Controlling Polyvariance for Specialization-Based Verification

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Verification via Reachability

**Forward Reachability**

Initial States \( \rightarrow \) \( t \rightarrow t^2 \rightarrow t^\omega \rightarrow \ldots \) Unsafe States

\[ t \quad t^2 \quad t^\omega \rightarrow \ldots \]

- = \( \emptyset \) safety
- \( \neq \emptyset \) unsafety

**Backward Reachability**

Initial States \( \rightarrow \) \( t^{-\omega} \rightarrow t^{-2} \rightarrow t^{-1} \rightarrow \ldots \) Unsafe States

\[ t^{-\omega} \rightarrow t^{-2} \rightarrow t^{-1} \rightarrow \ldots \]

- = \( \emptyset \) safety
- \( \neq \emptyset \) unsafety
Theorem:

The system is safe iff unsafe $\not\in M(Bw)$

\[ (S_{Bw})^\omega \]

\[ A \Theta \]

A $\leftarrow$ c with $c \Theta$ satisf.
**An Example of System Verification**

\[
\begin{align*}
\text{init}(<X_1,X_2>&>&) : & \quad X_1 \geq 1 \land X_2 = 0 \\
\text{t}(<X_1,X_2>, <X'_1,X'_2>&>&) : & \quad X'_1 = X_1 + X_2 \land X'_2 = X_2 + 1 \\
\text{u}(<X_1,X_2>&>&) : & \quad X_2 > X_1
\end{align*}
\]

**Bw:**

1. unsafe \( \leftarrow X_1 \geq 1 \land X_2 = 0 \land \text{bwReach}(X_1,X_2) \)
2. \( \text{bwReach}(X_1,X_2) \leftarrow X'_1 = X_1 + X_2 \land X'_2 = X_2 + 1 \land \text{bwReach}(X'_1,X'_2) \)
3. \( \text{bwReach}(X_1,X_2) \leftarrow X_2 > X_1 \)

Unfortunately, the computation of \( M(Bw) \) does not terminate.

**Verification via Specialization:**

(A) \( Bw \quad \longrightarrow \quad \text{SpBw} \)

(B) unsafe \( \notin \quad M(\text{SpBw}) \)
Specialization via Unfold/Definition/Fold

def-intro:
4. \( \text{new1}(X_1, X_2) \leftarrow X_1 \geq 1 \land X_2 = 0 \land \text{bwReach}(X_1, X_2) \)

fold:
1f. \( \text{unsafe} \leftarrow X_1 \geq 1 \land X_2 = 0 \land \text{new1}(X_1, X_2) \)

unfold:
4u. \( \text{new1}(X_1, X_2) \leftarrow X_1 \geq 1 \land X_2 = 0 \land X'_1 = X_1 \land X'_2 = 1 \land \text{bwReach}(X'_1, X'_2) \)

def-intro:
  \( \text{newp}(X'_1, X'_2) \leftarrow X'_1 \geq 1 \land X'_2 = 1 \land \text{bwReach}(X'_1, X'_2) \)

fold: ...
unfold: ...
def-intro:
  \( \text{newq}(X''_1, X''_2) \leftarrow X''_1 \geq 1 \land X''_2 = 2 \land \text{bwReach}(X''_1, X''_2) \)
  
  :  
  
  :  

\[ \text{Nontermination of specialization} \]
Need for Generalization

def-intro:
5. \( \text{new2}(X_1, X_2) \leftarrow X_1 \geq 1 \land X_2 \geq 0 \land \text{bwReach}(X_1, X_2) \) (generalization)

4uf. \( \text{new1}(X_1, X_2) \leftarrow X_1 \geq 1 \land X_2 = 0 \land X'_1 \geq X_1 \land X'_2 = 1 \land \text{new2}(X'_1, X'_2) \)

From 5 by unfold-fold:
6. \( \text{new2}(X_1, X_2) \leftarrow X_1 \geq 1 \land X_2 \geq 0 \land X'_1 = X_1 + X_2 \land X'_2 = X_2 + 1 \land \text{new2}(X'_1, X'_2) \)
7. \( \text{new2}(X_1, X_2) \leftarrow X_1 \geq 1 \land X_2 > X_1 \)

SpBw: 1f, 4uf, 6, 7.

Specialization has terminated (due to generalization).

The computation of \( M(\text{SpBw}) \) terminates:

- \( \text{unsafe} \notin M(\text{SpBw}) \)
- \( \text{new1}(X_1, X_2) \leftarrow \text{false} \)
- \( \text{new2}(X_1, X_2) \leftarrow X_1 \geq 1 \land X_2 > 1 \)
Need for Generalization

def-intro:
5. \[ \text{new2}(X_1, X_2) \leftarrow X_1 \geq 1 \land X_2 \geq 0 \land \text{bwReach}(X_1, X_2) \] (generalization)

4uf. \[ \text{new1}(X_1, X_2) \leftarrow X_1 \geq 1 \land X_2 = 0 \land X'_1 \geq X_1 \land X'_2 = 1 \land \text{new2}(X'_1, X'_2) \]

From 5 by unfold-fold:
6. \[ \text{new2}(X_1, X_2) \leftarrow X_1 \geq 1 \land X_2 \geq 0 \land X'_1 = X_1 + X_2 \land X'_2 = X_2 + 1 \land \text{new2}(X'_1, X'_2) \]
7. \[ \text{new2}(X_1, X_2) \leftarrow X_1 \geq 1 \land X_2 \geq X_1 \]

SpBw: 1f, 4uf, 6, 7.

- Specialization has terminated (due to generalization).
- The computation of \( M(\text{SpBw}) \) terminates:

\[ \text{unsafe} \not\in M(\text{SpBw}) \]

\[ \text{new1}(X_1, X_2) \leftarrow \text{false} \]

\[ \text{new2}(X_1, X_2) \leftarrow X_1 \geq 1 \land X_2 > 1 \]
**A Different Specialization**

new2 is *more general* than new1: use new2, instead of new1.

SpBw1:
1f’. unsafe ← $X_1 \geq 1 \land X_2 = 0 \land \text{new2}(X_1, X_2)$
6. new2($X_1, X_2$) ← $X_1 \geq 1 \land X_2 \geq 0 \land X'_1 = X_1 + X_2 \land X'_2 = X_2 + 1 \land \text{new2}(X_1, X_2)$
7. new2($X_1, X_2$) ← $X_1 \geq 1 \land X_2 > X_1$

SpBw1: 1f, 6, 7.

- Fold “immediately”: use of new1 and new2.
  More polyvariance (SpBw).
- Fold at the end “with a maximally general definition”: use of new2 only.
  Less polyvariance (SpBw1).

Polyvariance depends on generalization and folding and affects the specialization time and the size of the specialized program (and thus, the computation of the $M(\text{SpBw})$).
Constructing the Definition Tree: DefsTree

Initialization:

- a generic node $D$:
- Unfold using T’s and U’s:
- Partition of clauses into blocks:
- Generalize:

- Stop if node $D$ occurs earlier in DefsTree.
DefsTree for Our Verification

Initialization:

D1: 4. new1(X1, X2) ← X1 ≥ 1 ∧ X2 = 0 ∧ bwReach(X1, X2)

D2: 5. new2(X1, X2) ← X1 ≥ 1 ∧ X2 ≥ 0 ∧ bwReach(X1, X2)

Unfold:

4u. new1(X1, X2) ← X1 ≥ 1 ∧ X2 = 0 ∧ X'1 = X1 ∧ X'2 = 1 ∧ bwReach(X'1, X'2)

Generalize (ch+widen):

5. new2(X1, X2) ← X1 ≥ 1 ∧ X2 ≥ 0 ∧ bwReach(X1, X2)
Another generalization operator: (Convex-Hull and) WidenSum.
It takes into account the coefficients of the variables (in our case: 1).
Input: program Bw
Output: program SpBw such that unsafe ∈ M(Bw) iff unsafe ∈ M(SpBw)

Initialization: DefsTree := \{T \rightarrow D_1, \ldots, T \rightarrow D_k\}

while there exists a definition D in DefsTree which does not occur earlier
do - unfold using T_i’s and U_i’s and derive UnfD;
    - definition introduction:
      \textbf{Partition}(UnfD, \{B_1, \ldots, B_h\}) ;
      \textbf{Generalize}(D, B_i, DefsTree, G_i) and derive a new DefsTree
od

\textbf{Fold}(DefsTree, SpBw)

Polyvariance is controlled by choosing suitable \textbf{Partition}, \textbf{Generalize}, and \textbf{Fold}
procedures.
Various Partition Operators

UnfD: clauses $C_1, \ldots, C_m, C_{m+1}, \ldots, C_n$

(part constrained facts)

Partition:
1. Singleton: $\{C_1\}, \ldots, \{C_m\}$

($m$ blocks)

2. Finite Domain: clauses $C_i$ and $C_j$ in the same block iff $\text{con}(C_i)|_{X'} \approx_{fd} \text{con}(C_j)|_{X'}$

E.g., $X'_1=a \land X'_2=a \approx_{fd} X'_1=a \land X'_2=X'_1$

3. All: $\{C_1, \ldots, C_m\}$

(one block)

\vdots
### Reconstructing Known Techniques

<table>
<thead>
<tr>
<th>Technique by:</th>
<th>Partition</th>
<th>Generalization</th>
<th>Fold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cousot-Halbwachs:</td>
<td>Finite-Domain</td>
<td>Widen</td>
<td>----</td>
</tr>
<tr>
<td>Peralta-Gallagher:</td>
<td>All</td>
<td>Widen</td>
<td>Maximally General</td>
</tr>
<tr>
<td>FPPS (Lopstr 2010):</td>
<td>Singleton</td>
<td>Widen (or WidenSum)</td>
<td>Immediate</td>
</tr>
<tr>
<td>our <strong>new1-new2</strong>:</td>
<td>Singleton</td>
<td>Widen</td>
<td>Immediate</td>
</tr>
<tr>
<td>our <strong>new2</strong>:</td>
<td>Singleton</td>
<td>Widen</td>
<td>Maximally General</td>
</tr>
</tbody>
</table>
### Verification of System: Backward Reachability

<table>
<thead>
<tr>
<th></th>
<th>No-Specializat.</th>
<th>All_Widen</th>
<th>Singleton_WidenSum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bakery 4</td>
<td>130</td>
<td>Im 19 (6)</td>
<td>101 (1745)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MG 19 (6)</td>
<td>77 (1172)</td>
</tr>
<tr>
<td>Ticket 2</td>
<td>∞</td>
<td>Im ∞</td>
<td>0.02 (11)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MG ∞</td>
<td>0.02 (11)</td>
</tr>
<tr>
<td>Futurebus+</td>
<td>15</td>
<td>Im 17 (6)</td>
<td>2.4 (19)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MG 15 (3)</td>
<td>2.2 (15)</td>
</tr>
<tr>
<td>McCarthy91</td>
<td>∞</td>
<td>Im 4.13 (5)</td>
<td>∞</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MG 4.12 (3)</td>
<td>∞</td>
</tr>
</tbody>
</table>

29 protocols: 20 verified  | MG 21 verified  | 27 verified

- Similar results for Forward Reachability.

Times in milliseconds. Number of definitions between parentheses.

∞ means more than 200 seconds.
Conclusions

- A generic specialization algorithm reconstructing various techniques known in the literature (plus new ones), depending on:
  - partition operators (singleton, all, ...)
  - generalization operators (widen, ...)
  - folding procedure (immediate, maximally general)

- Specialization improves precision (i.e., the number of verified properties or systems) but may increment verification time

- Polyvariance control may allow fewer definitions and shorter verification times at the expense of possible loss of precision.
An implementation in SICStus Prolog as a module of the MAP transformation system.

http://map.uniroma2.it/mapweb
Future Work

- Perform more system verifications and check scalability of the approach.
- Use of polyvariance control outside the scope of the verification of reactive systems.
References


