Seminars in Computer Networks
Homework 1

May 1, 2013

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Instructions: You need to work on the homework individually.

Make sure that the solutions are typewritten or clear to read. *Give explanations for all of your claims.*

Some of the problems can be solved either by hand or by using Octave/Matlab. If you solve them by hand, include the calculations in your solutions. If you solve them by Octave/Matlab, include your Octave/Matlab code.

Hand in your solutions and keep a copy for yourself. After the due date we will post the solutions and in the final exam you may be asked to explain what were your mistakes.

Due date: 23/5/2013 (before the class).

Problem 1. *Power control infeasibility.* Consider the power control problem in a three-link cell with the link gains G_{ij} shown below. The receivers request $\gamma_1 = 1$,$\gamma_2 = 2$, and $\gamma_3 = 1$. The noise $n_i = 0.1$ for all i.

$$G = \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 1 \end{bmatrix}.$$

Prove this set of target SIRs is infeasible.

Problem 2. *Two-thirds of the mean.* Find the pure Nash equilibria of the following game, called "2/3 of the Mean": There are n players. Each player i selects a real number x_i between 0 and 100 (inclusive). Let $X = \sum_{i=1}^{n} x_i/n$ be the average of the numbers. The players whose number was closest to $2X/3$ get a payoff of 1, the others get 0.

Problem 3. *PageRank with different θ.*
Compute the PageRank vector π^* of the graph in Figure 1, for $\theta = 0.1, 0.3, 0.5$, and 0.85. What do you observe?

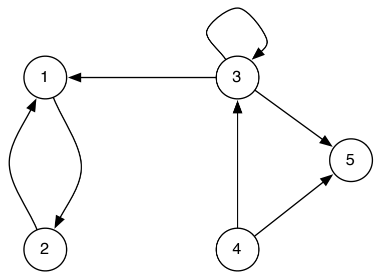

Figure 1: A graph.

Problem 4. *A statement about graphs.* True or false? Prove or find a counterexample to the following claim: every undirected graph with n vertices and $2n$ edges is connected.

Problem 5. *Baseline predictor.* Consider the collaborative filtering problem. Compute the baseline predictor $\hat{R}$ based on the following raw data matrix R:

$$R = \begin{bmatrix} 5 & - & 5 & 4 \\ - & 1 & 1 & 4 \\ 4 & 1 & 2 & 4 \\ 3 & 4 & - & 3 \\ 1 & 5 & 3 & - \end{bmatrix}.$$

Hint: You probably want to use Octave/Matlab. The function `pinv()` can be helpful. Include your Octave/Matlab code as part of the solution.

Problem 6. *Convex functions.* Determine whether the following functions are convex, concave, both, or neither:

(a). $f(x) = 3x + 4$, for all real x;

(b). $f(x) = 4\ln(x/3)$, for all $x > 0$;

(c). $f(x) = e^{2x}$, for all real x;

(d). $f(x, y) = -3x^2 - 4y^2$, for all real x and y;

(e). $f(x, y) = xy$, for all real x and y;

(f). $f(x, y) = x^{\alpha}y^{1-\alpha}$, where $0 \leq \alpha \leq 1$, for all $x > 0$, $y > 0$.

Problem 7. *Averaging a dependent crowd.* Consider three people making dependent estimates of a number, with the following expectations of errors and correlation of errors:

$$\begin{aligned}
\mathbb{E}[\epsilon_1^2] &= 1773, \\
\mathbb{E}[\epsilon_2^2] &= 645, \\
\mathbb{E}[\epsilon_3^2] &= 1796, \\
\mathbb{E}[\epsilon_1\epsilon_2] &= 1057, \\
\mathbb{E}[\epsilon_1\epsilon_3] &= 970, \\
\mathbb{E}[\epsilon_2\epsilon_3] &= 708.
\end{aligned}$$

Compute the average of errors and the error of the average in this case.

Problem 8. *Computing centrality and betweenness.*

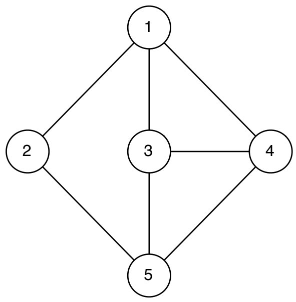

Figure 2: A graph.

(a). Compute the degree, closeness, and eigenvector centrality of each node in the graph in Figure 2.

(b). Compute the node betweenness centrality of nodes 2 and 3.

(c). Compute the link betweenness centrality of the links (3,4) and (2,5).

Problem 9. *Properties of DPC.* In the analysis of the Distributed Power Control (DPC) algorithm, we have seen that DPC converges to an optimal solution whenever the spectral radius of the matrix product DF is less than 1.

Recall that the *spectral radius* of A, $\rho(A)$, equals $\max|\lambda|$ where the maximum is taken over all the eigenvalues λ of A.

Can it happen that a set of target SIRs is feasible, but the spectral radius of DF is larger than 1? In this Problem, your goal is to show that this is not possible. In other words, you have to prove the following claim.

Claim: If a set of target SIRs is feasible, then the spectral radius $\rho(DF)$ is at most 1.

Hint: Use the following two results from linear algebra:

Proposition 1. If $A = (a_{ij})_{i,j\in\{1,\ldots,n\}}$ is a matrix in $\mathbb{R}^{n\times n}$ such that $a_{ij} \geq 0$ for all i, j with $i \neq j$, and if there exist positive numbers $t_1, \ldots, t_n$ such that

$$\sum_{j=1}^{n} t_j a_{ij} \leq 0 \qquad \text{(for all } i = 1, \ldots, n\text{)},$$

then the real part of any eigenvalue of A is less than or equal to zero.

Theorem 2 (Perron-Frobenius Theorem)**.** If $S \in \mathbb{R}^{n\times n}$ is a (strongly connected and aperiodic) matrix with nonnegative coefficients, then the spectral radius of S, $\rho(S)$, is an eigenvalue of S.