Just a few sentences...

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How is the modified cube?

(Beladi)
“... a computer scientist becomes
a logician and then
a philosopher”
(Luca Cardelli)
Modus Ponens, Implication

\[
 ((\text{having a boat} \land \text{having oars}) \rightarrow \text{I can cross the river}) \]

\[
 (\text{having a boat} \rightarrow \text{I can cross the river}) \land (\text{having oars} \rightarrow \text{I can cross the river})
\]

“... because something funny happens to the left of the arrow”

(John Reynolds)
Generalization

gcd(M,M,M).
gcd(M,N,D) :- M>N,  gcd(M-N, N, D).
gcd(M,N,D) :- N>M,  gcd(M, N-M, D).

Euclid invariant: if M>N then GCD(M,N) = GCD(M-N, M)
if N>M then GCD(M,N) = GCD(M, N-M)

Generalized Euclid invariant:
if M>N then AllComDivs(M,N) = AllComDivs(M-N, M)
if N>M then AllComDivs(M,N) = AllComDivs(M, N-M)

\[
\begin{align*}
  k \ D &= M & \text{and} & & h \ D &= N & \text{iff} & & (k-h) \ D &= M-N
\end{align*}
\]

“... No matter what you study there is always a Greek person involved”       (Kostas Stathis)

“... harder theorems may have simpler proofs”                               (Gordon Plotkin)
MORE GCD FORMULAS

1. Marcelo Polezzi
   \[ \text{GCD}(M,N) = 2 \sum_{k=1,...,M-1} \left\lfloor \frac{N}{M} \right\rfloor + M + N - M N \]

2. Donald Knuth
   \[ \text{GCD}(2^{\text{GCD}(M,N)} - 1) = \text{GCD}(2^M - 1, \ 2^N - 1) \]
How many deduction rules?

ONE RULE (for machines):
Resolution (Robinson, 1965)

MANY RULES (for humans):
Natural Deduction (Gentzen, 1935)

“... Everybody knew about resolution at that time,
I just wrote it down, together with the unification algorithm.”
(Alan Robinson)
What is a proof?

Proof of the 4-color conjecture by exhaustion:
- after 1000 hours of Fortran program computation
  examining 1,936 “reduced maps” (Appel & Haken. 1976)

- after a few hours of a general purpose theorem prover and
  proof checker (Robertson & Sanders & Seymour & Thomas. 2005)

Is the compiler correct? Is the hardware infallible?

“ We know only a small fractions of the theorems of Mathematics.
  Has $x^n - y^m = 1$ for integers $x$ and $y$, and positive
  numbers $n$ and $m$, others solutions besides $3^2 - 2^3 = 1$?

  (Maurice Nivat)
Deduction is limited

- because of the Language:
  
  the solution of the n-body problem cannot be expressed in closed form (Henry Poincaré, 1908)

- because of the Theory:

  Incompleteness Theorem of Peano Arithmetics (Kurt Gödel, 1936)
Computable functions may be very hard to compute.

“... For humans is easy to recognize the face of a friend in a crowd, while for a computer is not.”
(Andzrej Skowron)

“... computer science is an experimental science”
(Robin Milner)
Is Mathematical Intuition “Geometrical”? (1)

- Logic             Frege (1880), Russell (1900), etc.
- Formal Language (finitary)       Hilbert (1900), etc.
- Geometry

“For any given property $P(\_)$, there exists a minimal natural number $n$ such that $P(n)$ holds”

is equivalent to Complete Induction

“For any given property $P(\_)$, $(\forall n.(\forall k<n. P(k)) \rightarrow P(n)) \rightarrow \forall n. P(n)$”

$\neg\neg p \leftrightarrow p \quad \neg\forall x p \leftrightarrow \exists x \neg p \quad \neg(p \lor q) \leftrightarrow (\neg p \land \neg q)$
"... we use symmetries for understanding"
(Giuseppe Longo)
Brain vs. Mind Problem.

Is Gödel Incompleteness Theorem of PA relevant to the problem?

yes: Penrose, Searle, ...

no: Martin Davis, ...

Gödel (in the Gibbs Lecture, 1951)

"It might exist a theorem prover machine which is equivalent to mathematical intuition, but you cannot prove to be so, nor even be proved to yield only correct theorems of finitary number theory."

"... I wrote only one Prolog program. It was for computing the transitive closure. It did not terminate!"

(Martin Davis)
x = -y+2 \quad x=1/2 \quad \text{invent the Rationals (+, -, \times, /)}

y = x+1 \quad y=3/2

\text{fact(n) = if } n=0 \text{ then 1 else } n \times \text{fact(n-1)}

\text{invent the well-founded sets}

x = \{y, a\} \quad \text{(anti-foundation axiom)}

y = \{x, b\} \quad \text{ask the Mayor, while in Udine}

N = \{0\} + \{s\}\times N

B = N + B\times N \times B

X = X \rightarrow X \quad \text{invent Scott’s domains}
impredicative definitions:

\[ N = \bigcap \{ Y \mid N \subseteq Y \text{ and } 0 \in Y \text{ and } \forall y (y \in Y \rightarrow s(y) \in Y) \} \]

invent PER categories

“... you have to tie a knot” (Gordon Plotkin)
What is Bart the Moon Movement like?
“... be quick, because the Earth is moving”
(Robert Kowalski preparing his telescope for me to look at Saturn)
What is the Moon Movement like? (2)

Sunna Sunna Sunna

\[ x(t) = d \cos(t) + 1 \cos(m \ t) \]
\[ y(t) = d \sin(t) + 1 \sin(m \ t) \]

\( d = 400, \quad m = 13 \)
What is the Moon Movement like? (3)

Sun

Sun

Sun

d

1
Sun

Earth

Moon

concavity of Moon orbit is always towards the Sun, i.e., Moon’s steering wheel is always turned to the left!

How can the Moon turn around the Earth?
How big is our Solar System?

Assume \( d = 10 \) meters.
How many kilometers away is the nearest star (Proxima Centaury) (4.3 lightyears)?

(a): 100 Km  
(b): 1000 Km  
(c): more than 2700 Km

Milky Way galaxy: 100,000 lightyears long  
20,000 lightyears thick  
... and there are “at least” \( 10^9 \) galaxies.
As time goes on...

“... a computer scientist becomes
a logician and then
a philosopher
and finally
an astronomer”

“Dimmi che fai tu Luna in ciel,
dimmi che fai, o graziosa Luna,...”  (Giacomo Leopardi)

“Tell me what you are doing in the sky,
tell me, graceful Moon, what you are doing,...”