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**LOGIC CLASSIFICATION AND FEATURE  
SELECTION  
FOR BIOMEDICAL DATA**

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## Abstract

In this paper we investigate Logic Classification and related Feature Selection algorithms for large biomedical data sets. When the data is in binary/logic form, the Feature Selection problem can be formulated as a Set Covering problem of very large dimensions, whose solution is computationally challenging. We propose an alternative approximated formulation for Feature Selection that results in an extension of Set Covering of compact size, and use the logic classifier *Lsquare* to test its performances on two well known data sets. An ad hoc meta-heuristic of the GRASP type is used to solve efficiently the Feature Selection problem. A simple and effective method to convert rational data into logic data by interval mapping is also described. The computational results obtained are promising and the use of logic models, that can be easily understood and integrated with other domain knowledge, is one of the major strength of this approach.



## 1. Introduction

In past years research in molecular biology and molecular medicine has accumulated enormous amounts of data. Such large amount of information must be thoroughly analyzed to gain a better understanding of the underlying biological processes. Methods of knowledge-discovery and data mining are the best candidates for this challenging task.

Important examples of large data sets are found in: (i) genomic sequences gathered by the Human Genome Project and sequences of Single Nucleotide Polymorphisms (SNPs); (ii) gene expression data from microarray experiments; (iii) protein identification and quantification data from proteomics experiments.

For each of these problems, different modeling, algorithmic, and interpretation problems arise (see [1, 2, 3] for presentation and discussion of fundamental issues and methodologies).

The study of both genomic sequences and SNP sequences is aimed at identifying the positions of base pairs responsible of function, mechanism or interaction. The analysis of SNPs permits to understand the relationship between genotypic and phenotypic information, as well as to identify polymorphisms that can be related to specific genetic diseases. One application is the identification of gene/SNP patterns impacting cure/drug development for various diseases. Data sets are made of genomic or SNP sequences (in both cases, sequences of base pairs) of given individuals of the same species or of different but related species. The general analysis problem to be solved requires to associate diversities and commonalities among individuals to differences and identities between their base pairs sequences.

Microarrays are semiconductor devices used to detect the DNA makeup of a cell. They contains hundreds of thousands of tiny squares, each designed to hybridize with a sequence encoding a particular gene. The microarray squares react to the liquified human cells poured over them, and capture the sequences that they are targeted to hybridize with. After the reaction, the amount of captured gene sequences is detectable by a laser, that reads the “expression” of each gene at the corresponding square. Microarrays are used, e.g., to identify drug targets (the proteins with which drugs actually interact) and can also help to identify individuals with similar biological patterns. This way, drug companies can choose the most appropriate candidates for participating in clinical trials of new drugs. Data from microarray experiments are two dimensional arrays, in which each entry corresponds to the expression of one specific gene. Given a set of arrays (experiments on the same microarray), the analysis problem is to classify the experiments, taking into account the value of gene expressions for a small number of genes.

Finally, data bases of proteins exist that contain primary, secondary, and tertiary structures of each protein. There are protein families with common properties, whose function is characterized by patterns in their 3-dimensional structure. Here one wants to detect subset of aminoacids of the chains that are linked to the analyzed properties. In particular, to each aminoacid is associated the measure of the studied properties.

All these data sets are typically represented by matrices whose rows are associated with the objects, or records, while the columns are associated with the many measures taken on each object. Such measures are often referred to as *variables*, or *features*. For the analysis of such type of data there are analysis tools with different degree of complexity, where the complexity is to be found in the models that are needed and in the large dimension of the data that are to be processed. Such techniques are being referred as *Data Mining* (DM), indicating a collection of methods inherited mainly from the classical multivariate and non-parametric Statistics, the Computer Science oriented Learning Theory, Mathematical Programming, and Artificial Intelligence.

Data Mining is used to systematically explore the possibility of relations between variables when there are no (or incomplete) a-priori expectations as to the nature of those relations. Its use in biology and medicine has grown rapidly since 1997. When dealing with large data sets, it is often the case that the information available is somehow redundant for the scopes of the DM application; many mining tools deal with this issue by trying to provide classification or association rules that are as compact as possible.

In conjunction with Data Mining techniques it is common to apply *Feature Selection* (FS), by which we address a set of methods to identify, in a large set, those features that are best useful for the specific analysis task, be it the identification of a classification model, of association rules, or a statistical regression model. The role of Feature Selection is particularly important when computationally expensive Data Mining tools are used, or when the data collection process is difficult or costly, as it is the case in the

type of applications that we consider here.

In this paper, we test the efficacy of a particular class of feature selection methods for classification on data sets represented in binary, or logic form, in conjunction with a method to mine logic relations in the data, the *Lsquare* system (see [4], [5]). *Lsquare* models the classification problem as a sequence of minimum cost satisfiability problems (MINSAT) and is able to find separating formulas optimized according to certain criteria. To reduce the intractable dimensions of the data sets analyzed, we consider FS methods based on another well known combinatorial problem, *The Set Covering Problem* (SC). We propose a simplified version of such models that significantly reduces the dimension of the Set Covering problem to be solved showing that also in this case we are able to identify a good and small subset of the available features.

The paper is organized as follows.

Section 2 introduces the main issues in designing a FS method and the most basic and widely known techniques; then, describes the basic ideas of the Set Covering formulation, focusing both on the quadratic formulation already presented in [6], and on the simplified linear version proposed in this paper. A specific subsection deals with the complexity issues related with the solution of large SC problem and describes the metaheuristic algorithm used to solve the large instances associated with FS. In Section 3, the basic steps of the *Lsquare* method for logic learning are presented. Then, Section 4 describes the techniques used to preprocess the data, in particular to obtain data in logic form from measures expressed through real numbers, as it is the case for microarray data. Such process, known as *discretization* or *binarization*, is not to be overlooked in the entire process as it accomplishes a relevant transformation of the available information and influences the work of FS and DM. Then, in Section 5 we synthesize the whole analysis process, and report and discuss the experimental results obtained. Conclusions are drawn in Section 6 together with some consideration on future work in this challenging research area.

## 2. Feature Selection Methods

In many applications FS may be considered as an independent task in the mining process; it is used to reduce the data to a treatable size before it can be processed by a DM algorithm. A reason behind this strategy is that sophisticated DM algorithms often may fail, or have significant computational problems, when treating directly data set with a very large number of features, as in the case of biomedical data. A very extensive treatment of the use of FS in DM applications is given by Liu and Motoda [7], who provide a complete overview of the methods developed since the 70s, comparing the results of several applications and providing suggestions on how to drive the choice of the proper method for each specific problem. Feature Selection problems are typically solved in the literature using search techniques, where the evaluation of a specific subset is accomplished by a proper function (filter methods), or directly by the performance of a Data Mining tool (wrapper methods). A general overview of different methods is also available in [8] and [9]. In this paper, we focus on data represented by binary features and try to exploit particular methods that are designed for such cases. As we will see later in Section 4, our proposal is to convert, by proper *binarization* techniques, rational features into binary features in order to apply logic based classification methods. Such binarization process induces a further proliferation of the number of features, and thus demands even more strongly a valid method to reduce the size of the problem to a treatable one. In the following sections we describe such methods.

### 2.1. Feature Selection for Biological Data Analysis

Many FS methods have been applied in biological data analysis. Most of such work concerns microarrays classification, where the number of features is in the range of several thousands. In [10] the authors propose a hybrid of filter and wrapper approaches to Feature Selection, based on Unconditional Mixture Modeling, Information Gain Ranking, and Markov blanket Filtering. In [11, 12] Redundancy based methods are applied. Support Vector Machines are used as a classification method to prove the goodness of a selected feature set in [13, 14]. In [15], Nearest Neighbor methods and Support Vector Machines are used for predicting protein functional classes from binary vectors obtained by comparing functional domains in the SBASE database to each protein sequence. In [16] a global search mechanism, weighted decision tree,

decision-tree-based wrapper, a correlation-based heuristic, and the identification of intersecting feature sets are employed for selecting significant genes/SNPs for predicting drug effectiveness.

## 2.2. Set Covering formulation for Feature Selection: The Minimal Test Collection

When dealing with binary features, the problem of selecting a subset of features of minimal size that guarantees the separation between two sets can be formulated as an Integer Linear Programming Problem, more specifically as a Set Covering problem. Here we consider a known formulation, that suffers from the fact that its size, in terms of number of rows, increases quadratically with the number of rows of the data matrix.

A formal definition of the problem of feature selection (called *test cover*) is presented in [17]. The input is a set of items  $\{1, \dots, m\}$  (e.g., arrays of gene expression from a microarray experiment) and a collection  $\mathbf{F}$  of features (e.g. a feature could be a gene)  $\{f_1, \dots, f_n\}$ . The item set is divided into two classes, (e.g., microarray obtained from DNA of sick patients and microarrays obtained from DNA of healthy ones). By convention, we indicate the two classes with *class A* and *class B*. For each item  $h$ , each feature  $f_i$  takes a value in a given metric. In a binary setting each feature has two possible values  $\{1, 0\}$ , representing the presence or the absence of a given characteristic, associated with that feature, in item  $h$ . To represent the value of feature  $i$  for item  $h$  we use the notation  $f_{ih}$ , that would then take value 0 or value 1. A feature  $f_i$  differentiates (covers) item pair  $\{k, h\}$  if  $f_{ik} \neq f_{ih}$ . If we consider all the pairs of items  $\{k, h\}$  where  $k$  belongs to a class (say, "healthy patients") and  $h$  belongs to the other class (say, "sick patients"), then a subcollection  $\mathcal{F} \subset \mathbf{F}$  of features is a cover if each of such pairs  $\{k, h\}$  are covered by at least one element in  $\mathcal{F}$ . Obviously, the number of pairs is equal to the product of the cardinalities of the two classes, than grows quadratically with  $m$ . The problem of finding a subset of features of minimal size that covers all the pairs of distinct elements is called *Combinatorial Feature Set* or *Minimal Test Collection*. A mathematical formulation of the problem is given below:

$$\begin{aligned} \min \quad & \sum_{i=1}^n x_i \\ & \sum_{i=1}^n a_{ij} x_i \geq 1 \quad j = 1 \dots M \\ & x_i \in \{0, 1\} \quad i = 1 \dots n, \end{aligned} \tag{1}$$

where  $x_i = 1$  if  $f_i$  is chosen and 0 otherwise; each of the  $M$  constraints is associated with a pair of items belonging to different classes; e.g., if row  $j$  is associated with the item pair  $\{k, h\}$  then we have that for feature  $i$   $a_{ij} = 1$  if and only if  $f_{ik} \neq f_{ih}$ . Here we refer to the above problem as *QSC* (Quadratic Set Covering).

For the QSC problem, in [6] a branch-and-bound procedure is presented, based on a new definition of branching rules and lower bounds. Nevertheless, when problem size is significantly large, the use of optimization algorithms to produce guaranteed optimal solutions becomes impractical, and one has to resort to heuristics schemes, such as, among the others, the one proposed in [18]. The above approach to the FS problem presents also additional drawbacks. Its purpose is to find a minimal set that can separate the given data, that plays the role of *training* data in the general process. The features associated with the minimal set are then used to project the training data and to derive classification rules, that are then applied to *test* data, once the latter has been projected using the same feature set. When the data is noisy or not well sampled, it may happen that the very thrifty representation of the data overlooks some features that, although not strictly needed for separating the training data, may play a role in formulas that behave well on testing data. In other words, maintaining a certain measure of redundancy in the information that is retained by the FS selection process may be a good strategy for the production of rules with good predictive power. This is particularly true when FS is followed by classification algorithms that can perform an additional selection of the features based on the separating model. For this reason, we want to evaluate the quality of the QSC formulation also when the requirement on the number of covering features is raised, that is when the *rhs* of the covering constraints is greater than 1. This would

result in the selection of a larger set that brings more information in the classification step. In the present setting, such modification is easy to make on the modelling side, as it is sufficient to rewrite the covering constraints of QSC as follows:

$$\begin{aligned} \min \quad & \sum_{i=1}^n x_i \\ & \sum_{i=1}^n a_{ij} x_i \geq \alpha \quad j = 1 \dots M \\ & x_i \in \{0, 1\} \quad i = 1 \dots n, \end{aligned} \quad (2)$$

where  $\alpha$  is an integer representing the degree of redundancy required. In such case, the standard Set Covering problem is transformed into a Generalized Set Covering (GSC) problem and many of the known results and algorithms cannot be applied in a straightforward fashion. To any extent, the QSC formulation is computationally very expensive to solve, and we propose a more compact model based on a GSC formulation that does not guarantee to find a subset of the features that performs perfect separation in the training set, but identifies a small set of highly informative and loosely correlated features. This is done with a consistently smaller computational effort as compared with QSC.

### 2.3. An approximated formulation of the Feature Selection: The “Linear” Set Covering

The approximated Set Covering formulation described in this section is based on a very simple idea. Given a feature  $f_i$ , let  $P_A(i)$  and  $P_B(i)$  be the proportion of items where feature  $f_i$  has value 1 in sets  $A$  and  $B$ , respectively. If  $P_A(i) > P_B(i)$ , then the presence of  $f_i$  with value 1 is likely to characterize items that belong to  $A$ , and viceversa. Let us assume that we are in this case, that is,  $P_A(i) > P_B(i)$ , and define for  $f_i$  the following vector  $d_{ij}$ ,  $j = 1, \dots, m$  (note that here  $j$  is used to refer a single item, and not a pair of items as in models (1) and (2)):

$$d_{ij} = \begin{cases} 1, & \text{if item } j \text{ is in class } A \text{ and } f_i \text{ has value } 1; \\ 0, & \text{if item } j \text{ is in class } A \text{ and } f_i \text{ has value } 0; \\ 1, & \text{if item } j \text{ is in class } B \text{ and } f_i \text{ has value } 0; \\ 0, & \text{if item } j \text{ is in class } B \text{ and } f_i \text{ has value } 1. \end{cases}$$

For the case  $P_A(i) < P_B(i)$ , the corresponding vector  $a_{ij}$  is computed in the same way exchanging the roles of  $A$  and  $B$ . A feature  $f_i$  with  $a_{ij} = 1$  for all  $j \in 1, \dots, m$  would be able to discriminate perfectly between  $A$  and  $B$ , as it would suffice to tell whether an item belongs to one of the two classes. Obviously, such feature is not likely to exist, but we may assume that the number of ones in vector  $a_{ij}$  is positively correlated with the discriminating power of feature  $f_i$ . Moreover, we would like to select a subset of the features that exhibits, as a set, a good discriminating power for all the items considered, so that we may use more features combined together, among the ones in the set, to obtain a complete separation between  $A$  and  $B$ . According to this line of reasoning, we formulate the following Set Covering problem:

$$\begin{aligned} \min \quad & \sum_{i=1}^n x_i \\ & \sum_{i=1}^n d_{ij} x_i \geq \alpha \quad j = 1 \dots m \\ & x_i \in \{0, 1\} \quad i = 1 \dots n, \end{aligned} \quad (3)$$

where, as in QSC,  $x_i = 1$  implies that feature  $f_i$  is selected in the final set and  $\alpha$  represents the degree of redundancy of the information provided by the selected features. Although such formulation does not guarantee exact separation, it identifies good features that exhibit their discriminating power in a distributed way over the item set, due to the presence of the same *rhs*  $\alpha$  for each item. The problem above is referred to as LSC (Linear Set Covering) and has the same nature as QSC but only a linear

```

procedure GRASP(MaxIterations)
1   for  $i = 1, \dots, \text{MaxIterations}$  do
2       Build a greedy randomized solution  $x$ ;
3        $x \leftarrow \text{LocalSearch}(x)$ ;
4       if  $i = 1$  then  $x^* \leftarrow x$ ;
5       else if  $w(x) > w(x^*)$  then  $x^* \leftarrow x$ ;
6   end;
7   return  $(x^*)$ ;
end GRASP;

```

Figure 1: Pseudo-code of a generic GRASP for a minimization problem.

number of constraints. Some of the results conducted on large data sets show experimentally that the trade off between the approximation of the LSC model and its computational simplicity may be worth considering.

In the above QSC and LSC formulations we have presented very trivial objective functions, that simply minimize the size of the feature sets. It is important to note that both models are designed to host additional information by means of proper weights associated with the features, that can be taken into account to drive the solution algorithm towards the choice of particular features, according to contextual consideration. For example, we may want to drive the algorithm towards particular features that, a priori, have a special role in the genetic phenomenon under study; or we may choose the weights in such a way that those features associated with the extremes of the measuring scale of a gene expression in microarray data are preferred as they are less prone to measuring errors.

#### 2.4. GRASP Heuristic

To solve the generalized set covering problems we pursue a non-deterministic and heuristic method known in literature as GRASP, acronym of Greedy Randomized Adaptive Search Procedure. GRASP is a randomized multistart iterative metaheuristic initially proposed in Feo and Resende [19, 20]. For a comprehensive study of GRASP strategies and variants, the reader is referred to the survey chapter by Resende and Ribeiro [21], as well as to the annotated bibliography of Festa and Resende [22] for a survey of applications. Generally speaking, GRASP is a randomized heuristic having two phases: a construction phase and a local search phase. The construction phase adds one element at a time to a set that ends up with a representation of a feasible solution. At each iteration, an element is randomly selected from a *restricted candidate list*  $RCL$ , whose elements are among the best ordered, according to some greedy function. Once a feasible solution is obtained, the local search procedure attempts to improve it by producing a locally optimal solution with respect to some neighborhood structure. The construction and the local search phases are repeatedly applied. The best solution found is returned as an approximation of the optimal one. Figure 1 depicts the pseudo-code of a generic GRASP heuristic for a minimization problem.

The construction phase makes use of an adaptive greedy function, a construction mechanism for the restricted candidate list, and a probabilistic selection criterion. The greedy function takes into account the contribution to the objective function achieved by selecting a particular element. In the case of the generalized unweighted set cover problem, the selection involves candidate features and it is intuitive to relate the greedy function to the number of constraints still to be fully covered that a feature not yet chosen would cover if selected. More formally, at a generic iteration of the GRASP construction phase let  $\overline{F} \subset F$  be the subset of features already selected as partial solution and let  $\overline{C} \subseteq C$  be the subset of constraints still to be fully covered. Then, for each  $f \in F \setminus \overline{F}$  we define  $\sigma(f) = |\overline{C}_f|$ , where  $\overline{C}_f \subseteq \overline{C}$  is the subset of constraints not yet fully covered that feature  $f$  would cover. This greedy function measures how much additional cover will result from the selection of  $f$ . The greedy choice consists in selecting the feature  $f \in F \setminus \overline{F}$  with the highest greedy function value. To define the construction mechanism for the

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restricted candidate list  $RCL$ , let

$$\sigma_{min} = \min\{\sigma(f) \mid f \in F \setminus \overline{F}\}$$

and

$$\sigma^{max} = \max\{\sigma(f) \mid f \in F \setminus \overline{F}\}.$$

Denoting by  $\mu = \sigma_{min} + \beta \cdot (\sigma^{max} - \sigma_{min})$  the cut-off value, where  $\beta$  is a parameter such that  $0 \leq \beta \leq 1$ , the restricted candidate list  $RCL$  is made up by all features whose value of the greedy function is greater than or equal to  $\mu$ . A feature is randomly selected from the restricted candidate list and the sets  $\overline{F}$  and  $\overline{C}$  are updated accordingly to the just made selection (adaptive component).

Since we are dealing with the problem in its unweighted variant, the local search procedure only checks for redundancy of the current solution built by the GRASP construction phase, which is replaced by its best improving neighbor. The search stops after all possible moves have been evaluated and no improving neighbor has been found.

### 3. The Learning System *Lsquare*

The learning tool used in this application, *Lsquare*, is a particular learning method that operates on data represented by logic variables and produces rules in propositional logic that classify the items in one of two classes. The general scheme of this method is the one of automatic learning, where the items presented in a *training set* are used to infer the rules that link the class of an item with the value of some of its features; these rules are then used to classify new records and predict their class.

The choice of *Lsquare* is motivated by the fact that it uses a logic representation of the description variables, that are to all extents logic variables, and of the classification rules, that are logic formulas in Disjunctive Normal Form (DNF). Such property enables to analyze and interpret the classification results also from the semantic point of view, as the classification rules determined by the method express combination of the features that can be interpreted by domain experts and bring to light new knowledge in an easily understandable format.

The learning of propositional formulas from data is tackled also by other learning methods, such as the strongly heuristic, simple and widely used decision trees (originally proposed in [23]), to the more sophisticated LAD system, originally proposed by Hammer in [24], [25], with complexity and mathematical programming issues similar to those faced in *Lsquare*, to the greedy approach proposed in [26].

The *Lsquare* system and some of its additional components have been described in other papers ([4], [27], [5]) and its detailed description is out of the scope of this paper. Here, we simply mention the fact that the rules are determined using a particular problem formulation that amounts to be a well know and hard combinatorial optimization problem, the *minimum cost satisfiability problem*, or MINSAT, that is solved using a very sophisticated solver based on decomposition and learning techniques [5]. The DNF formulas identified have the property of being created by conjunctive clauses that are searched for in order of coverage of the training set. Therefore, they usually are formed by few clauses with large coverage (the interpretation of the trends present in the data) and several clauses with smaller coverage (the interpretation of the outliers).

### 4. Preprocessing and data binarization

The preprocessing phase is conceived to transform input data from a problem dependent format to a problem independent format. In general, biological data sets are large arrays of integer or real numbers, that corresponds to measures on the items. Such data are not suitable for logic methods, and a transformation is needed to adapt the data sets. This transformation consists of the identification of a set of intervals of values for each feature, the computation of the number of samples in each interval, and a reduction of the number of these intervals through the elimination of empty intervals and unification of contiguous intervals with the same meaning. In binarized data, the new features can be viewed as

binary, or logic, variables, that indicate whether the measure of one of the original real features belongs to a certain interval.

To obtain an initial set of intervals for feature  $f_i$  we consider its mean  $\mu_i$  and variance  $\sigma_i$  over the training items, and create a number of equal sized intervals symmetrical with respect to  $\mu$  and proportional in size to  $\sigma_i$ . Once such intervals have been created, we iterate a set of steps that merge two adjacent classes if one of them is empty, if the proportion of elements in  $A$  and  $B$  is not altered when the two classes are merged (class entropy), and finally if the reduction obtained in the entropy of the feature is negligible.

For a given feature  $f_i$ , let  $K_i$  be the set of the intervals in which  $f_i$  is discretized; its entropy  $h_i$  is given by  $\sum_{k \in K_i} f_{ik} \log f_{ik}$ , where  $f_{ik} = p_{ik}/n$ , and  $p_{ik}$  is the number of samples included in the interval  $k$ ; since  $h_i = 0$  if the number of intervals  $K_i$  is equal to 1, the goal is to obtain a good tradeoff between a high level of entropy and a small number of intervals. The procedure performs the following steps on the training data set:

1. For each feature  $f_i$  the mean value  $\mu_i$  and the variance  $\sigma_i$  of the values of the feature over the items of the training set are evaluated;
2.  $N$  intervals around  $m_i$  are computed, so that each interval width  $w_i$  is equal to  $\sqrt{\sigma_i}/N$  (such intervals are indicated with  $C_{ik}$ , for  $k = 1, \dots, N$ );
3. For each interval  $C_{ik}$ , the total number  $p_{ik}$  of samples that are included in the interval is determined, together with the number of samples that are in class  $A$ ,  $p_{ik}^A$  and the number of samples that are in class  $B$ ,  $p_{ik}^B$ .
4. The  $N$  intervals are reduced on the basis of the following three criteria:
  - if an interval is empty then it is unified with one of the smaller of its adjacent intervals;
  - if in two adjacent intervals class  $A$  ( $B$ ) samples are strongly prevalent over those of  $B$  ( $A$ ) samples, the two intervals are unified;
  - if one interval is poorly populated it is unified with one of the two adjacent classes if the entropy level of the feature does not fall below a given threshold.

Given the final set of intervals, a binary representation of the values of the feature is obtained by mapping the rational value of that feature into its corresponding interval, and setting the corresponding binary variable to 1.

## 5. Experiments and results

We have applied the classification procedures described above to two data sets characterized by a very large number of features, that have already been considered in other similar works. Such choice is driven by the intention to test the efficacy of the methods for problems in this application area, in order to gain the sufficient thrust to apply them in real problems where the results may be of some potential interest. The experiments described below have the same structure: a *training* set of data is analyzed with the FS set covering models to extract a significant and small subset of the features; then, *Lsquare* is applied to extract a logic model that separates at best the two sets  $A$  and  $B$  with the available features. Finally, such model is applied on *test* data, i.e., data of the same nature of the training set, for which we know the classification in  $A$  and  $B$ , but that has not been considered in the training phase. The percentage of correct recognition on the *test* set is a reasonable measure of the quality of the whole method, and can be compared with other ones already presented in the literature. According to the purpose of the paper, we investigate the behavior of the performances of the system when the main experimental parameters are varied.

In Section 5.1, we describe the experiments conducted on the Thrombin data set. Here no preprocessing and binarization is needed, as the data is already made available in binary form. The problem is of large size and we were able to solve the QSC problem optimally and with the proposed heuristic procedure, and compare its results with those obtained with the LSC feature selection method.

value of $\alpha$	% correct on $A$	% correct on $B$	total correct
10	0.673	0.399	0.426
20	0.629	0.617	0.618
30	0.588	0.757	0.740
40	0.609	0.790	0.772
50	0.622	0.829	0.809
60	0.622	0.822	0.803
70	0.620	0.858	0.835
80	0.620	0.857	0.834

Table 1: Correct Recognition rate for 50% percentage split testing with LSC FS and increasing value of  $\alpha$ .

In Section 5.2, we consider a typical case of classification of microarray data where the genes should differentiate between patients with two different types of leukemia. Here the whole process is tested, including the binarization. The behavior of the LSC and QSC models with different values of  $\alpha$  is analyzed.

### 5.1. Thrombin data set

Thrombin data set is extracted from KDD competition [28] for testing the performance of various procedures for data mining. The problem of analyzing the molecular bioactivity of drugs w.r.t. a receptor, in order to separate the active (binding) compounds from the inactive (non-binding) ones is proposed. The data set<sup>1</sup> includes compounds that binds to thrombin, a key receptor in blood clotting and compounds that do not bind. The problem was to identify a small set of sites of the compound molecules that allow to separate active vs inactive compounds. Two sets of data are provided: the training data set includes 1909 known molecules (42 actively binding thrombin), the test data set includes 639 new compounds with unknown capacity to bind thrombin; 139.351 binary features describe the 3-D properties of each compound. The chemical structures of these compounds are not relevant for our analysis and are not included. The definitions of the individual bits are not included - we don't know what each individual bit means, only that they are generated in an internally consistent manner for all compounds. Thrombin data set was not preprocessed since it is a binary data set and was used straight-forwardly to generate the LSC problem. The performance of our method is compared with the Jie Cheng method, the winner of the KDD cup 01 [28], that used a Bayesian network predictive model.

Table 5.1 reports the results obtained when the LSC method is used, for values of  $\alpha$  ranging from 10 to 120. Each experiment is obtained by a 50% random split of the available data into training and testing, repeated 8 times and then averaged. The level of the recognition percentages compares with those characterizing the best results obtained on the same data available in the literature ([28]), when the value of  $\alpha$  reaches 80. The best formulas obtained use approximately 60 features, and it explains why the good results are obtain when  $\alpha$  reaches that value.

It is of some interest to compare the results of Table 5.1 with those obtained using a subset of features selected by the optimal solution of the associated QSC. Such solutions have been obtained at an unpractical computational cost by a Branch & Cut algorithm that uses the latest version of the ILOG Cplex solver for integer programming. The minimal test is composed of only 41 features, but the performances measured with the same metric are significantly worse than the ones obtained with LSC. Moreover, the optimal solution of 41 features is not unique, and the many solutions of that dimension show a high variance in their performances. It so appears that better FS selection solutions are obtained enforcing a significant level of redundancy in the information retained by the selected feature, devolving to the classification algorithm (*Lsquare*, in this case) the task of selecting a good separating model and the features that support it.

<sup>1</sup>The data set has been provided by DuPont Pharmaceuticals for the KDD Cup 2001 competition.

value of $\alpha$	dim. of features set	used features	correct on $A$	correct on $B$	total correct
1	2	1	0.900	0.500	0.735
2	3	3	0.950	0.500	0.765
3	5	4	1.000	0.643	0.853
4	6	3	0.700	0.571	0.647
5	7	3	0.850	0.857	0.853
10	14	4	0.950	0.643	0.824
15	20	3	0.950	0.714	0.853
20	26	2	0.900	0.929	0.912
30	39	3	0.900	0.643	0.794
35	46	2	0.900	0.929	0.912
65	85	2	0.900	0.929	0.912
75	97	2	0.900	0.929	0.912

Table 2: Correct Recognition rate and solution dimensions for testing set with LSC FS and increasing value of  $\alpha$ .

value of $\alpha$	dim. of features set	used features	correct on $A$	correct on $B$	total correct
1	2	2	0.650	0.643	0.647
2	4	2	0.850	0.929	0.882
3	6	2	0.850	0.429	0.676
4	7	3	1.000	0.357	0.735
5	9	2	0.950	0.500	0.765
10	17	3	0.850	0.929	0.882
15	23	3	0.850	0.929	0.882
30	31	2	0.850	0.571	0.735
35	49	2	0.900	0.286	0.647
50	58	3	1.000	0.214	0.676
65	75	2	0.900	0.929	0.912

Table 3: Correct Recognition rate and solution dimensions for testing set with QSC FS and increasing value of  $\alpha$ .

## 5.2. Leukemia data set

The other data set is derived from microarray experiments. It is a collection of 72 samples from leukemia patients; each sample gives the expression level of 7130 gene [29]. The collection includes 47 sample from type I Leukemias (called *ALL*) and 25 from type II Leukemias (called *AML*) The collection is split into two sets, the first with 38 samples ( $ALL/AML = 27/11$ ) serving as a training set, and the other 34 (20/14) as test set. Here, we deployed also the preprocessing and the binarization stage. Each feature was initially divided into intervals with high granularity (according to the scheme described in Section 4). From each feature we derived 30 intervals, obtaining 213,900 initial features, that were then reduced to approx 120,000 using the proposed procedure. Such still large set of features was used to build the formulation of the LSC and QSC, that were then solved with different values of  $\alpha$ . A synthesis of the results obtained is given in Tables 5.2 and 5.2, where we report the value of  $\alpha$ , the correct recognition percentages, the size of the feature set selected by the solution of the LSC and QSC models respectively, and also the number of features effectively used by the classification algorithm to build the separating logic model.

Comparing the two tables, it is easy to see that the LSC model performs as good as the QSC one, at times even better, although computationally less expensive. But the very interesting result in this

case is that, besides having recognition percentages that match in quality those obtained by many other methods presented in the literature ([10]), the size of the good formulas obtained is very small, usually composed of only 2 features, regardless of the dimension of the set of features selected by the solution of LSC/QSC. This implies that we can indicate not only the two genes that seem to differentiate the two types of Leukemia, but also the levels at which such genes should express in order to be significant. For example, if we consider one of the separating formulas obtain on 2 features, we may “translate” it into its original values and describe as follows:

```
IF the expression of gene in position 5 is less than -429.99
OR larger than -420.77
AND the expression of gene in position 6 is below -148.1325
THEN the item is of type ALL,
ELSE is of type AML
```

Obviously such statement may lack significance for a genetist, but its compactness surely makes it understandable and potentially interesting from the semantic point of view. We recall that the rule above is able to separate exactly the observations in the training set, while it achieves an overall precision of 0.922 on provided testing data.

## 6. Conclusions

In this work, an approach for the solution of classification problems in data sets with a large number of features has been presented. The method is based on the representation of data in binary/logic form; when the data is not naturally available in this form, discretization procedures as the one presented in Section 4 of this paper can be used. The Feature Selection step is the main focus of the paper. We propose an alternative and more compact formulation as a Generalized Set Covering (where the *rhs* may assume values larger than 1) and solve it using a GRASP metaheuristic for the quite large data sets used for the experiments. The results obtained show that the method has some potential and its simplicity seems not to affect the quality of the results. Additional considerations have to be drawn as conclusive remarks to the work done.

First, we note that the computationally challenging Set Covering formulation, known also as the minimal test collection, does not always produce guaranteed quality feature subsets. The objective of minimizing the number of features that guarantee separability may results too strict, and produce solutions that completely overlook certain aspects of the data that are useful for predictive and interpretative purposes. Some degree of redundancy may come in use, as proposed and experimentally validated in this paper. Moreover, the simpler and lighter formulation here presented as LSC turns out to be experimentally effective and more robust.

Second, the use of mathematical formulations as (1) and (3) allow the use of costs associated with features in the objective function. Such cost may very well represent prior knowledge of domain experts, and could be used to drive the solution towards those features that are more interesting or significant in the particular application. Another interesting aspect in the use of costs with binarized data is that they can be used to mitigate the effect of noise in the data, by assigning higher costs to those features that are associated with intervals closed to the center of the measuring scale of the original numeric feature, and smaller costs to those with extreme values. In this way, the solution would tend to use features whose binary value is less affected by measuring errors and noise.

Last but not least, the approach presented is strongly characterized by the representation of both data and separating model in logic form. This implies that the knowledge extracted from the data is easier to understand, to combine with previously existing knowledge, and to interact with by domain experts. In our opinion, such characteristic is extremely important in a research field where the previous experience has to be combined with knowledge extracted directly from data, and where the results of a mathematical/algorithmic tool have to be understood and validated ex-post before they may be transferred to real applications.

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