for 0.05% of Optimality Gap and Different Time Spans

	MIP time (s)		Optimality Gap (%)		B&C Nodes	
hours	118-bus	RTS-96	118-bus	RTS-96	118-bus	RTS-96
24	3.45	3.54	0.030	0.045	0	5
48	9.25	7.94	0.036	0.032	0	0
72	68.2	13.09	0.034	0.049	625	0
96	167.44	45.76	0.041	0.049	560	490

any of the cases. Notice that due to the tightness, *TC* could prove optimality exploring considerably fewer B&C nodes less than (an order of magnitude) *3bin* and *1bin*, which could not even converge to optimality.

For the IEEE 118-bus system, *TC* always found better integer solutions (reported in Table 6) than the other formulations. *3bin* and *1bin* could not prove optimality for any of the cases. *TC* could prove optimality for the two smallest cases, where *TC* explored fewer nodes than the others, which could not even reach optimality. For the two largest cases, none of the formulations could reach optimality, but *TC* was an order of magnitude nearer to optimality. Also notice that for these two large cases, *TC* could explore more nodes within the time limit due to its compactness, which lower the LP complexity solved during the iterations.

Table 6 and Table 7 show the computational performance of the UC formulations trying to reach optimality (more precisely to 10^{-4}% of relative optimality gap) within a 10000 seconds time limit. Notice that *1bin* could only reach optimality gaps above 0.13% for 7 out of 8 cases, and in the best case the optimality gap was above 0.09%. Similarly, *3bin* presented optimality gaps above 0.09% for 7 of the cases. In short, only *3bin* could reach an optimality gap below 0.09% in just one case. To observe the performance of *TC* around these orders of magnitude of optimality gaps, Table 8 shows the performance of *TC* for a requiered optimality gap of 0.05% for the two IEEE test systems. Notice that 4 cases could even be solved before branching (0 B&C nodes), 5 cases were solved in less than 15 seconds, and all the cases could be solved in less than 170 seconds, unlike *3bin* and *1bin* which could not reach that low optimality gaps within 10000 seconds. Due to the simultaneous tightness and compactness, *TC* could reach 0.05% optimality tolerance for four cases (one for the IEEE 118-bus system and three for the IEEE 73-bus system) in less time than that required by *1bin* and *3bin* to solve their LP problem.

Furthermore, for the IEEE 73-bus system, *TC* presented better (higher) lower bounds

Table 9: IEEE 73-bus System: Initial vs. Final Lower Bounds of UC formulations for Different Time Spans

	LP Relaxations (M$)			Final best lower bound (M$)		
hours	*TC*	*3bin*	*1bin*	*TC*	*3bin*	*1bin*
24	1.695161	1.691586	1.675454	1.695434	1.693621	1.693167
48	3.326716	3.319535	3.289971	3.327422	3.322417	3.32144
72	4.958264	4.947482	4.904489	4.958887	4.951332	4.947532
96	6.589812	6.575429	6.519006	6.590607	6.57985	6.569458

in the initial LP relaxation than the final lower bounds found by *3bin* and *1bin* within the time limit, as shown in Table 9 (this was not the case for the IEEE 118-bus system). Thanks to the convex hull provided in this paper, for the IEEE 73-test system, *TC* could provide initial lower bounds, in less than 2 seconds (see LP time in Table 7), which were better than the final lower bounds obtained by *3bin* and *1bin* within 10000 seconds.

6. Conclusion

This paper presented the convex hull description of the single thermal Unit Commitment problem with the following basic constraints: generation limits, start-up and shut-down capabilities, and minimum up and down times. The model does not include some crucial constraints, such as ramping, but the proposed constraints can be used as the core of any UC formulation and they can help to tighten the final UC model.

Computational experiments have been carried out among the new proposed formulation and two previous formulations called *1bin* and *3bin* considering two Unit Commitment variants: the self-UC and the network-constrained UC problems. For both problems, the new proposed formulation presents a dramatic improvement in computation in comparison with *3bin* and *1bin* due to its tightness; and it also presents a lower LP burden due to its compactness (see Table 4 and Table 5).

Acknowledgments

The authors thank Laurence Wolsey, Santanu Dey, Antonio Frangioni, and Paolo Ventura for useful discussions on the paper.

AppendixA. 1bin and 3bin UC formulations

This section presents the basic constraints for the *1bin* and *3bin* UC formulations. The nomenclature used here is the same one presented in Section 2, the new nomenclature is defined once it is introduced. It is important to highlight that *1bin* and *3bin* formulations consider the total energy production variable $\widehat{p}_t$ from 0 to $\overline{P}$, unlike the formulation presented in this paper where p_t represents the energy production above $\underline{P}$.

AppendixA.1. 1bin formulation

The *1bin* formulation is the following (see Carrion and Arroyo [1]):

$$
\begin{array}{ll}
\underline{P}u_t \leq \widehat{p}_t \leq \overline{P}u_t & t = 1, \ldots, T \\
\widehat{p}_t \leq \widehat{p}_{t-1} + RUu_{t-1} + SU(u_t - u_{t-1}) + \overline{P}(1 - u_t) & t = 2, \ldots, T \\
\widehat{p}_{t-1} \leq \widehat{p}_t + RDu_t + SD(u_{t-1} - u_t) + \overline{P}(1 - u_{t-1}) & t = 2, \ldots, T \\
\sum_{j=1}^{G}(1 - u_j) = 0 & \\
\sum_{j=t}^{t+TU-1} u_j \geq TU(u_t - u_{t-1}) & t = G + 1, \ldots, T - TU + 1 \\
\sum_{j=t}^{T} [u_j - (u_t - u_{t-1})] \geq 0 & t = T - TU + 2, \ldots, T \\
\sum_{j=1}^{L} u_j = 0 & \\
\sum_{j=t}^{t+TD-1}(1 - u_j) \geq TD(u_{t-1} - u_t) & t = L + 1, \ldots, T - TD + 1 \\
\sum_{j=t}^{T} [1 - u_j - (u_{t-1} - u_t)] \geq 0 & t = T - TD + 2, \ldots, T \\
suc_t \geq C^{SU}(u_t - u_{t-1}) & t = 2, \ldots, T \\
sdc_t \geq C^{SD}(u_{t-1} - u_t) & t = 2, \ldots, T \\
0 \leq u_t \leq 1 & t = 1, \ldots, T
\end{array}
\tag{A.1}
$$

where $G = min\{T, (TU - \tau_0)u_0\}$ and $L = \min\{T, (TD + \tau_0)(1 - u_0)\}$ are the minimum number of time instants the unit must be initially on or off, respectively (τ_0 indicates the number of time instants the unit has been on prior to time 0 if $\tau_0 > 0$, while $-\tau_0$ indicates the number of time instants the unit has been off prior to time 0 if $\tau_0 < 0$).

Note that *1bin* models the unit's start-up and shut-down capabilities inside the ramping constraints. For the set of experiments presented in 5.1, where no ramping constraints are considered, the ramping constraints of *1bin* were adapted to only model the start-up and shut-down capabilities. Therefore, the constraints for the unit's start-up and shut-down capability become $\widehat{p}_t \leq SU(u_t - u_{t-1}) + \overline{P}(1 + u_{t-1} - u_t)$ and $\widehat{p}_{t-1} \leq SD(u_{t-1} - u_t) + \overline{P}(1 + u_t - u_{t-1})$, respectively.

AppendixA.2. 3bin formulation

The *3bin* formulation is the following Ostrowski et al. [18]:

$$
\begin{array}{ll}
\underline{P}u_t \leq \widehat{p}_t \leq \overline{P}u_t & t = 1, \ldots, T \\
\widehat{p}_t \leq \widehat{p}_{t-1} + RUu_{t-1} + SUv_t & t = 2, \ldots, T \\
\widehat{p}_{t-1} \leq \widehat{p}_t + RDu_t + SDw_t & t = 2, \ldots, T
\end{array}
\tag{A.2}
$$

where the minimum up and down constraints are guaranteed using (1)-(3), and the initial conditions of those constraints are ensured in the same way as *1bin* (see AppendixA.1).

Similarly to *1bin*, *3bin* also models the unit's start-up and shut-down capabilities inside the ramping constraints. Then, for the set of experiments presented in 5.1, the ramping constraints of *3bin* were adapted to only model the start-up and shut-down capabilities. Therefore, the constraints for the unit's start-up and shut-down capability become $\widehat{p}_t \leq \overline{P}u_{t-1} + SUv_t$ and $\widehat{p}_{t-1} \leq \overline{P}u_t + SDv_t$, respectively.

Note that, unlike *1bin*, *3bin* and *TC* do not need extra variables suc_t and std_t for the start-up and shut-down costs since these costs can be directly expressed with variables v_t and w_t and included in the objective function, see (41).

AppendixB. Network-Constrained UC Formulation

Here, we present the network-constrained UC formulation, of which core is based on the tight and compact model presented in Section 3. Although some nomenclature and constraints were introduced before, for the sake of clarity and completeness, this section provides the complete nomenclature and set of constraints. In the following, we present the additional needed notations beyond the ones presented in Section 2.

AppendixB.1. Nomenclature

Indexes and Sets

$b \in \mathcal{B}$ Buses, running from 1 to B.

$\mathcal{B}^{\mathrm{W}}$ Set of buses in $\mathcal{B}$ with wind power injection.

$l \in \mathcal{L}$ Transmission lines, running from 1 to L.

$t \in \mathcal{T}$ Hourly periods, running from 1 to T hours.

System Parameters

D_{bt} Energy demand on bus b at the end of hour t [MW].

D_t^- / D_t^+ System requirements for downward/upward reserve for hour t [MW].

$\overline{F}_l$ Power flow limit on transmission line l [MW].

$\Gamma_{lb}/\Gamma_{lg}^{\mathrm{G}}$ Shift factor for line l associated with bus b / unit g [p.u.].

P_{bt}^{W} Nominal forecasted wind energy for hour t [MW].

Unit's Parameters

C_g^{LV} Linear variable production cost [\$/MWh].

C_g^{NL} Non-load cost [\$/h].

$C_g^{\mathrm{SD}}/C_g^{\mathrm{SU}}$ Shut-down / Sart-up cost [\$].

RD_g/RU_g Ramp-down/ramp-up capability [MW/h].

Decision Variables

p_{bt}^{W} Wind energy output for hour t [MW].

r_{gt}^-/r_{gt}^+ Downwards/upwards power reserve [MW].

AppendixB.2. Objective Function

The UC seeks to minimize all production costs:

$$\min \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} \left[C_g^{\mathrm{LV}}\left(\underline{P}_g u_{gt} + p_{gt}\right) + C_g^{\mathrm{NL}} u_{gt} + C_g^{\mathrm{SU}} v_{gt} + C_g^{\mathrm{SD}} w_{gt}\right] \tag{B.1}$$

The proposed formulation also takes into account variable start-up costs, which depend on how long the unit has been offline. The reader is referred to [15, 14] for further details.

AppendixB.3. System-wide Constraints

Energy demand balance and upward/downward reserves requirements are guaranteed as follows:

$$\sum_{g\in\mathcal{G}} \left(\underline{P}_g u_{gt} + p_{gt}\right) = \sum_{b\in\mathcal{B}} D_{bt} - \sum_{b\in\mathcal{B}^{\mathrm{W}}} p^{\mathrm{W}}_{bt} \quad \forall t \tag{B.2}$$

$$\sum_{g\in\mathcal{G}} r^{+}_{gt} \geq D^{+}_{t} \quad \forall t \tag{B.3}$$

$$\sum_{g\in\mathcal{G}} r^{-}_{gt} \geq D^{-}_{t} \quad \forall t \tag{B.4}$$

Transmission limits are ensured with:

$$-\overline{F}_l \leq \sum_{g\in\mathcal{G}} \Gamma^{\mathrm{G}}_{lg} \left(\underline{P}_g u_{gt} + p_{gt}\right) + \sum_{b\in\mathcal{B}^{\mathrm{W}}} \Gamma_{lb} p^{\mathrm{W}}_{bt} - \sum_{\forall b\in\mathcal{B}} \Gamma_{lb} D_{bt} \leq \overline{F}_l \quad \forall l,t \tag{B.5}$$

AppendixB.4. Individual Unit Constraints

The commitment, start-up/shut-down logic and the minimum up/down times are guaranteed by constraints (1)-(3) replicated for each generation unit g and where the initial conditions for the minimum up/down constraints are detailed in [14]. Basically, u_{gt} is fixed (become constant) to 0 or 1 for the initial periods where the unit must remain offline or online, respectively.

The energy production and reserves must be within the power capacity limits:

$$p_{gt} + r^{+}_{gt} \leq \left(\overline{P}_g - \underline{P}_g\right) u_{gt} - \left(\overline{P}_g - SD_g\right) w_{g,t+1} \\ - \max\left(SD_g{-}SU_g, 0\right) v_{g,t} \quad \forall g\in\mathcal{G}^1, t \tag{B.6}$$

$$p_{gt} + r^{+}_{gt} \leq \left(\overline{P}_g - \underline{P}_g\right) u_{gt} - \left(\overline{P}_g - SU_g\right) v_{gt} \\ - \max\left(SU_g{-}SD_g, 0\right) w_{g,t+1} \quad \forall g\in\mathcal{G}^1, t \tag{B.7}$$

$$p_{gt} + r^{+}_{gt} \leq \left(\overline{P}_g - \underline{P}_g\right) u_{gt} - \left(\overline{P}_g - SU_g\right) v_{gt} \\ - \left(\overline{P}_g - SD_g\right) w_{g,t+1} \quad \forall g\notin\mathcal{G}^1, t \tag{B.8}$$

$$p_{gt} - r^{-}_{gt} \geq 0 \quad \forall g,t \tag{B.9}$$

where $\mathcal{G}^1$ is defined as the units in $\mathcal{G}$ with $TU_g{=}1$.

Ramping capability limits are ensured with:

$$\left(p_{gt} + r^{+}_{gt}\right) - p_{g,t-1} \leq RU_g \quad \forall g,t \tag{B.10}$$

$$-\left(p_{gt} - r^{-}_{gt}\right) + p_{g,t-1} \leq RD_g \quad \forall g,t \tag{B.11}$$

notice that by modeling the generation output p_{gt} above $\underline{P}_g$, the proposed formulation avoids introducing binary variables into the ramping constraints (B.10) and (B.11), unlike *1bin* and *3bin*, see AppendixA.1 and AppendixA.2, respectively. In other words, when the generation output variable is defined between 0 and $\overline{P}_g$, then the ramping constraints should consider the case when a generator's output level should not be limited by the ramp rate, when it is starting up or shutting down; such complicating situations are usually tackled by introducing big-M parameters together with binary variables into the ramping constraints.

Wind production limits are represented by:

$$p_{bt}^{\mathrm{W}} \leq P_{bt}^{\mathrm{W}} \quad \forall b \in \mathcal{B}^{\mathrm{W}}, t \tag{B.12}$$

Finally, non-negative constraints for all decision variables:

$$p_{gt}, r_{gt}^{+}, r_{gt}^{-} \geq 0 \quad \forall g, t \tag{B.13}$$

$$p_{bt}^{\mathrm{W}} \geq 0 \quad \forall b \in \mathcal{B}^{\mathrm{W}}, t \tag{B.14}$$

References

[1] Carrion, M., Arroyo, J., 2006. A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem. IEEE Transactions on Power Systems 21 (3), 1371–1378.

[2] Damci, P., Kucukyavuz, S., Rajan, D., Atamtu, A., 2015. A polyhedral study of production ramping. Mathematical Programming, to appear.

[3] Frangioni, A., Gentile, C., 2006. Solving nonlinear single-unit commitment problems with ramping constraints. Operations Research 54 (4), 767–775.

[4] Frangioni, A., Gentile, C., 2015. An extended mip formulation for the single-unit commitment problem with ramping constraints. 17th British-French-German conference on Optimization, London, June 15-17.

[5] Frangioni, A., Gentile, C., Lacalandra, F., 2008. Solving unit commitment problems with general ramp constraints. International Journal of Electrical Power & Energy Systems 30 (5), 316–326.

[6] Frangioni, A., Gentile, C., Lacalandra, F., 2009. Tighter approximated MILP formulations for unit commitment problems. IEEE Transactions on Power Systems 24 (1), 105–113.

[7] Hedman, K., Ferris, M., O'Neill, R., Fisher, E., Oren, S., May 2010. Co-Optimization of Generation Unit Commitment and Transmission Switching With N-1 Reliability. Power Systems, IEEE Transactions on 25 (2), 1052 –1063.

[8] Knueven, B., Ostrowski, J., Wang, J., 2015. Constrained minkowski sums of polyhedra with an application in unit commitment. Optimization Online (http://www.optimization-online.org/DB_FILE/2015/09/5099.pdf).

[9] Lee, J., Leung, J., Margot, F., 2004. Min-up/min-down polytopes. Discrete Optimization 1 (1), 77–85.

[10] Malkin, P., 2003. Minimum runtime and stoptime polyhedra. manuscript.

[11] Morales-España, G., Sep. 2014. Unit Commitment: Computational Performance, System Representation and Wind Uncertainty Management. Ph.D. thesis, Pontifical Comillas University, KTH Royal Institute of Technology, and Delft University of Technology, Spain.

[12] Morales-Espana, G., Correa-Posada, C. M., Ramos, A., 2015. Tight and Compact MIP Formulation of Configuration-Based Combined-Cycle Units. IEEE Transactions on Power Systems PP (99), 1–10.

[13] Morales-España, G., Gentile, C., Ramos, A., 2015. Tight MIP formulations of the power-based unit commitment problem. OR Spectrum 37 (4), 929–950.

[14] Morales-Espana, G., Latorre, J., Ramos, A., 2013. Tight and compact MILP formulation for the thermal unit commitment problem. IEEE Transactions on Power Systems 28 (4), 4897–4908.

[15] Morales-Espana, G., Latorre, J. M., Ramos, A., 2013. Tight and compact MILP formulation of start-up and shut-down ramping in unit commitment. IEEE Transactions on Power Systems 28 (2), 1288–1296.

[16] Morales-Espana, G., Ramos, A., Garcia-Gonzalez, J., 2014. An MIP formulation for joint market-clearing of energy and reserves based on ramp scheduling. IEEE Transactions on Power Systems 29 (1), 476–488.

[17] Nemhauser, G. L., Wolsey, L. A., 1999. Integer and combinatorial optimization. John Wiley and Sons, New York.

[18] Ostrowski, J., Anjos, M. F., Vannelli, A., 2012. Tight mixed integer linear programming formulations for the unit commitment problem. IEEE Transactions on Power Systems 27 (1), 39–46.

[19] Rajan, D., Takriti, S., 2005. Minimum Up/Down polytopes of the unit commitment problem with start-up costs. Research Report RC23628, IBM.
URL `http://domino.research.ibm.com/library/cyberdig.nsf/1e4115aea78b6e7c85256b360066f0d4/cdcb02a7c809d89e8525702300502ac0?OpenDocument`

[20] Silbernagl, M., Huber, M., Brandenberg, R., 2014. Improving accuracy and efficiency of start-up cost formulations in mip unit commitment by modeling power plant temperatures. arXiv:14082644 [math] http://arxiv.org/abs/1408.2644.

[21] Tahanan, M., Ackooij, W. v., Frangioni, A., Lacalandra, F., 2015. Large-scale Unit Commitment under uncertainty. 4OR, 1–57.

[22] Williams, H. P., 2013. Model Building in Mathematical Programming, 5th Edition. John Wiley & Sons Inc.

[23] Wolsey, L., 1998. Integer Programming. Wiley-Interscience.

[24] Wong, P., Albrecht, P., Allan, R., Billinton, R., Chen, Q., Fong, C., Haddad, S., Li, W., Mukerji, R., Patton, D., Schneider, A., Shahidehpour, M., Singh, C., Aug. 1999. The IEEE Reliability Test System-1996. A report prepared by the Reliability Test System Task Force of the Application of Probability Methods Subcommittee. IEEE Transactions on Power Systems 14 (3), 1010–1020.