Seminars in Computer Networks
Homework

May 21, 2014

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Instructions: You need to work on the homework individually. You agree to the Honor Code, according to which answers will be your own work.

Make sure that the solutions are typewritten or clear to read. *Give explanations for all of your claims.*

Send your solutions by email to `bonifaci@dis.uniroma1.it` (please write `[SEMCN]` in the mail subject) and keep a copy for yourself.

Due date: 29/5/2014.

Problem 1. Let $\mathbf{A}$ be the adjacency matrix of an undirected network and let $\mathbf{1}$ be the column vector whose elements are all 1. In terms of these quantities write expressions for:

(a). the vector $\mathbf{k}$ whose elements are the degrees k_i of the vertices;

(b). the number m of edges in the network;

(c). the matrix $\mathbf{N}$ whose element N_{ij} is equal to the number of common neighbors of vertices i and j;

(d). the total number of triangles in the network, where a triangle means three vertices, each connected by edges to both of the others.

Problem 2. Consider a bipartite network, with its two types of vertex, and suppose that there are n_1 vertices of type 1 and n_2 vertices of type 2. Show that the mean degrees c_1 and c_2 of the two types are related by

$$c_2 = \frac{n_1}{n_2} c_1.$$

(The mean degree c of a set of nodes S is $\frac{1}{|S|}\sum_{i \in S} k_i$, where k_i is the degree of node i.)

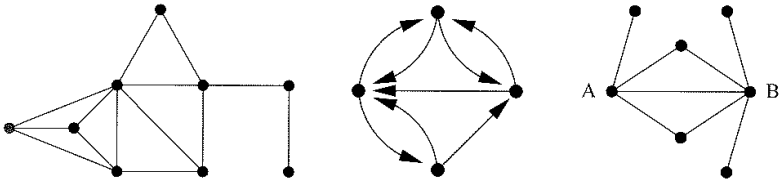
Problem 3. Consider an undirected (connected) tree of n vertices. Suppose that a particular vertex in the tree has degree k, so that its removal would divide the tree into k disjoint regions, and suppose the sizes of those regions are $n_1, \ldots, n_k$.

(a). Show that the betweenness centrality x of the vertex is

$$x = n^2 - \sum_{m=1}^{k} n_m^2.$$

(b). Hence, or otherwise, calculate the betweenness of the ith vertex from the end of a "line graph" of n vertices, that is, n vertices in a row like this:

Problem 4. Consider these three networks:



(a). Find a 3-core in the first network.

(b). If there is a directed edge from vertex i to vertex j in a directed network and there is also an edge from j to i then we say the edge from i to j is *reciprocated*. The *reciprocity* r of a network is defined as the fraction of edges that are reciprocated. What is the reciprocity of the second network?

(c). What is the cosine similarity of vertices A and B in the third network?

Problem 5. Consider a general $n \times n$ matrix $\mathbf{M}$ with eigenvalues μ_i where $i = 1 \ldots n$. Show that the matrix $\mathbf{M} - a\mathbf{I}$ has the same eigenvectors as $\mathbf{M}$ and eigenvalues $\mu_i - a$.

Problem 6. (a). True or false: the cascading capacity of every infinite k-regular network (a network where every node has degree k) is $1/k$. Prove it or find a counterexample.

(b). Find the cascading capacity of this infinite network: