Ottimizzazione dei Sistemi Complessi

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Penalità esterne o sequenziali

Per il problema

$$\begin{aligned} &\min\ f(x) \\ &s.t.\ h(x) = 0 \\ &\qquad g(x) \leq 0 \end{aligned}$$

Si definisce

$$\begin{aligned} P(x;\epsilon) &= f(x) + \frac{1}{\epsilon}\left(\|h(x)\|^2 + \|\max\{0, g(x)\}\|^2\right) \\ \nabla_x P(x;\epsilon) &= \nabla f(x) + \frac{2}{\epsilon}\Big(\nabla h(x) h(x) + \nabla g(x) \max\{0, g(x)\}\Big) \end{aligned}$$

$$\frac{2}{\epsilon_k} h(x_k) = \mu_k \to \mu^*, \qquad \frac{2}{\epsilon_k} \max\{0, g(x_k)\} = \lambda_k \to \lambda^*$$

Lagrangiani aumentati

Per il problema

$$\begin{array}{ll} \min & f(x) \\ s.t. & h(x) = 0 \\ & g(x) \le 0 \end{array}$$

Si definisce

$$\begin{aligned} L_a(x,\mu,\lambda;\epsilon) &= f(x) + \mu^\top h(x) + \lambda^\top \max\left\{g(x), -\frac{\epsilon}{2}\lambda\right\} \\ &+ \frac{1}{\epsilon}\left(\|h(x)\|^2 + \left\|\max\left\{g(x), -\frac{\epsilon}{2}\lambda\right\}\right\|^2\right) \\ \nabla_x L_a(x,\mu,\lambda;\epsilon) &= \nabla_x L(x,\mu,\lambda) + \frac{2}{\epsilon}\Big(\nabla h(x)h(x) + \nabla g(x)\max\{g(x), -\epsilon\lambda/2\}\Big) \end{aligned}$$

$$\mu_k + \frac{2}{\epsilon_k} h(x_k) = \mu_{k+1} \to \mu^*, \qquad \lambda_k + \frac{2}{\epsilon_k}\max\left\{g(x_k), -\frac{\epsilon_k}{2}\lambda_k\right\} = \lambda_{k+1} \to \lambda^*$$

Funzioni con barriera logaritmica

Per il problema

$$\begin{aligned} &\min \; f(x) \\ &s.t. \; h(x) = 0 \\ &\qquad g(x) \geq 0 \end{aligned}$$

Si definisce

$$\begin{aligned} B(x;\rho) &= f(x) - \rho \sum_{i=1}^{m} \log g_i(x) + \frac{1}{2\rho}\|h(x)\|^2 \\ \nabla_x B(x;\rho) &= \nabla f(x) - \sum_{i=1}^{m} \frac{\rho}{g_i(x)} \nabla g_i(x) + \frac{1}{\rho}\nabla h(x) h(x) \end{aligned}$$

$$\frac{1}{\rho_k} h(x_k) = \mu_k \to \mu^*, \qquad \frac{\rho_k}{g_i(x_k)} = (\lambda_k)_i \to \lambda_i^*$$

Funzioni con barriera logaritmica

Per il problema

$$\begin{aligned} \min\ & f(x) \\ s.t.\ & h(x) = 0 \\ & g(x) \leq 0 \end{aligned}$$

Si definisce

$$\begin{aligned} B(x;\rho) &= f(x) - \rho \sum_{i=1}^{m} \log -g_i(x) + \frac{1}{2\rho}\|h(x)\|^2 \\ \nabla_x B(x;\rho) &= \nabla f(x) - \sum_{i=1}^{m} \frac{\rho}{g_i(x)} \nabla g_i(x) + \frac{1}{\rho} \nabla h(x) h(x) \end{aligned}$$

$$\frac{1}{\rho_k} h(x_k) = \mu_k \to \mu^*, \qquad \frac{-\rho_k}{g_i(x_k)} = (\lambda_k)_i \to \lambda_i^*$$

Funzioni con barriera logaritmica – variabili slack

Per il problema

$$\begin{array}{ll} \min & f(x) \\ s.t. & h(x) = 0 \\ & g(x) \geq 0 \end{array} \quad \rightsquigarrow \quad \begin{array}{ll} \min & f(x) \\ s.t. & h(x) = 0 \\ & g(x) - s = 0 \\ & s \geq 0 \end{array}$$

Si definisce

$$\begin{aligned} B(x, s; \rho) &= f(x) - \rho \sum_{i=1}^{m} \log s_i + \frac{1}{2\rho} \left(\|h(x)\|^2 + \|g(x) - s\|^2 \right) \\ \nabla_x B(x, s; \rho) &= \nabla f(x) + \frac{1}{\rho} \nabla h(x) h(x) + \frac{1}{\rho} \nabla g(x)(g(x) - s) \end{aligned}$$

$$\frac{1}{\rho_k} h(x_k) = \mu_k \to \mu^*, \qquad -\frac{1}{\rho_k}(g(x_k) - s_k) = \lambda_k \to \lambda^*$$