Synthesis of Strategies for Impartial Two-Person Games

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Initial Program

```
win(0,M).                       % wins who finds the table empty
win(s(N),1)    :-    ¬ win(N,1), ¬ win(N,2).
win(s(s(N)),2) :-    ¬ win(N,1), ¬ win(N,2).
```

Initial Program with Types

1. win(0,M). % wins who finds the table empty
2. win(s(N),1) :- nat(N), ¬ win(N,1), ¬ win(N,2).
3. win(s(s(N)),2) :- nat(N), ¬ win(N,1), ¬ win(N,2).

4. nat(0).
5. nat(s(N)) :- nat(N).

6. move(1).
7. move(2).

By definition, unfolding, folding steps we get:

Definite, Nondeterministic Final Program

```
win(0,M).                          % definite program:
win(s(N),1) :- new1(N).            %    no negation in the bodies
win(s(N),2) :- new2(N).            % nondeterministic program

new1(s(N)) :- new3(N).             % nondeterministic program
new1(s(N)) :- new4(N).             % nondeterministic program
new2(s(N)) :- new1(N).
new3(0).
new4(0).
new4(s(N)) :- new5(N).
new5(s(N)) :- new1(N).
```

The Derivation ...

- initial definition -

w(N,M) :- win(N,M).

- unfold -

w(0,M).

w(s(N),1) :- nat(N), ¬ win(N,1), ¬ win(N,2).

w(s(s(N)),2) :- nat(N), ¬ win(N,1), ¬ win(N,2).

- define -

new1(N) :- nat(N), ¬ w(N,1), ¬ w(N,2).

new2(s(N)) :- nat(N), ¬ w(N,1), ¬ w(N,2).

- fold -

w(0,M).

w(s(N),1) :- new1(N).

w(s(N),2) :- new2(N).

...

After the Determinization Strategy we get:

```
det_win(0,M).                           % definite program
det_win(s(N),M) :- new6(N,M).           % deterministic program

new6(s(N),M) :- new7(N,M).

new7(0,1).
new7(s(N),M) :- new8(N,M).

new8(0,2).
new8(s(N),M) :- new9(N,M).

new9(s(N),M) :- new7(N,M).
```

The idea of Determinization

```
a :- b
a :- c
```

% a is nondeterministic

becomes

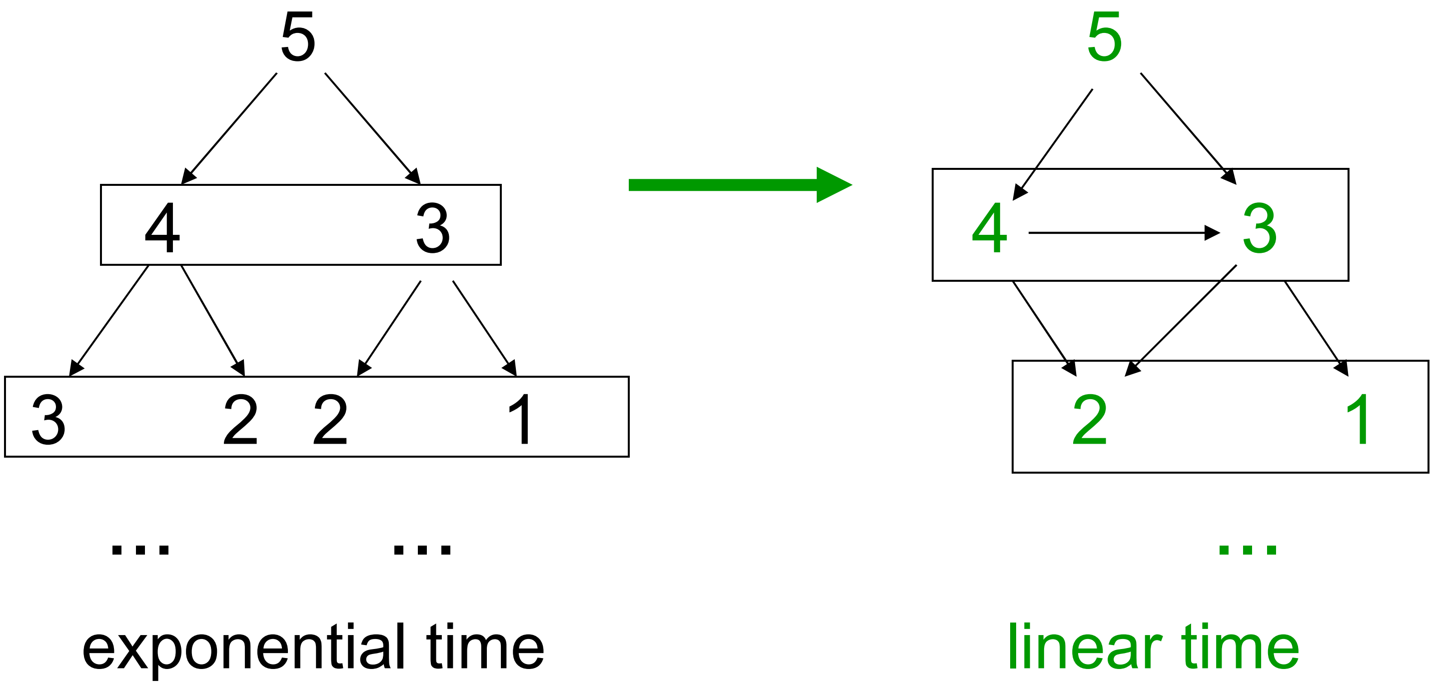
```
a :- new
new :- b
new :- c
```

% a is deterministic

If we can fold new then we avoid nondeterminism.

Determinization: from exponential to linear

(as in Fibonacci)



Further Improvements (1)

Actually, ... new6 is equal to new9. Eliminating new9:

```
det_win(0,M).
det_win(s(N),M) :- new6(N,M).

new6(s(N),M) :- new7(N,M).
new7(0,1).
new7(s(N),M) :- new8(N,M).
new8(s(N),M) :- ~~new9(N,M).~~   new6(N,M).
new8(0,2).
~~new9(s(N),M) :- new7(N,M).~~
```

Further Improvements (2)

Eliminating transient clauses by unfolding, we get:

```
det_win(0,M).
det_win(s(N),M) :- new6(N,M).

new6(s(0),1).
new6(s(s(0)),2).
new6(s(s(s(N))),M) :- new6(N,M).
```

Conclusions

Automatic derivation of a winning strategy

http://www.iasi.cnr.it/~proietti/system.html

Invariants are captured by folding steps

(see R. Backhouse et al.)

Computation of the next move in constant (or log) time after an initial linear cost (see R. Bird: Loopless Functional Algorithms)

For the future: - more experiments

- other classes of games