

Ottimizzazione Non Lineare

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Problemi con soli vincoli di disuguaglianza – 1

Consideriamo il problema

$$\begin{array}{ll} \min & f(x) \\ \text{c.v.} & g(x) \leq 0 \end{array}$$

Usiamo la trasformazione:

- $g_i(x) \leq 0 \rightsquigarrow g_i(x) + y_i^2 = 0, i = 1, \dots, m$

$$\begin{aligned} L_a(x, y, \lambda; \epsilon) &= f(x) + \sum_{i=1}^m \lambda_i (g_i(x) + y_i^2) + \frac{1}{\epsilon} \sum_{i=1}^m (g_i(x) + y_i^2)^2 \\ &= f(x) + \lambda^\top g(x) + \frac{1}{\epsilon} \|g(x)\|^2 + \frac{1}{\epsilon} \sum_{i=1}^m (\epsilon \lambda_i y_i^2 + 2g_i(x)y_i^2 + y_i^4) \end{aligned}$$

Problemi con soli vincoli di disuguaglianza – 2

Quindi

$$\begin{array}{ll} \min & f(x) \\ \text{c.v.} & g(x) \leq 0 \end{array} \quad \text{ovvero} \quad \begin{array}{ll} \min & f(x) \\ \text{c.v.} & g_i(x) + y_i^2 = 0, \quad i = 1, \dots, m \end{array}$$

abbiamo

$$L_a(x, y, \lambda; \epsilon) = f(x) + \lambda^\top g(x) + \frac{1}{\epsilon} \|g(x)\|^2 + \frac{1}{\epsilon} \sum_{i=1}^m (\epsilon \lambda_i y_i^2 + 2g_i(x)y_i^2 + y_i^4)$$

fissati ϵ e λ , bisogna “risolvere”

$$\min_{x,y} L_a(x, y, \lambda; \epsilon)$$

Gradiente della Lagrangiana

$$\nabla_x L_a = \nabla f(x) + \nabla g(x)\lambda + \frac{2}{\epsilon} \sum_{i=1}^m (g_i(x) + y_i^2) \nabla g_i(x)$$

$$\nabla_\lambda L_a = g(x) + y^2$$

$$\nabla_{y_i} L_a = \frac{4}{\epsilon} y_i \left(\epsilon \frac{\lambda_i}{2} + g_i(x) + y_i^2 \right)$$

Quindi, ponendo $\nabla_{y_i} L_a = 0$ otteniamo

$$y_i \left(\epsilon \frac{\lambda_i}{2} + g_i(x) + y_i^2 \right) = 0$$

ovvero

$$y_i^2 = \max \left\{ 0, -\epsilon \frac{\lambda_i}{2} - g_i(x) \right\}$$

Espressione per vincoli di disuguaglianza

$$L_a(x, \lambda; \epsilon) = f(x) + \lambda^\top \left(g(x) + \max \left\{ 0, -\epsilon \frac{\lambda}{2} - g(x) \right\} \right) + \frac{1}{\epsilon} \left\| g(x) + \max \left\{ 0, -\epsilon \frac{\lambda}{2} - g(x) \right\} \right\|^2$$

Ovvero

$$L_a(x, \lambda; \epsilon) = f(x) + \lambda^\top \max \left\{ g(x), -\epsilon \frac{\lambda}{2} \right\} + \frac{1}{\epsilon} \left\| \max \left\{ g(x), -\epsilon \frac{\lambda}{2} \right\} \right\|^2$$

$$\nabla_x L_a(x, \lambda; \epsilon) = \nabla f(x) + \nabla g(x) \left(\lambda + \frac{2}{\epsilon} \max \left\{ g(x), -\epsilon \frac{\lambda}{2} \right\} \right)$$

$$\nabla_\lambda L_a(x, \lambda; \epsilon) = \max \left\{ g(x), -\epsilon \frac{\lambda}{2} \right\}$$

Metodo di soluzione - Metodo dei Moltiplicatori

Algoritmo SEQLAGR

Dati: $\epsilon_0 > 0$, $\beta > 1$, $\{\tau_k\} \rightarrow 0$, $\rho > 0$, λ_0 , μ_0 , maxit

for $k = 0, 1, \dots, \text{maxit}$

Calcola x_k t.c. $\|\nabla_x L_a(x_k, \mu_k, \lambda_k; \epsilon_k)\| \leq \tau_k$

if $\|\nabla L_a(x_k, \mu_k, \lambda_k; \epsilon_k)\| < \rho$ **then**

$x^* \leftarrow x_k$, $\mu^* \leftarrow \mu_k$, $\lambda^* \leftarrow \lambda_k$ STOP

endif

$\epsilon_{k+1} = \epsilon_k / \beta$

$$\mu_{k+1} = \mu_k + \frac{2h(x_k)}{\epsilon_k}, \quad \lambda_{k+1} = \lambda_k + \frac{2 \max\{g(x_k), -\epsilon_k \lambda_k / 2\}}{\epsilon_k}$$

endfor

Return: miglior coppia trovata (x^*, μ^*, λ^*)